Structural Programming and Data Structures

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CMPUT 102: Searching

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Course Content

• Introduction	• Vectors
• Objects	• Testing/Debugging
• Methods	• Arrays
• Tracing Programs	• **Searching**
• Object State	• Files I/O
• Sharing resources	• Sorting
• Selection	• Inheritance
• Repetition	• Recursion

Objectives of Lecture 21

Searching

- Introduce two techniques for searching for an element in a collection;
- Learn sequential search algorithm;
- Learn the binary search algorithm for ordered collections.
- Learn how to evaluate the complexity of an algorithm and compare between algorithms.

Outline of Lecture 21

- Review the simple array examples
- Sequential search approach
- Complexity of sequential search
- Binary search approach
- Complexity of binary search
- Compare sequential search and binary search

Array Example

```
// Find the largest element in an array of ints

int  markArray[] = {50, 37, 71, 99, 63};
int  index;
int  max;
index = 0;
max = markArray[index];
for (index = 1; index < markArray.length; index++)
        if (markArray[index] > max)
            max = markArray[index];
System.out.println(max);
```

markArray

50	0
37	1
71	2
99	3
63	4

index=5

max

99

Array Example2

```
// Find the index of the largest element in an
array of ints
int  markArray[] = {50, 37, 71, 99, 63};
int  index;
int  indexOfMax;
index = 0;
indexOfMax = 0;
for (index = 1; index < markArray.length; index++)
   if (markArray[index] > markArray[indexOfMax])
      indexOfMax = index;
System.out.println(indexOfMax);
```

markArray

50	0
37	1
71	2
99	3
63	4

index = 5

indexOfMax

3

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The Search Problem

- Given a container, find the index of a particular element, called the key.
- Technique applies for vectors, arrays, files, etc.
- Applications: information retrieval, database querying, etc.

Element sought for: 30

Collection:

0	1	2	3	4	5	6	7	8	9
25	50	10	95	75	30	70	55	60	80

Sequential Search

- Compare the key to each element in turn, until the correct element is found, and return its index.

0	1	2	3	4	5	6	7	8	9
25	50	10	95	75	30	70	55	60	80
30	30	30	30	30	30				

Sequential Search Code

Compare all elements of the collection until we find the key.

```
/*   a sequential search code (first tentative)  */
public static int sequential_search( int data[], int key ) {
  boolean found = false;
  int index = 0;

  while ( !found) {
    if ( key == data[index] )
      found = true;
    else
      index = index + 1;
  }
  return index;
}
```


Element not found

- We must take into account that the key we are searching for may not be in the array.
- In this case we must return a special index, say -1.

0	1	2	3	4	5	6	7	8	9	-1
25	50	10	95	75	30	70	55	60	80	
35	35	35	35	35	35	35	35	35	35	35

Search Algorithm

INPUT: data: array of int; key: int;
OUTPUT: index : an int such that
data[index] == key if key is in data,
or -1 if key is not stored in data.
Method:
1. index = 0; found=false;
2. While (not found and index < data.length)
   check similarity data[index] and key
   index = index + 1
3. if not found then index = -1;

```
/*   a sequential search method   */
public static int sequential_search( int data[], int key ) {
  boolean found = false;
  int index = 0;

  while ( !found && index < data.length ) {
   if ( key == data[index] )
     found = true;
   else
     index = index + 1;
  }

  if  (!found)   index = -1;
  return index;
}
```

Revised Sequential Search Code

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- Review the simple array examples
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Complexity Analysis

- How efficient is this algorithm?
- In general if we have an algorithm that does something with ***n*** objects, we want to express the time efficiency of the algorithm as a function of ***n***.
- Such an expression is called the **time complexity** of the algorithm.
- In the case of search, we can count the number of comparison operations between the key and the elements.

Worst, Best and Average cases

- In fact, we usually have multiple expressions:
  - the worst case complexity,
  - the best case complexity
  - the average case complexity.

Complexity of Sequential Search

- How many comparison operations are required for a sequential search of an n-element container?
- In the worst case ➔ n.
- In the best case ➔ 1.
- In the average case:

$$\frac{1+2+3+\ldots+n}{n} = \frac{n(n+1)}{2n} = \frac{(n+1)}{2}$$

- In this case, we say the complexity of Search is in the order of *n*, denoted as ***O(n)***.
- **Can we improve this algorithm?**

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Binary Search

- If the elements are ordered, we can do better.
- Guess the middle and adjust accordingly.

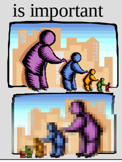
0	1	2	3	4	5	6	7	8	9
10	25	30	50	55	60	70	75	80	95

② ③ ①

30 30 30

too small too big

The order is important



Binary Search Algorithm

30

guess = (low + high) / 2

L G H

0	1	2	3	4	5	6	7	8	9
10	25	30	50	55	60	70	75	80	95

high = guess - 1

guess = (low + high) / 2

L G H

0	1	2	3	4	5	6	7	8	9
10	25	30	50	55	60	70	75	80	95

low = guess + 1

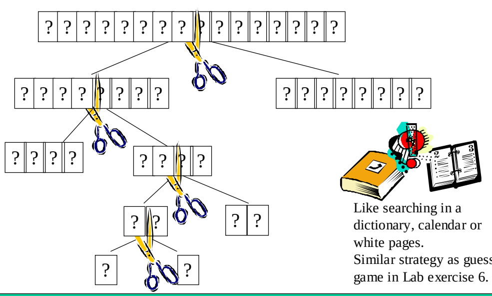
guess = (low + high) / 2

G H

0	1	2	3	4	5	6	7	8	9
10	25	30	50	55	60	70	75	80	95

Strategy of Binary Search:

Given an ordered array of integers, and a value of integer, search for the value in the array using an approach of **Divide and Conquer .**



Like searching in a dictionary, calendar or white pages.
Similar strategy as guess game in Lab exercise 6.

