CMPUT 675 - Assignment #0

Fall 2014, University of Alberta

Pages: 2

This assignment does not count toward your final grade. It is completely optional, you may choose to skip it without having your grade negatively affected. You can use this to get some experience with how I grade assignments. Feel free to only attempt some questions.

I will "grade" any submission within 3 days of receiving it as long as you submit it before the due date of Assignment #1. I offer no guarantees if you submit after.

Problem 1)

Marks: 5

Let $G = (V, E)$ be an undirected graph. For a vertex $v \in V$, let $\deg(v)$ be the number of edges $e \in E$ having v as an endpoint.

A *colouring* of G with k colours is an assignment $\phi : V \to \{1, \ldots, k\}$ such that $\phi(u) \neq \phi(v)$ for every edge $(u, v) \in E$.

Let $\Delta = \max_{v \in V} \deg(v)$. Describe a simple greedy algorithm to colour G with $\Delta + 1$ colours.

Problem 2)

Marks: 5

Let $G = (V, E)$ be an undirected graph. A *matching* is a subset of edges $F \subseteq E$ such that no two edges in F share an endpoint. Unlike most optimization problems we will see in this course, a maximum-size matching can be computed in polynomial time.

Consider the following simpler algorithm.

Algorithm 1 Greedy Matching

```
F ← ∅
while there is some edge e ∈ E − F such that F ∪ {e} is a matching do
    F ← F ∪ {e}
end while
return F
```

Show that the returned matching F satisfies $|F| \geq OPT/2$ where OPT is the size of the largest matching.

Problem 3)

Marks: 5

Let $G = (V, E)$ be an undirected graph. For any subset of vertices $S \subseteq V$, let $\delta(S)$ be the set of edges with precisely one endpoint in V. That is,

$$\delta(S) = \{(u, v) \in E : |S \cap \{u, v\}| = 1\}.$$

Show that there is some subset of vertices S such that $|\delta(S)| \geq |E|/2$.