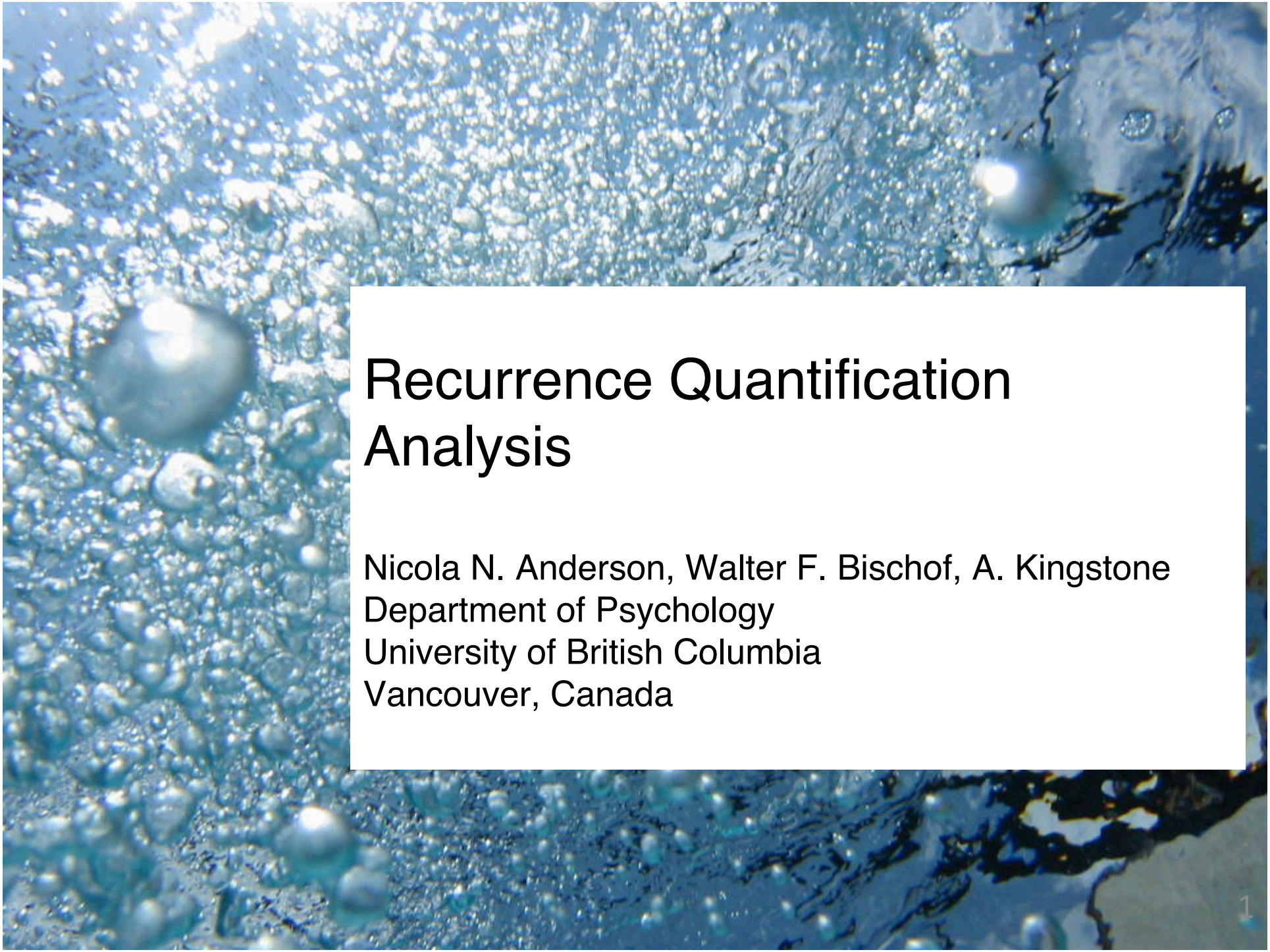
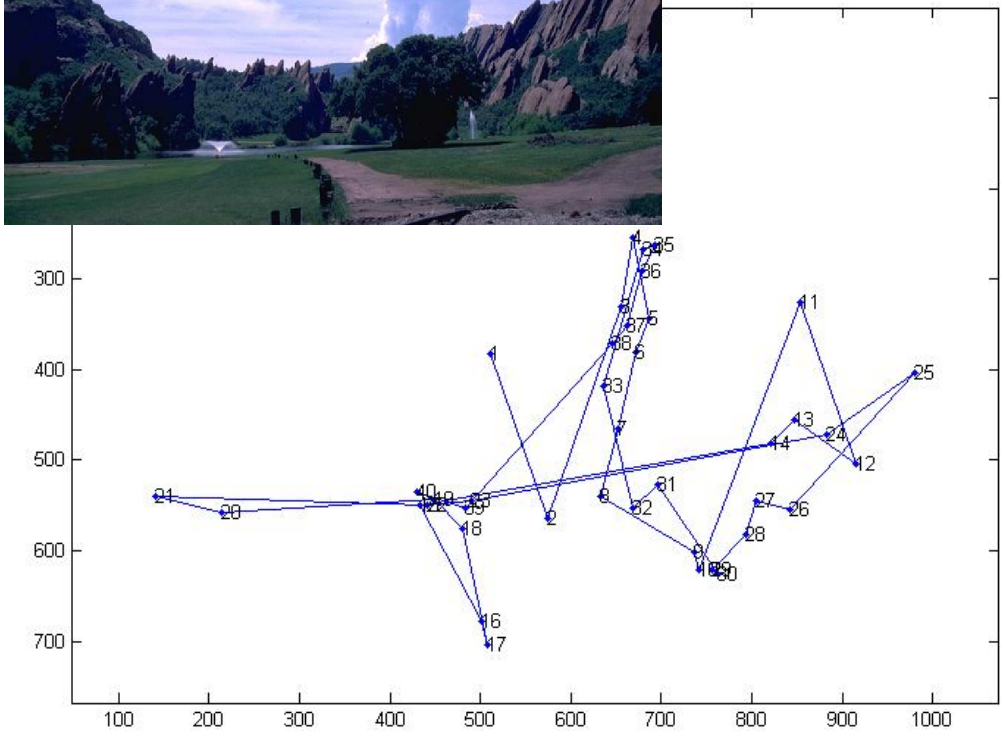
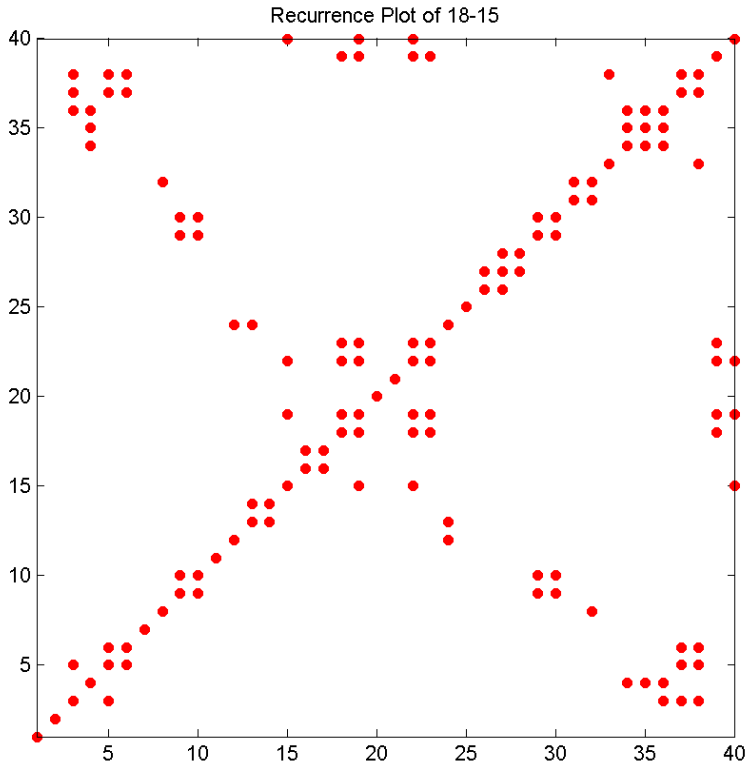


Recurrence Quantification Analysis

Nicola N. Anderson, Walter F. Bischof, A. Kingstone
Department of Psychology
University of British Columbia
Vancouver, Canada