Binary Search Code

Divide in 2 between lower and upper bounds until we find the key.

```
/*   a binary search code of ordered array (first tentative)  */
  public static int binary_search( int data[], int key ) {
    boolean found = false;
    int guess; int low = 0; int high=data.length-1;

    while ( !found) {
      guess = (high+low)/2;
      if ( key == data[guess] )   found = true;
      else if (key < data[guess])  high=guess-1;
      else low = guess+1;
    }
    return guess;
  }
```


Element not found

35

guess = (low + high) / 2

L G H

0	1	2	3	4	5	6	7	8	9
10	25	30	50	55	60	70	75	80	95

high = guess - 1

guess = (low + high) / 2

L G H

0	1	2	3	4	5	6	7	8	9
10	25	30	50	55	60	70	75	80	95

low = guess + 1

guess = (low + high) / 2

G H

0	1	2	3	4	5	6	7	8	9
10	25	30	50	55	60	70	75	80	95

Element not found (con’t)

35

G H

0	1	2	3	4	5	6	7	8	9
10	25	30	50	55	60	70	75	80	95

low = guess + 1

guess = (low + high) / 2

GH

0	1	2	3	4	5	6	7	8	9
10	25	30	50	55	60	70	75	80	95

high = guess - 1

guess = (low + high) / 2

H G

0	1	2	3	4	5	6	7	8	9
10	25	30	50	55	60	70	75	80	95

~~low <= high~~

Binary Search Algorithm

INPUT: data: array of ordered int; key: int;
OUTPUT: index : an int such that
data[index] = = key if key is in data,
or -1 if key is not stored in data.

Method:

1. lower = 0; upper = length;
2. While (not found && low < =upper)
   index = (lower + upper) /2;
   check similarity data[index] and key
   if similar then found, otherwise
   if key < data[index]
   upper = index-1;
   else lower = index +1;
3. If (data[index] != key) index = -1;


```
/*  a binary search code of ordered array  */
 public static int binary_search( int data[], int key ) {
  boolean found = false;
  int guess; int low = 0; int high=data.length-1;

  while ( !found && low <= high) {
   guess = (high+low)/2;
   if ( key == data[guess] )  found = true;
   else if (key < data[guess])  high=guess-1;
   else low = guess+1;
  }
  if (! found) guess = -1;
  return guess;
 }
```

Revised Binary Search Code

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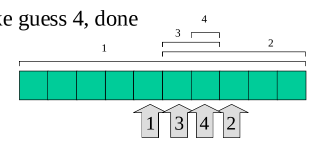
Worst-case Binary Search

- Each time we guess, we divide the list in half:
- In the worst case:
  - 10 elements, make guess 1, then
  - 5 elements, make guess 2, then
  - 2 elements, make guess 3, then
  - 1 element, make guess 4, done



Worst-case Binary Search (con't)

- If there were 15 elements:
  - 15 elements, make guess 1, then
  - 7 elements, make guess 2, then
  - 3 elements, make guess 3, then
  - 1 elements, make guess 4, done
- These results are the same, but if we have from 16 to 31 elements it takes 5 guesses.
- This formula is: $\lfloor \log_2(n)+1 \rfloor$
- $\log_2(n)$ is number of times you have to divide n by 2 to get 1

Average-case Binary Search

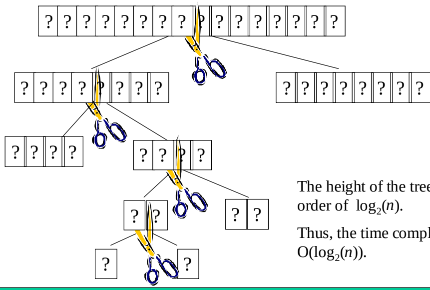
- If there were 15 elements:
  - 1 element takes 1 guess
  - 2 elements take 2 guesses
  - 4 elements take 3 guesses
  - 8 elements take 4 guesses
- The average is:

$$(1*1)+(2*2)+(4*3)+(8*4) = 49 \approx 3$$

- The average case is about one less than the worst case, so this is: $\lfloor \log_2(n) \rfloor$

Time Complexity of Binary Search

The number of comparisons is proportional to the height of the following search tree:



The height of the tree is in the order of $\log_2(n)$.

Thus, the time complexity is $O(\log_2(n))$.

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Sequential and Binary Search

- For average and worst case sequential search, it takes: $\frac{(n+1)}{2}$ and n.
- For average and worst case binary search, it takes: $\lfloor \log_2(n) \rfloor$ and $\lfloor \log_2(n)+1 \rfloor$.

list size	Sequential average	Sequential worst	Binary average	Binary worst	Ratio
10	6	10	3	4	2
100	51	100	6	7	8
1000	501	1000	9	10	55
10000	5001	10000	13	14	384