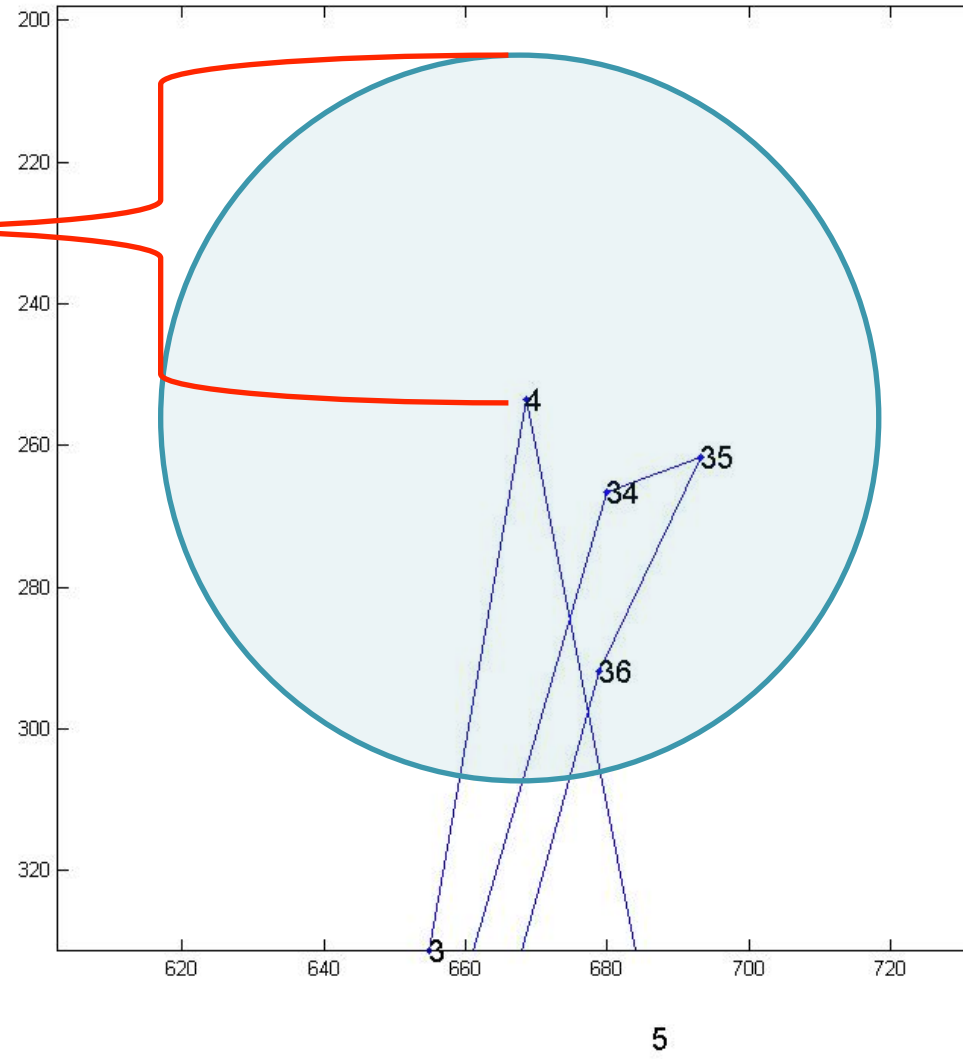
Building Recurrence Plots



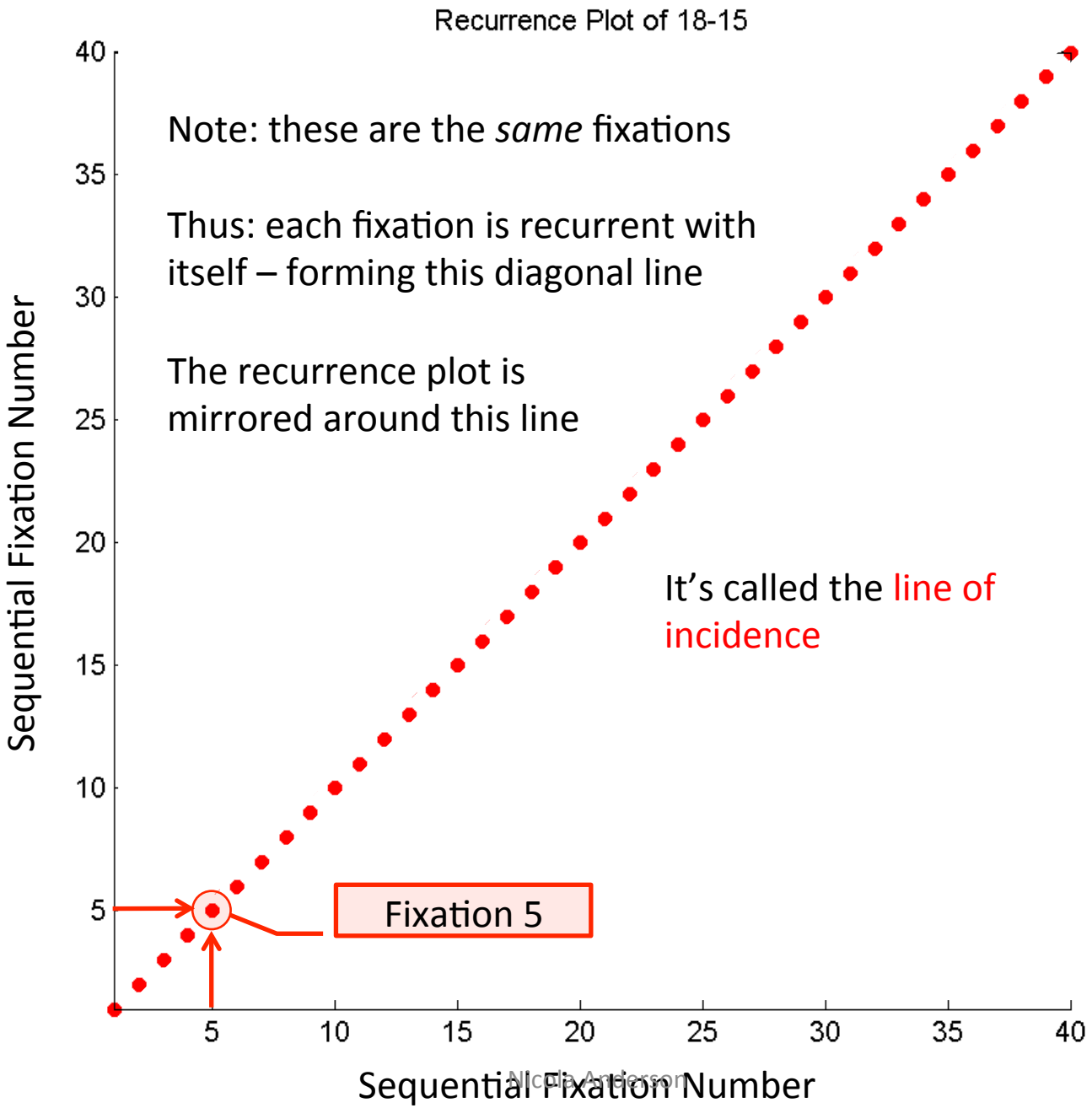


Building Recurrence Plots

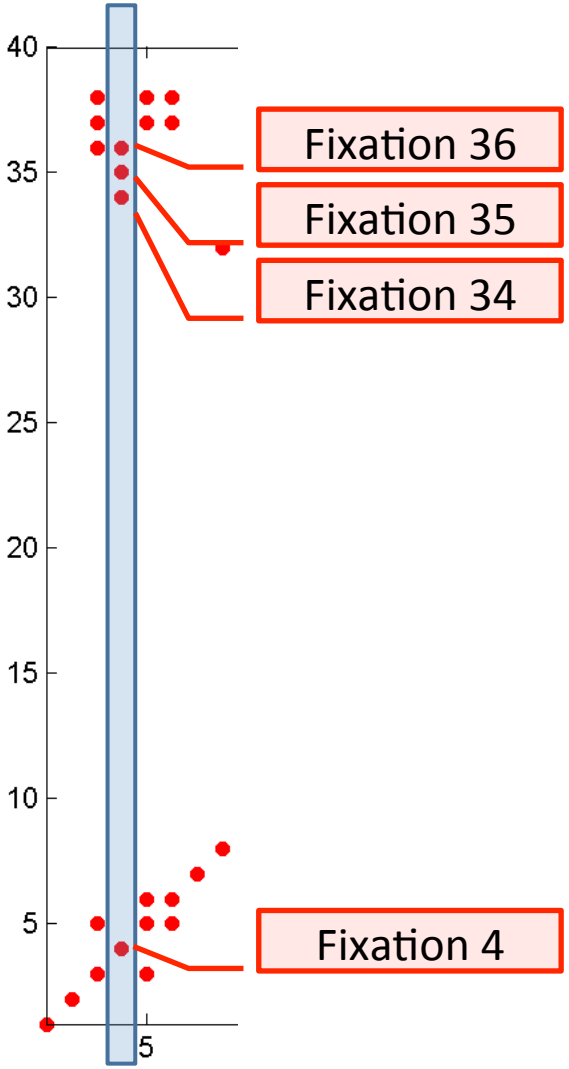
Radius = 50 pixels

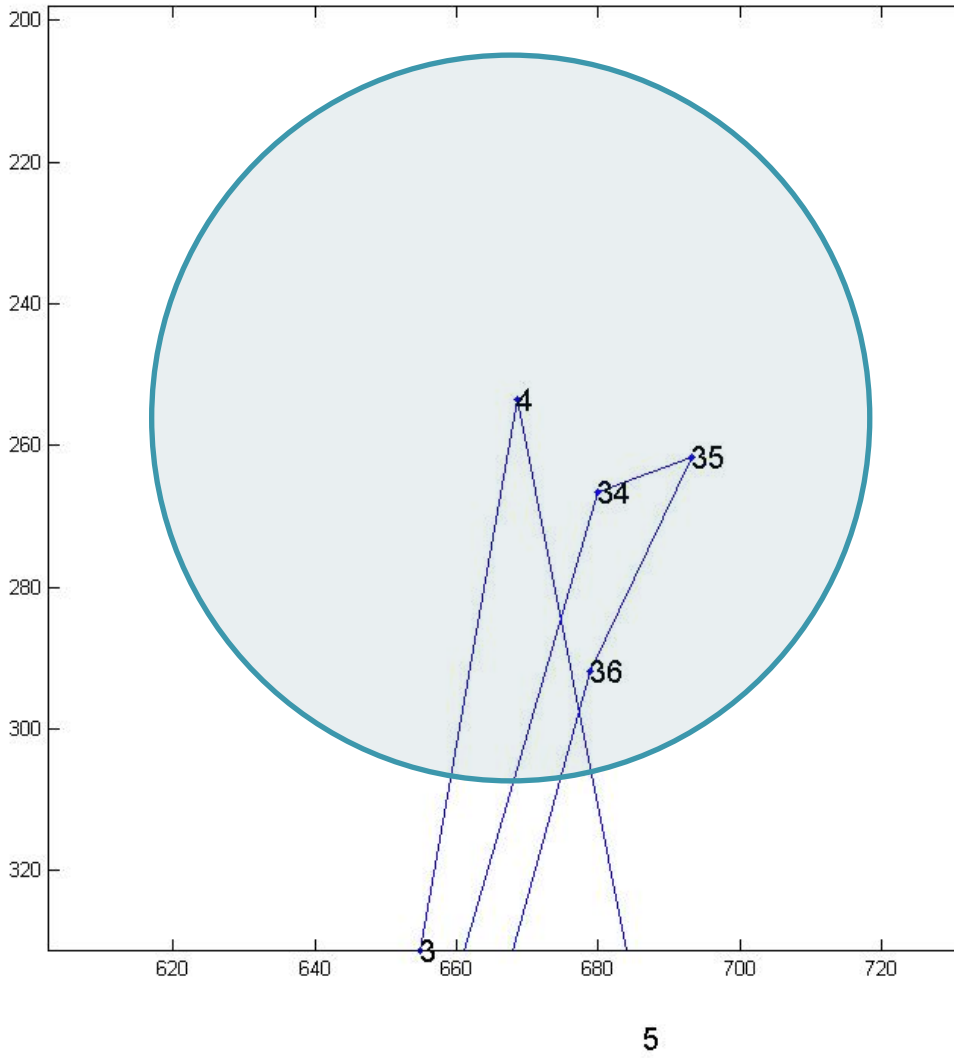


Any two fixations that fall within this radius is considered "recurrent"

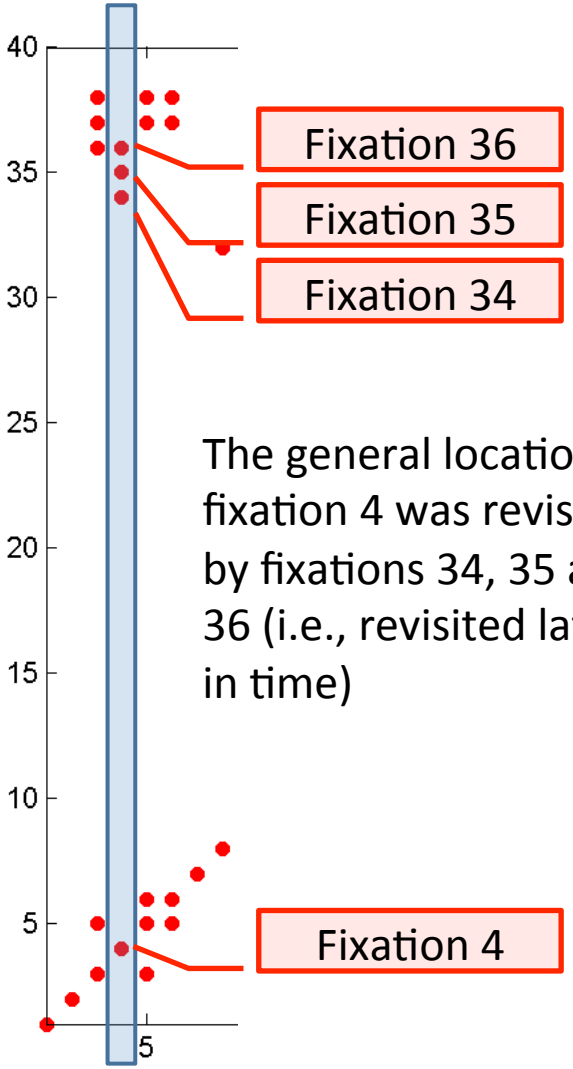

Recurrence Plot of 18-15
Note: these are the *same* fixations
Thus: each fixation is recurrent with itself – forming this diagonal line
The recurrence plot is mirrored around this line
It's called the line of incidence
Fixation 5
Sequential Fixation Number
Sequential Fixation Number
5
10
15
20
25
30
35
40

Building Recurrence Plots

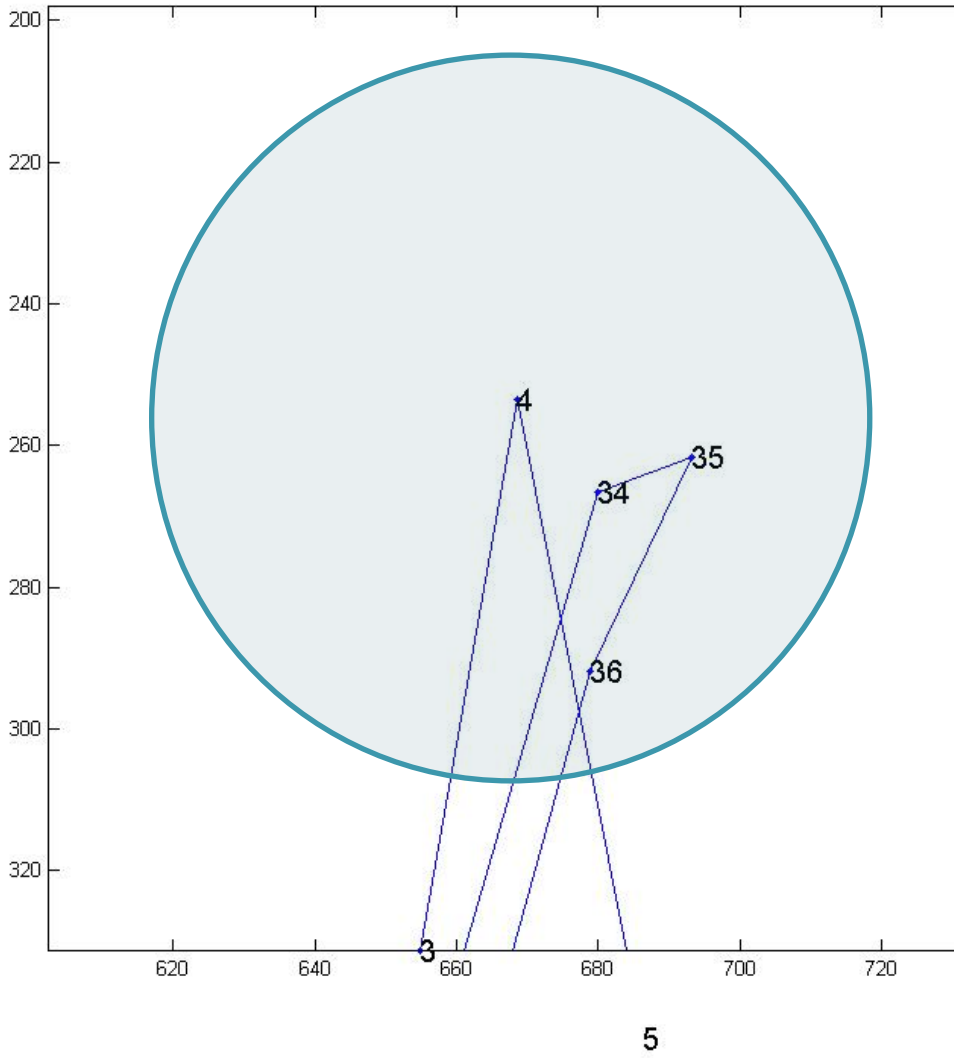




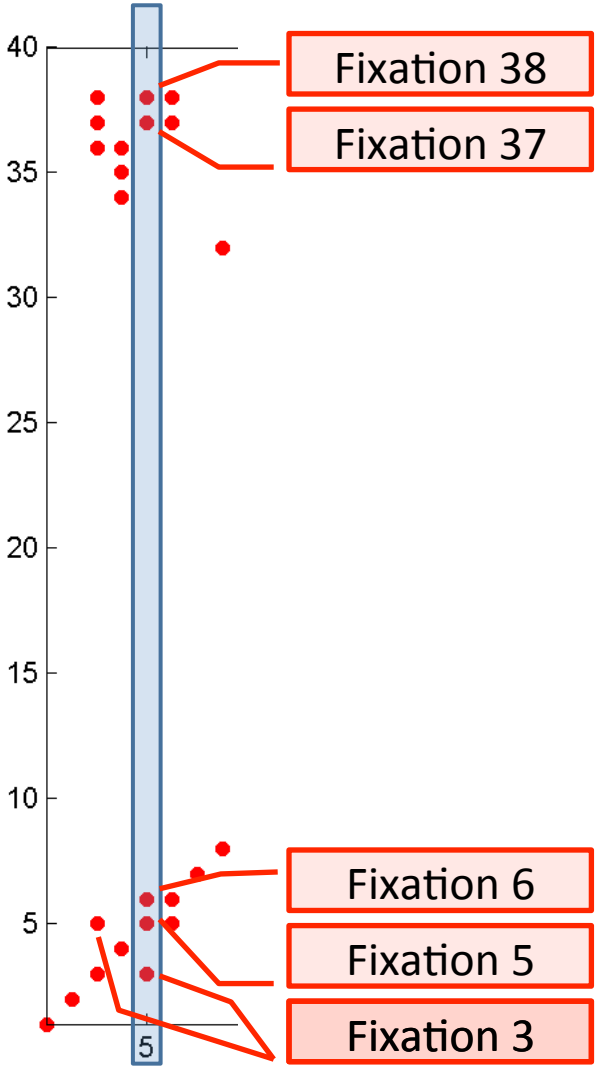
Building Recurrence Plots

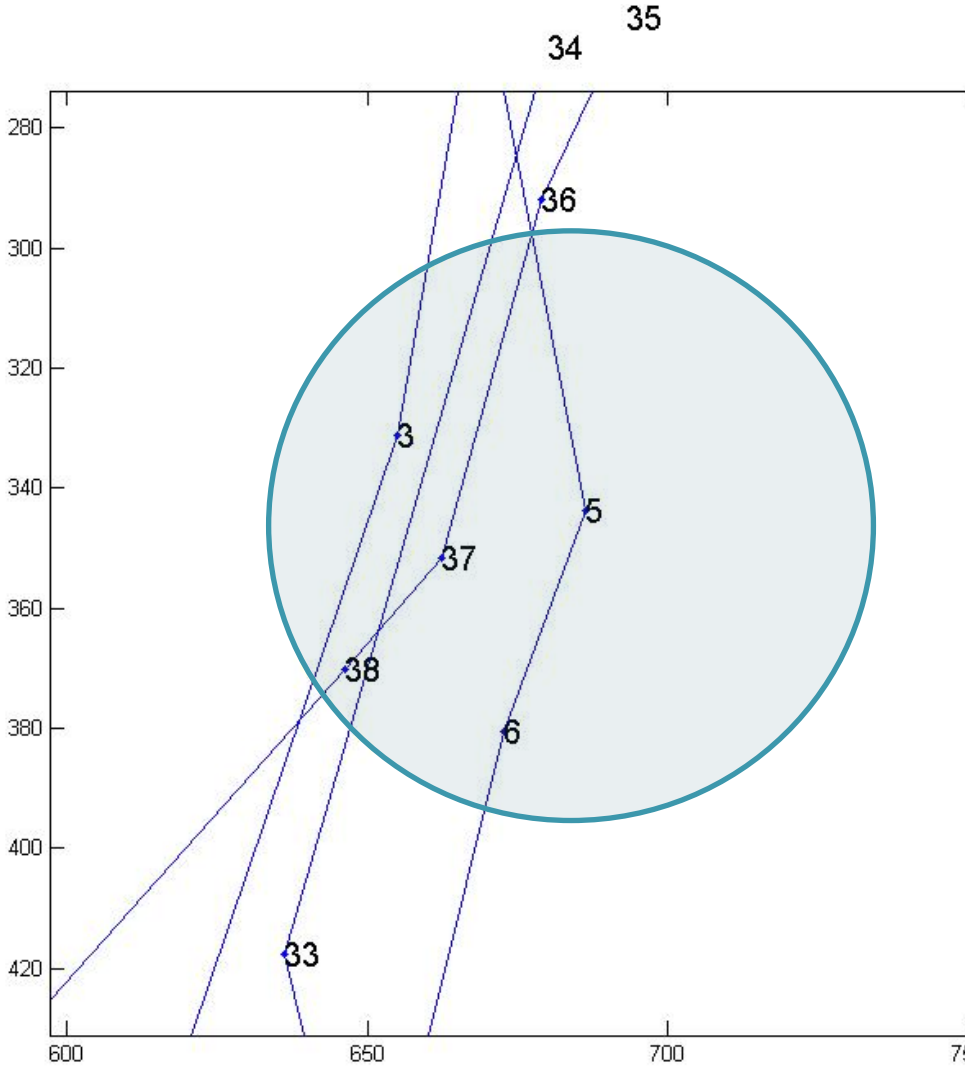


The general location of fixation 4 was revisited by fixations 34, 35 and 36 (i.e., revisited later in time)

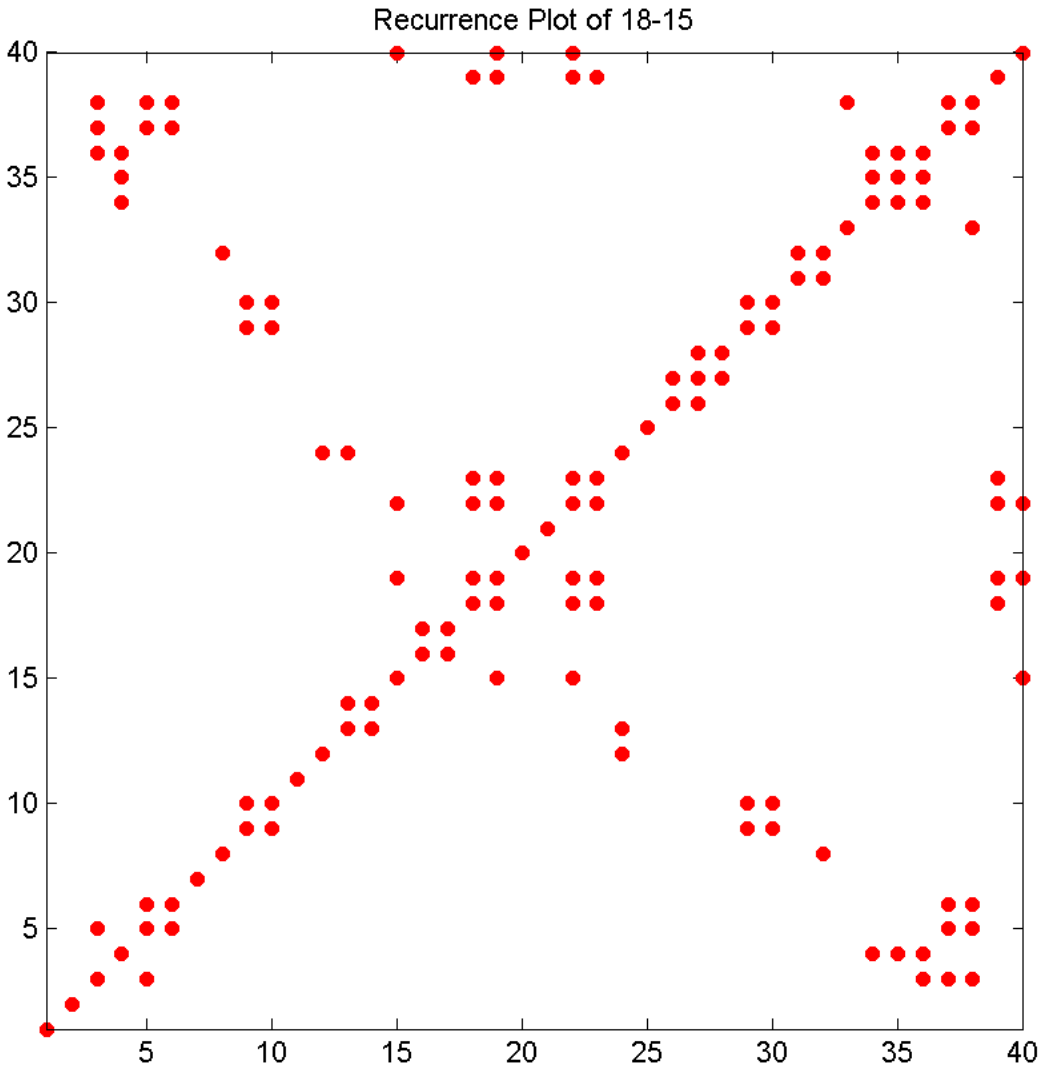


Building Recurrence Plots

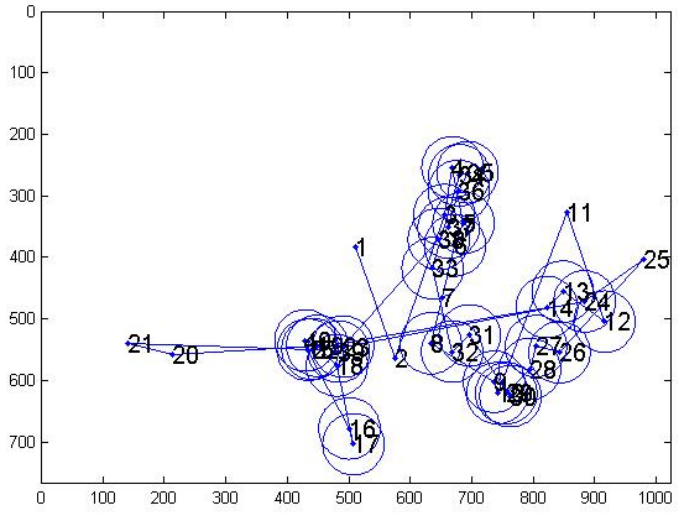




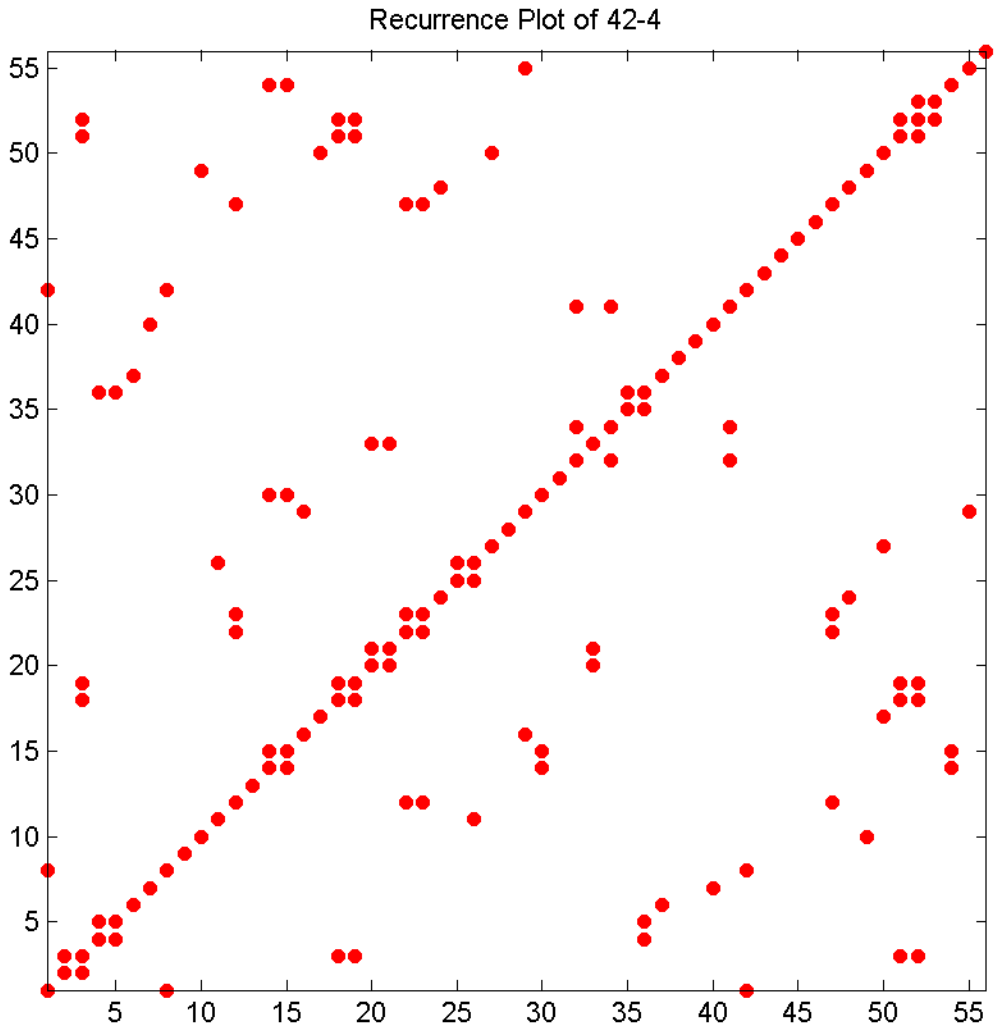
Building Recurrence Plots



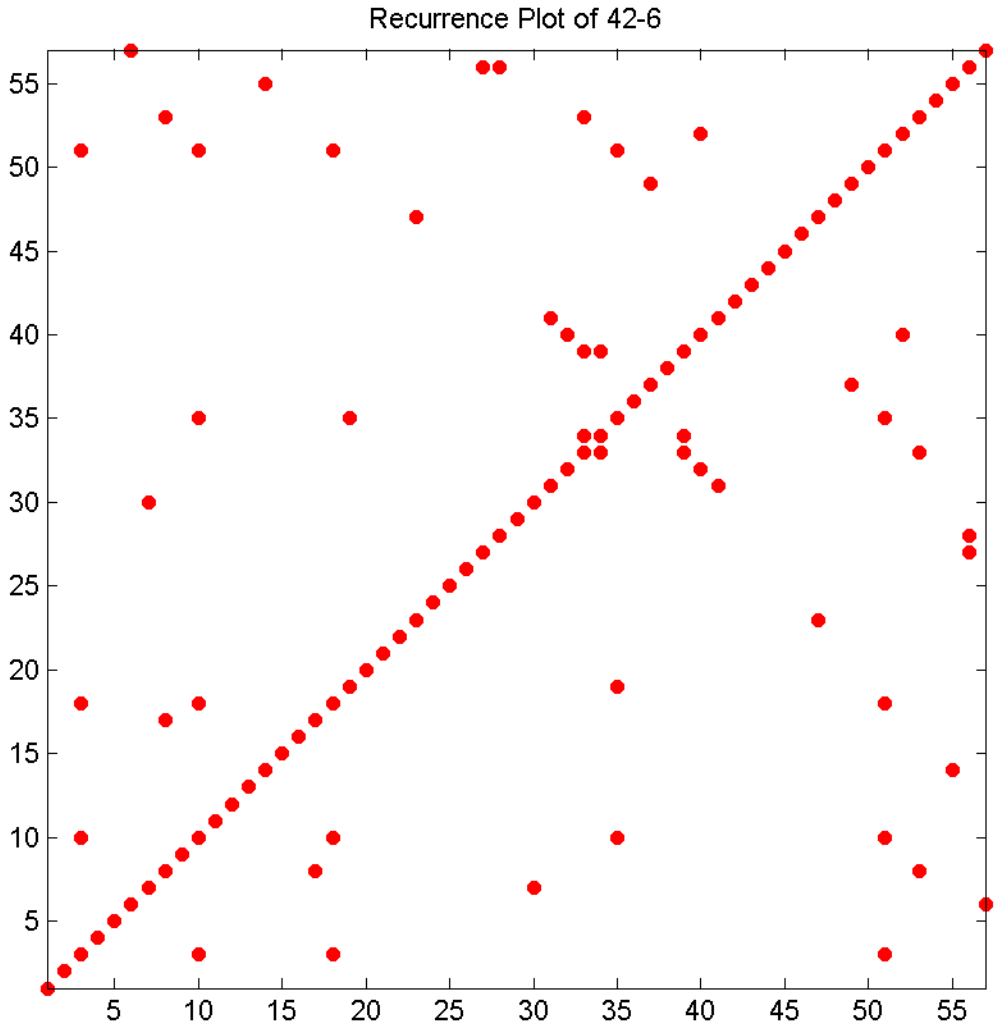
Compare every fixation with every other fixation to make the entire plot



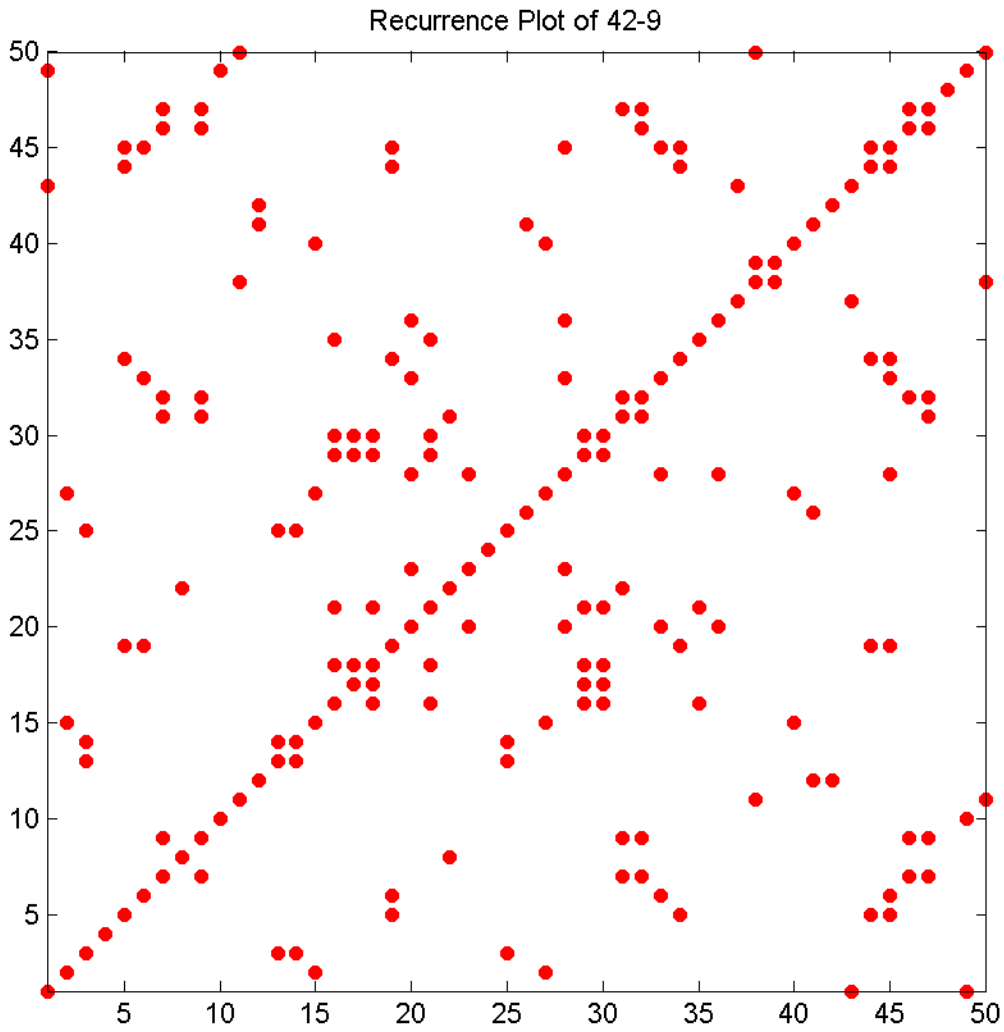
A few examples



A few examples



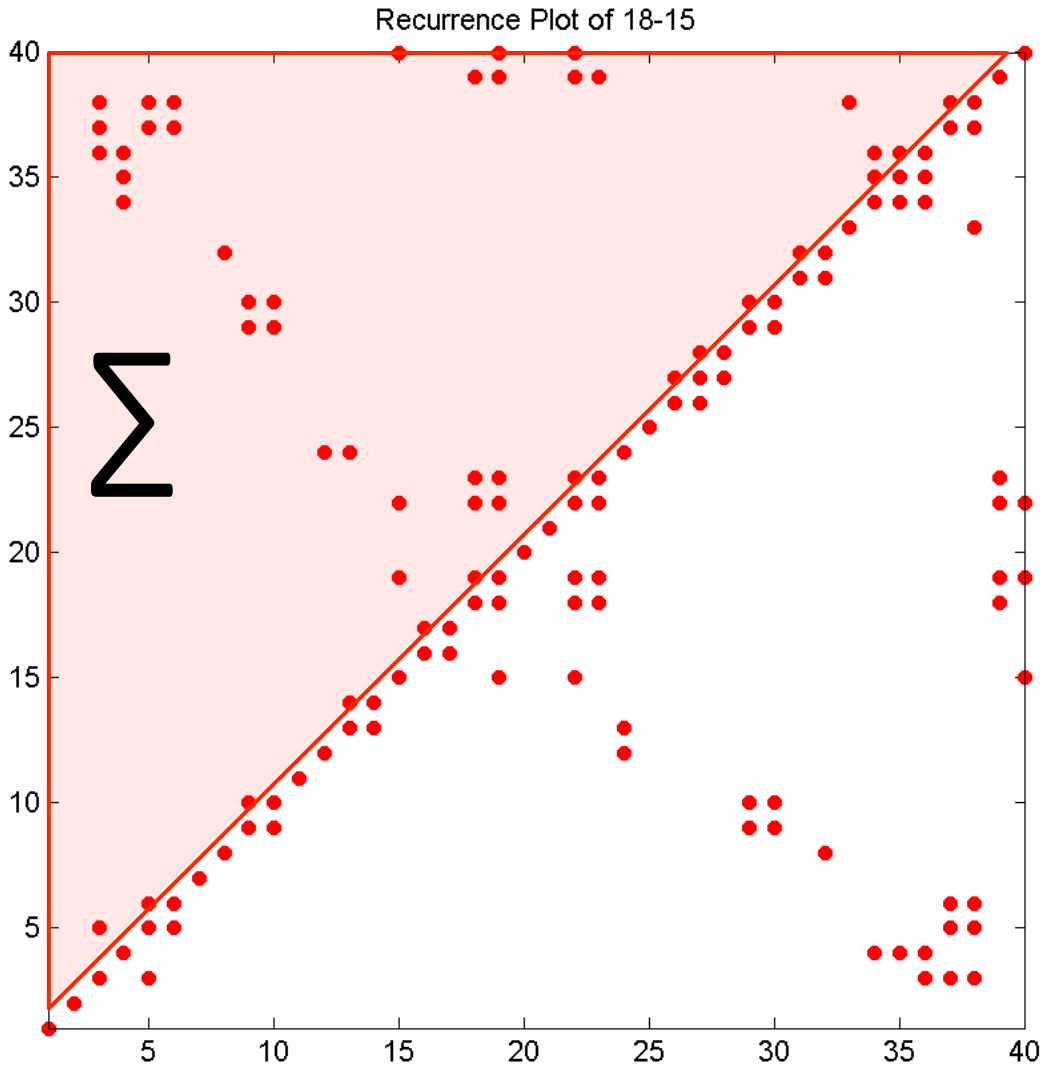
A few examples



Recurrence Quantification Analysis

- We generate these plots for every subject on every trial
- Then we quantify them:
  - Number of recurrent points
  - Percentage recurrent points
  - Determinism
  - Laminarity/Trapping Time
  - Center of Recurrence Mass

Number/Percentage of Recurrent Points



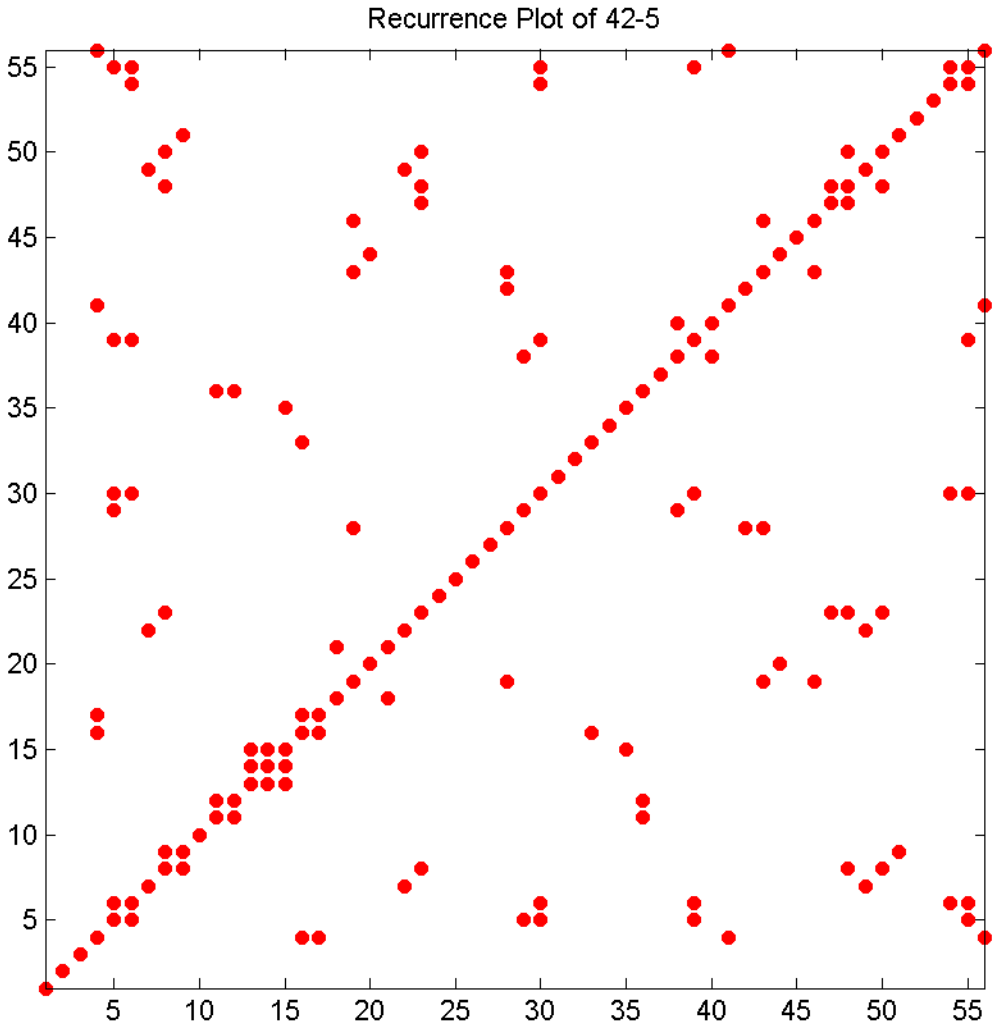
Count up all the points above the diagonal, i.e. the line of incidence is not included

Number of recurrences = 46

Express as percentage of total number of possible recurrent points (40*39/2)

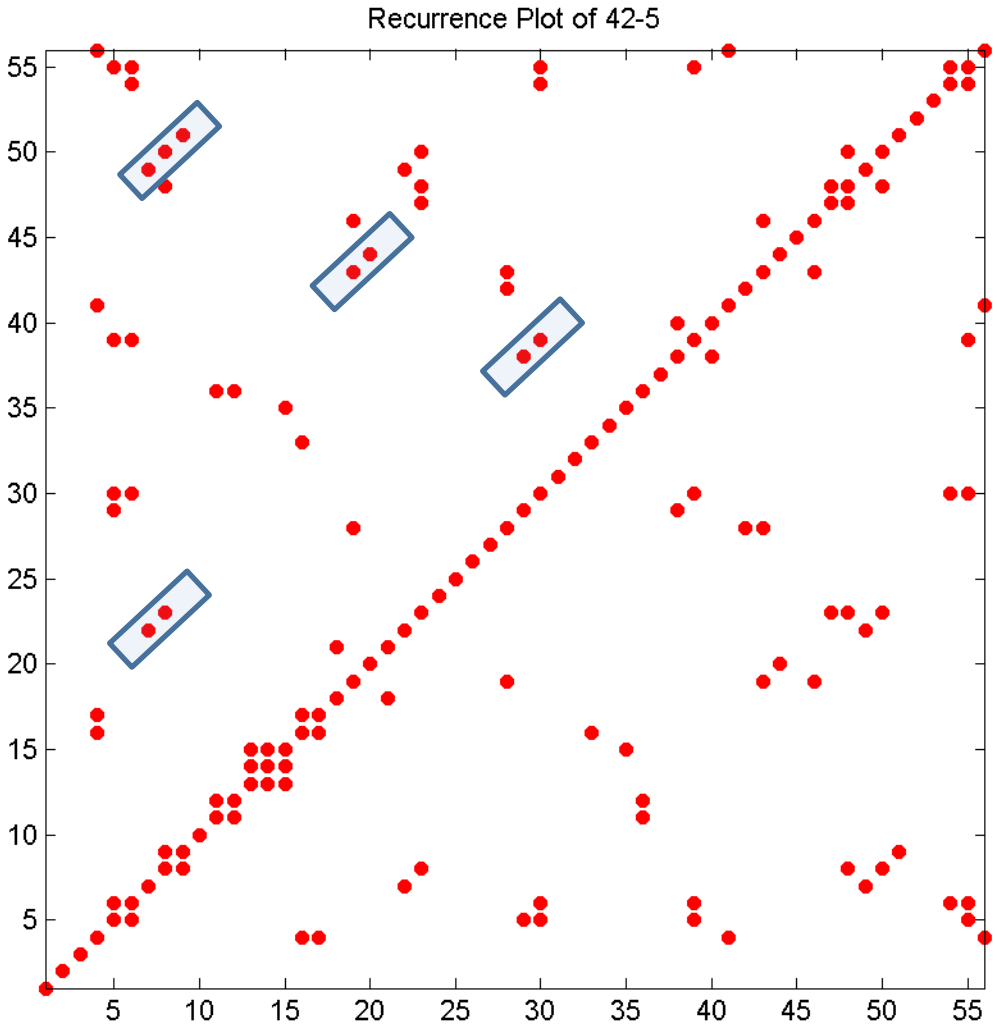
recurrence = 5.90

Determinism



Determinism: The percentage of points that fall on diagonal lines

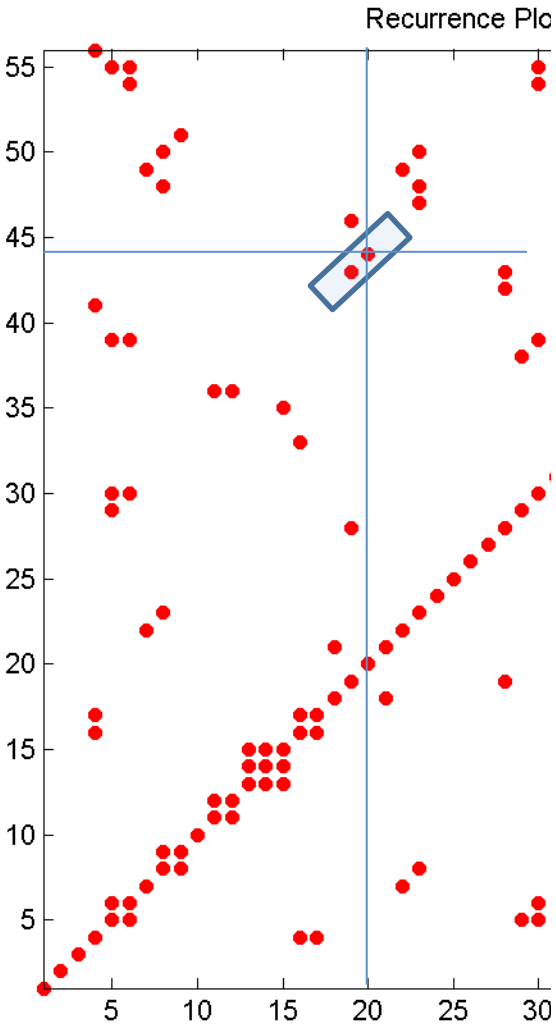
Determinism

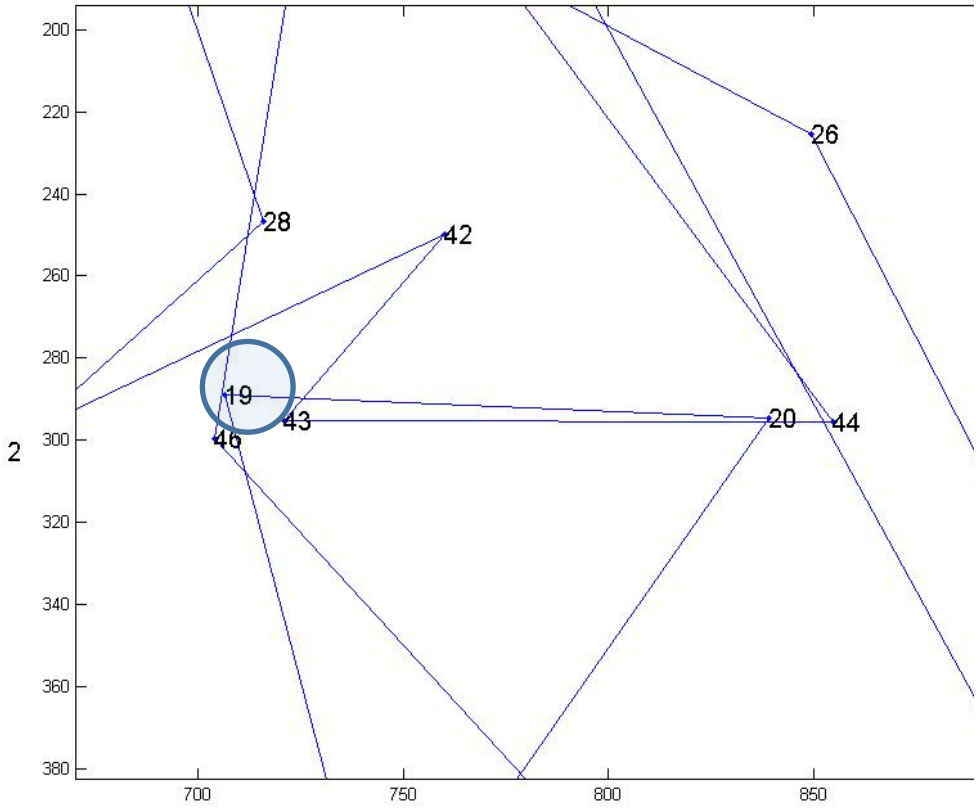


Determinism: The percentage of recurrent points that fall on diagonal lines

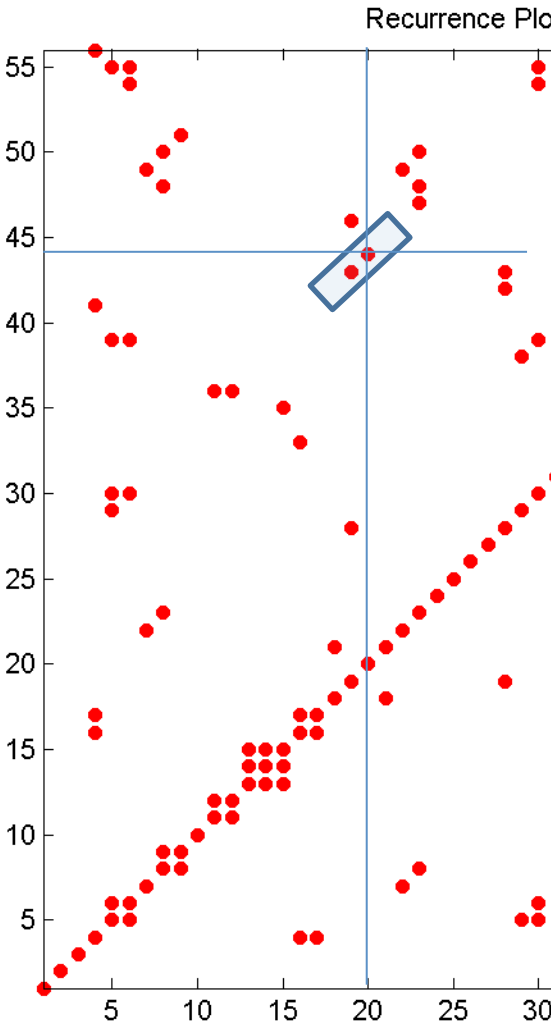
Diagonal lines represent the recurrence of fixation trajectories through space (or specific sequences of fixations that repeat)

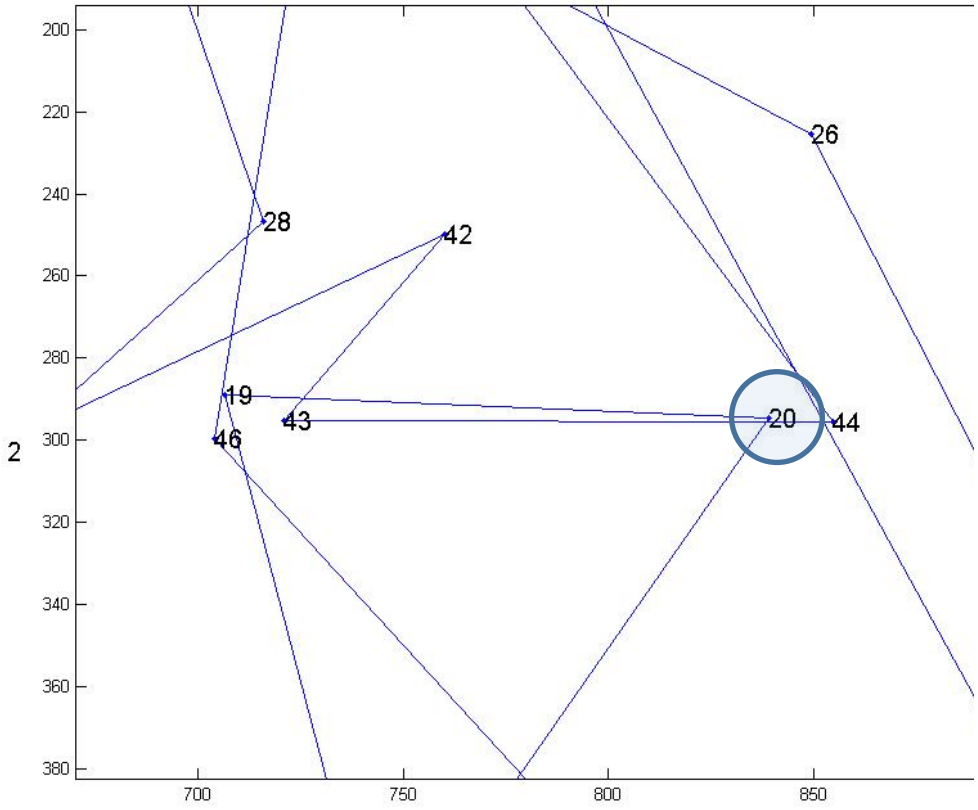
Determinism



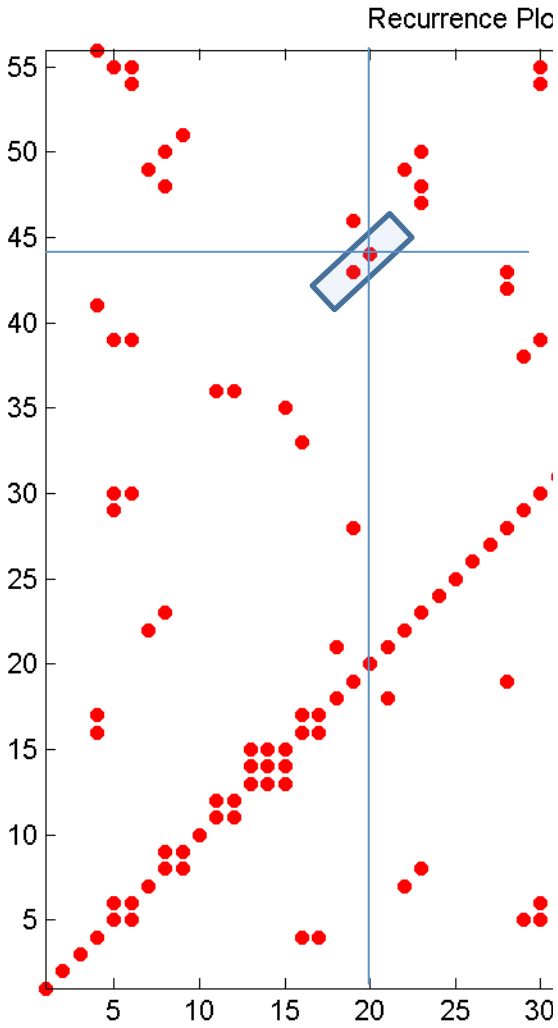


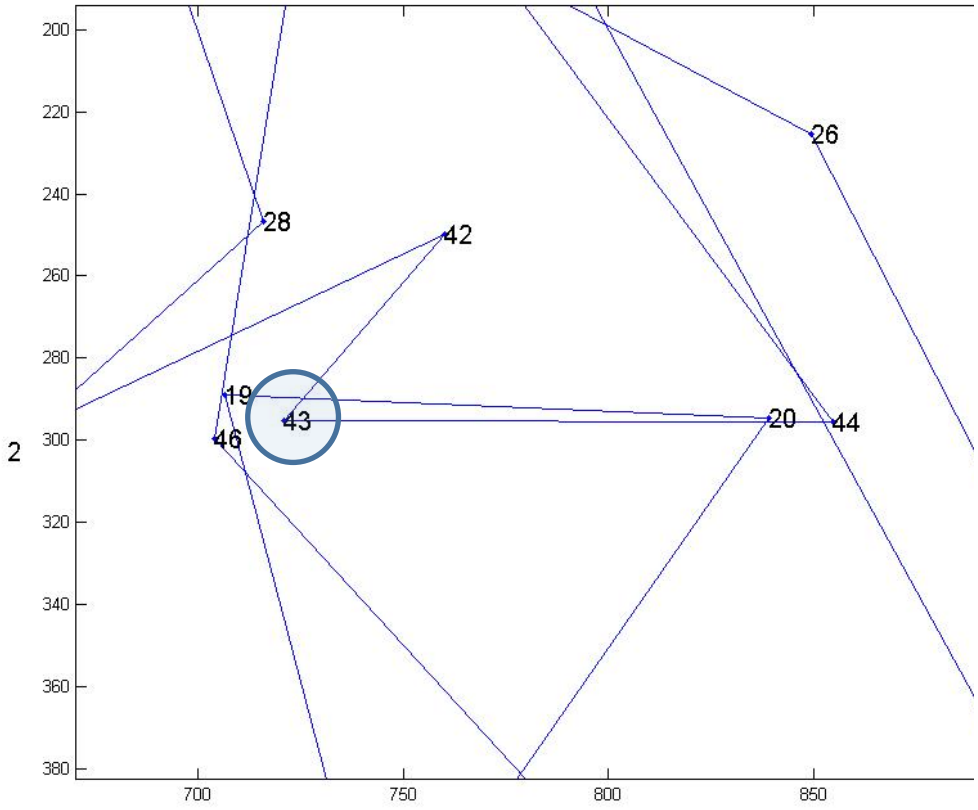
Determinism



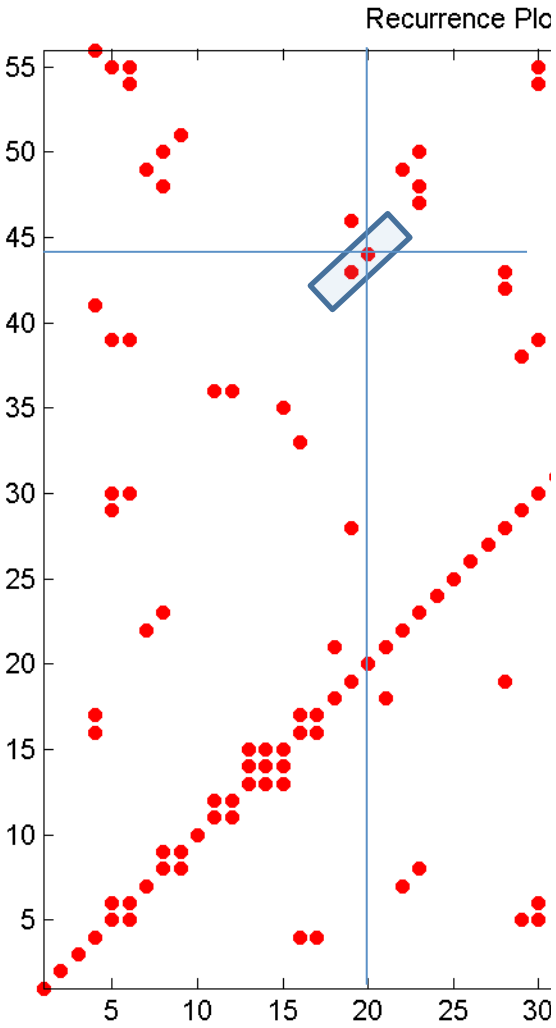


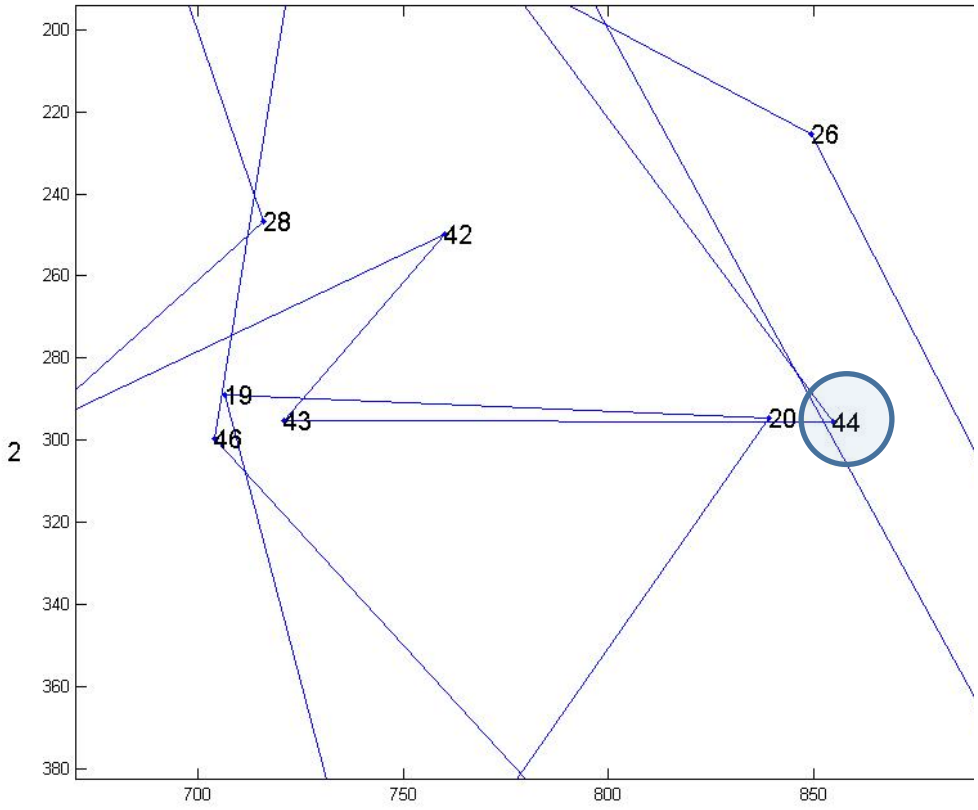
Determinism



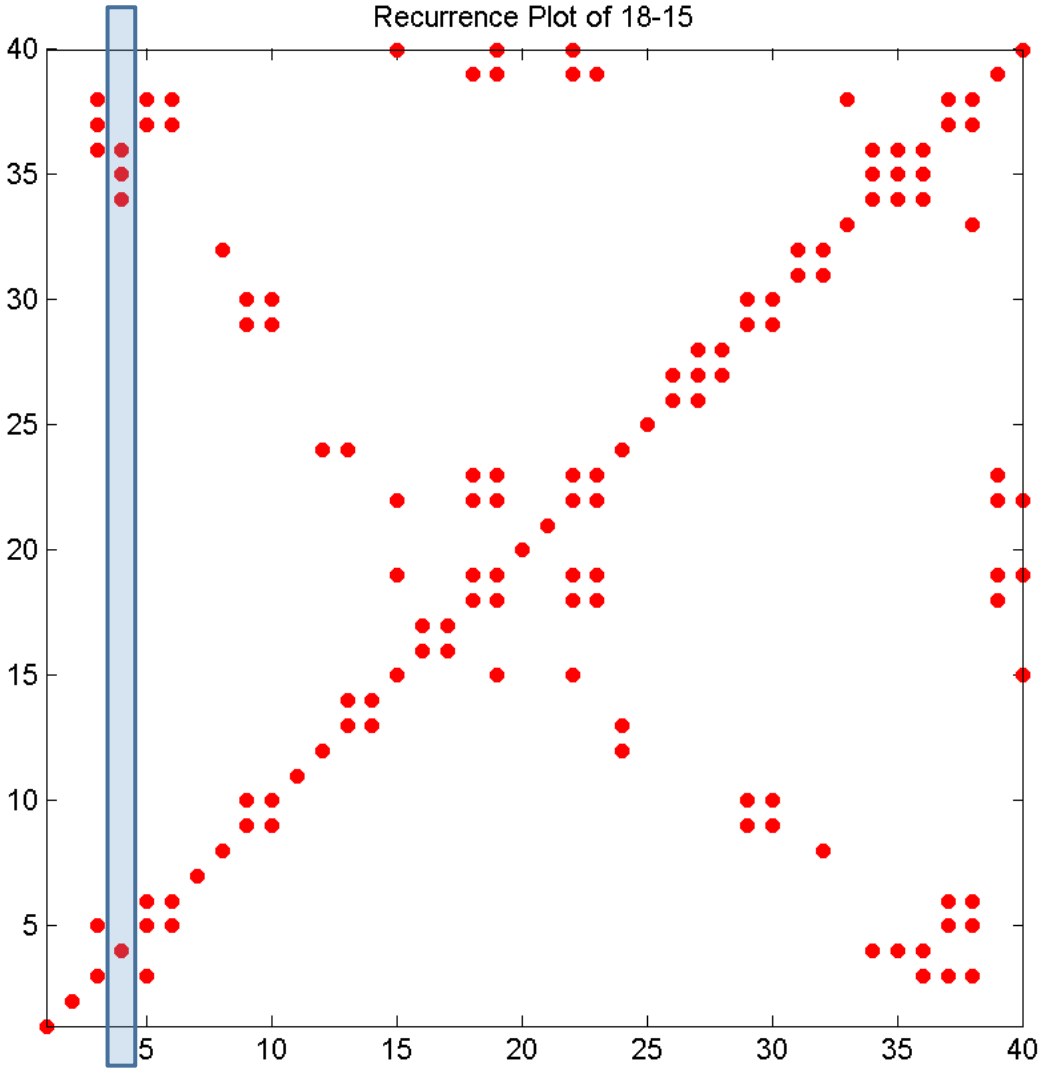


Determinism



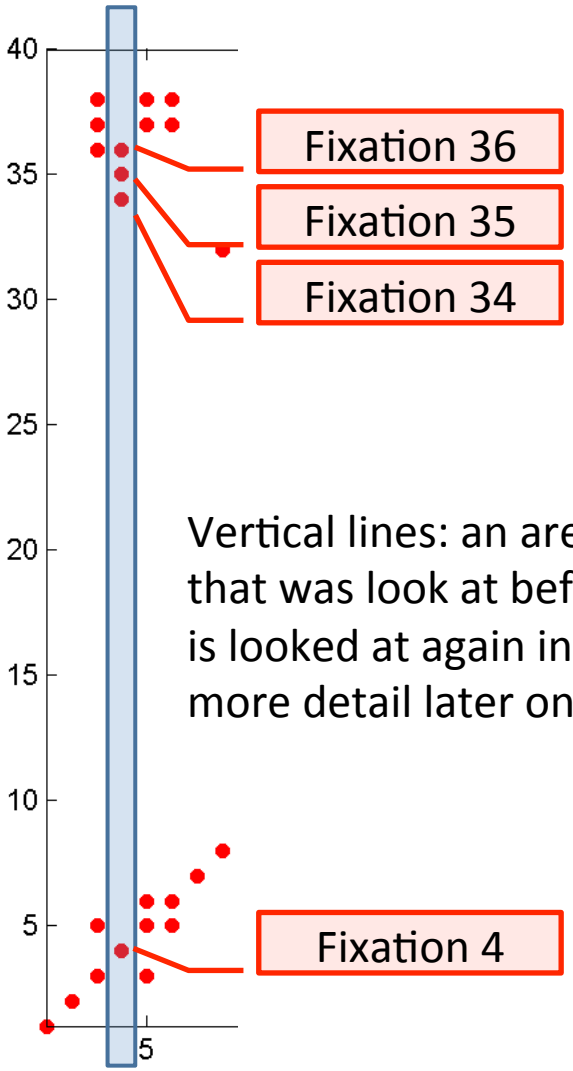


Laminarity/Trapping Time

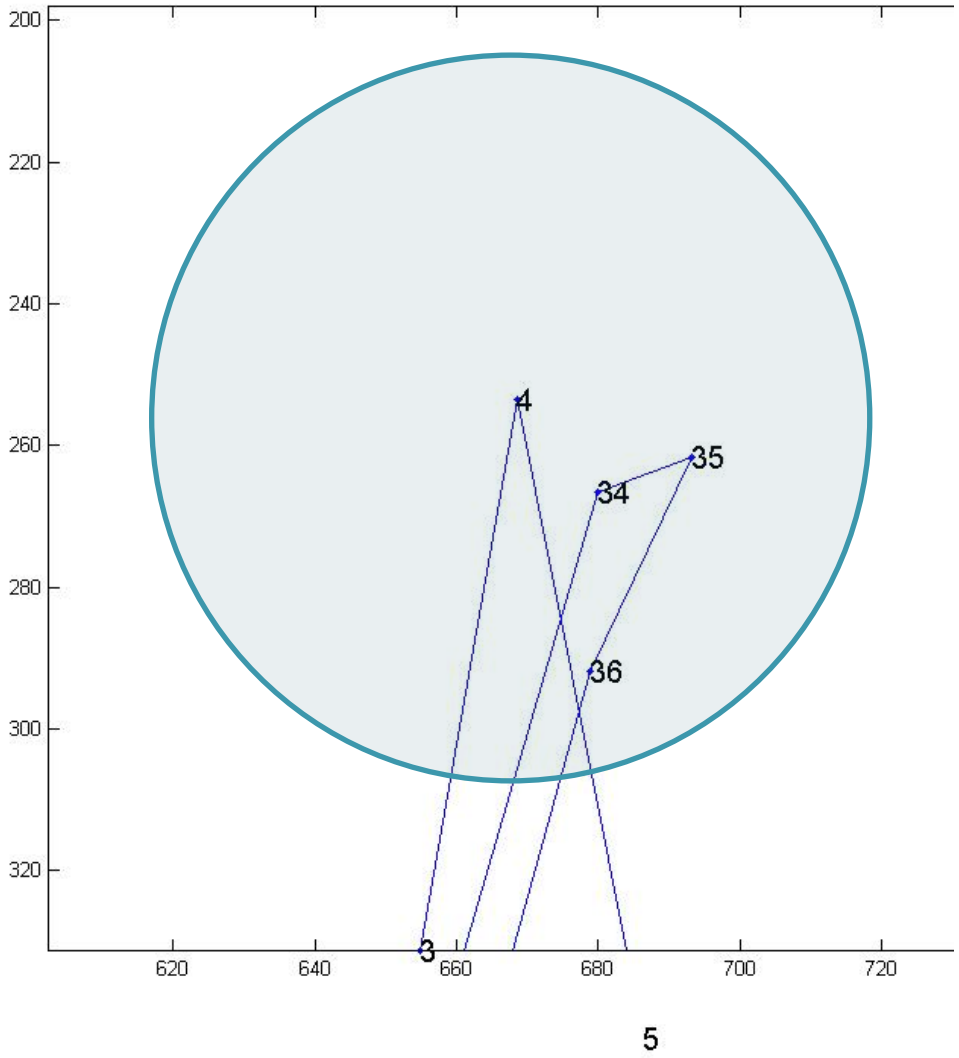


Laminarity: number of recurrent points that form vertical lines

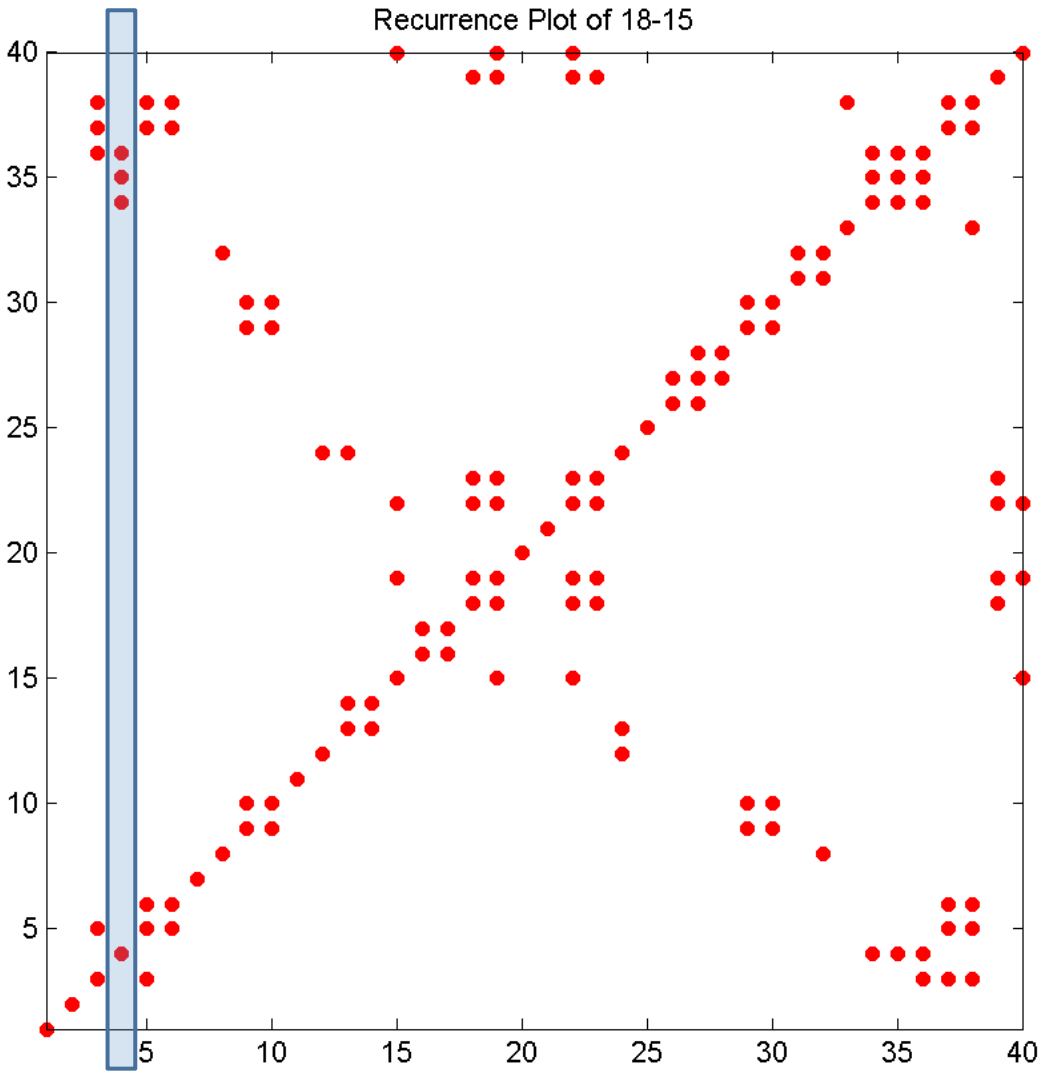
Laminarity/Trapping Time



Vertical lines: an area that was look at before, is looked at again in more detail later on



Laminarity/Trapping Time

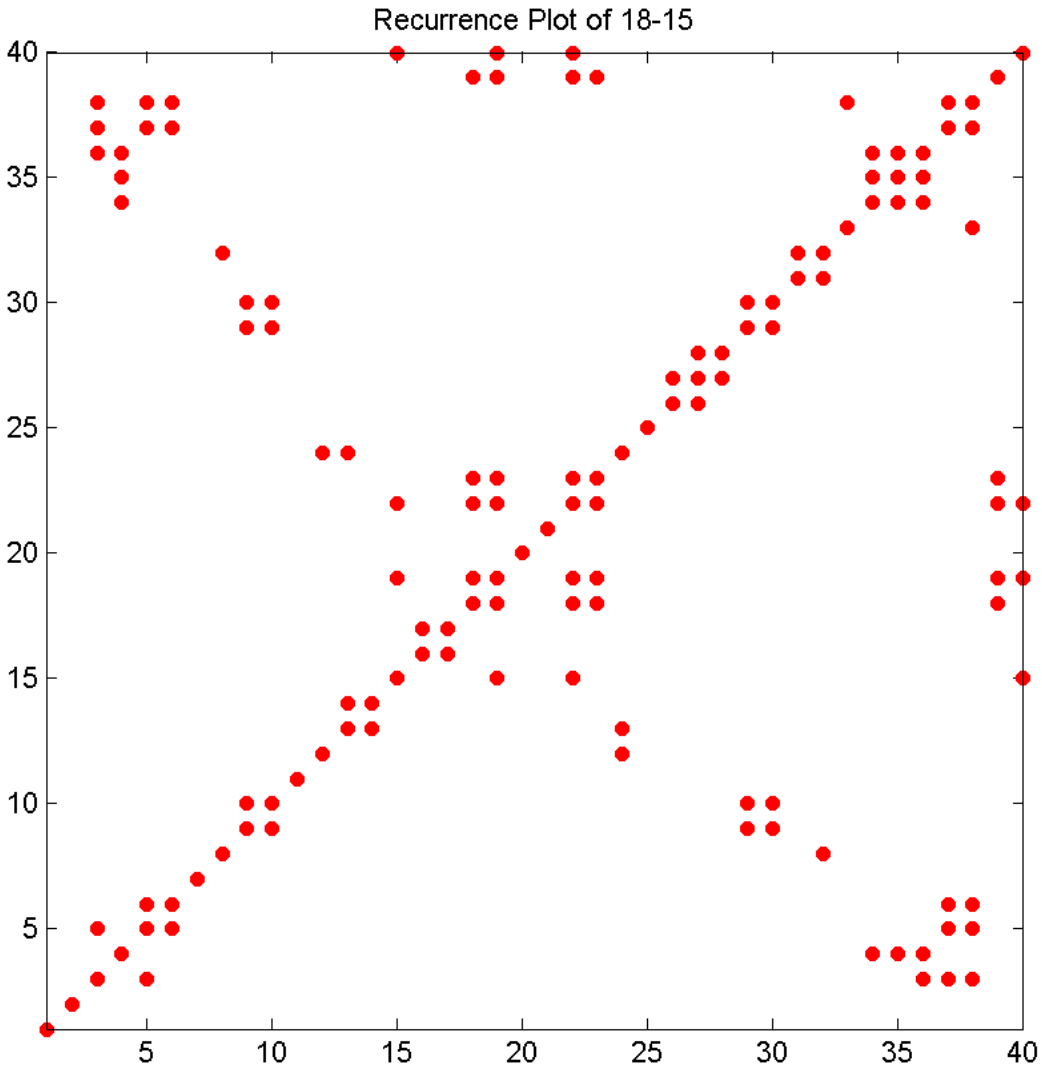


Laminarity: number of recurrent points that form vertical lines

A measure of clusters of eye movements (sequential fixations that come back to a previously seen location)

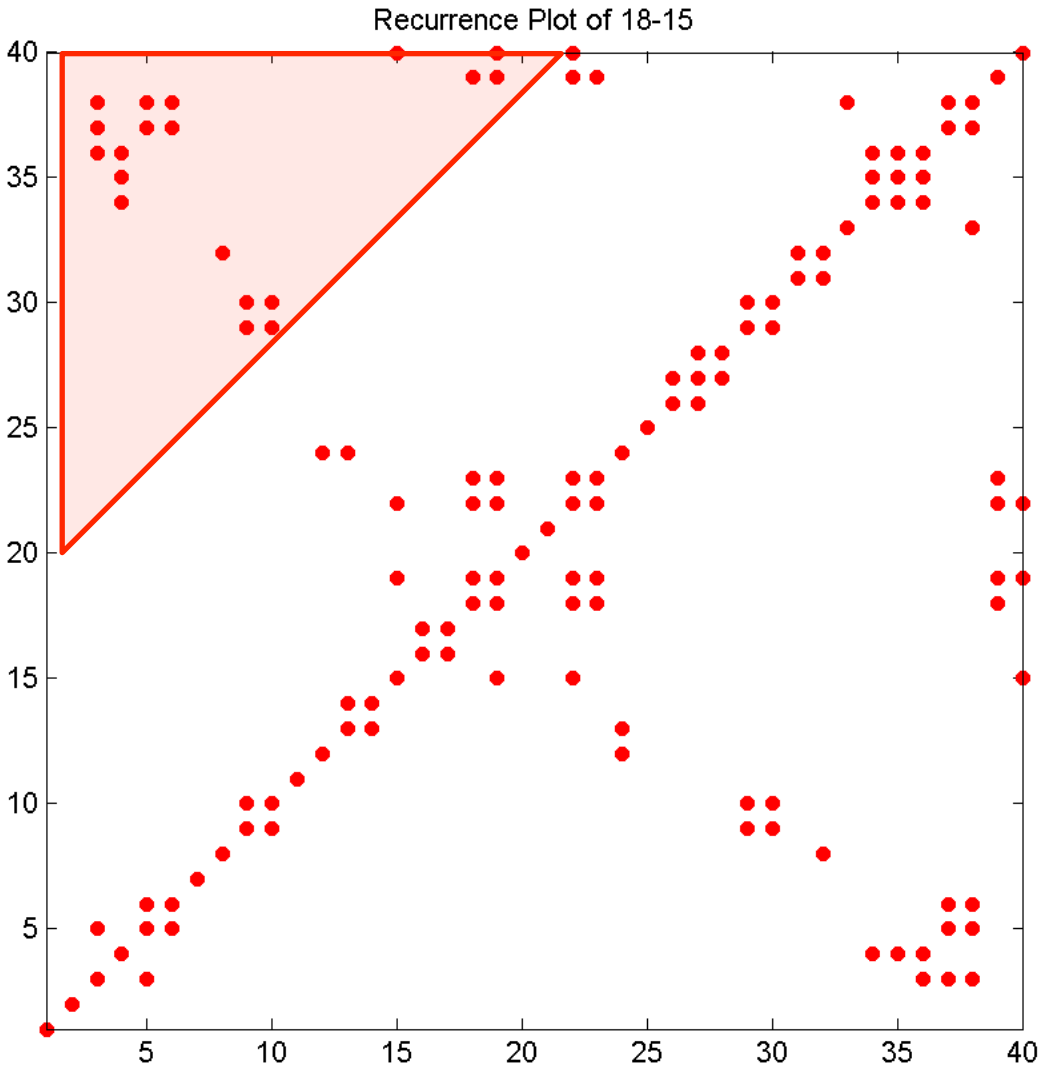
Trapping Time: average length of the vertical lines (for this trial, this value is an average of 2.18 fixations)

Center of Recurrence Mass



CoRM: Where are most of the recurrent points?

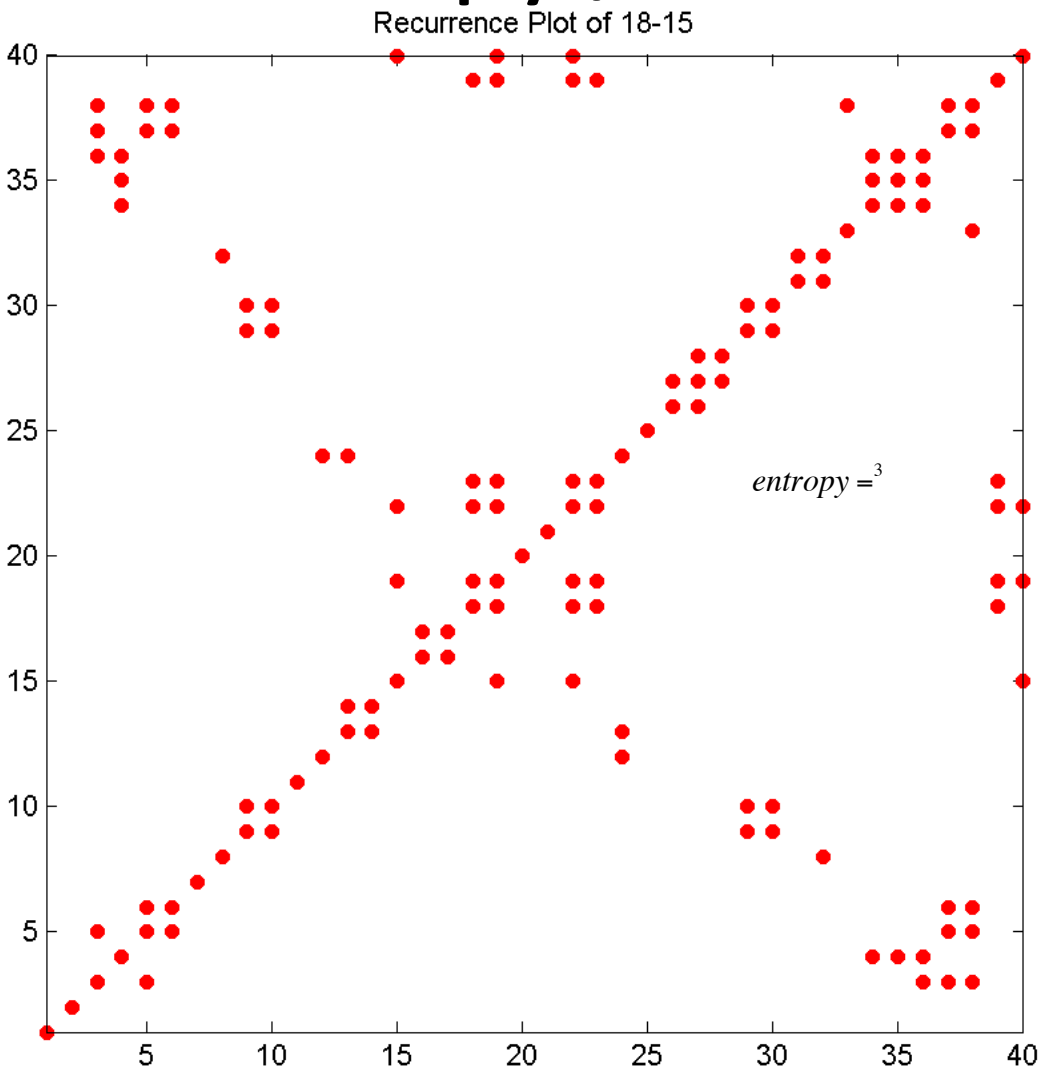
Center of Recurrence Mass



CORM: Where are most of the recurrent points?

If here – means that most of the recurrence is happening later in the trial (i.e., far apart in time)

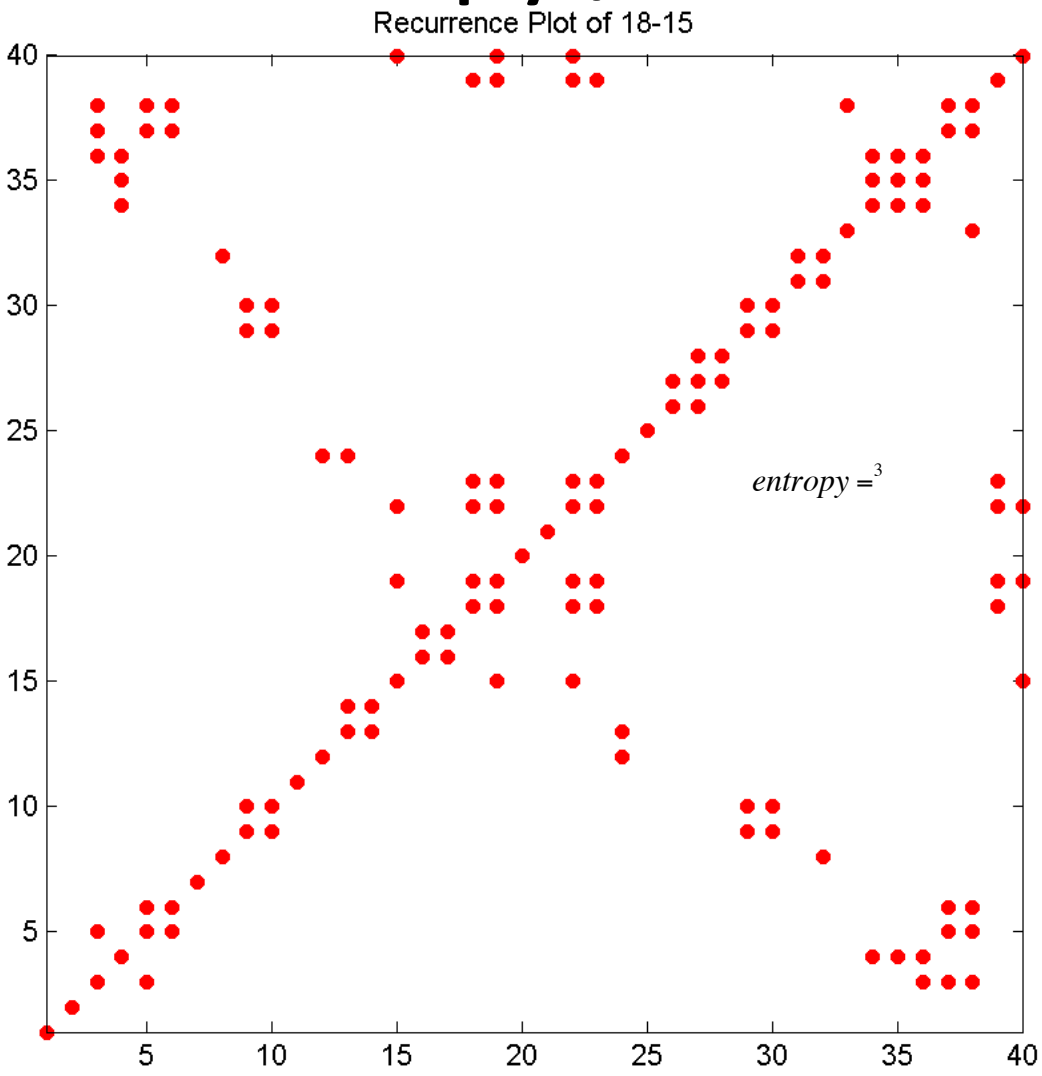
Entropy / Relative Entropy



Shannon entropy of distribution of diagonal line lengths.

A measure of the recurrence plot complexity

Entropy / Relative Entropy



Create histogram of diagonal line lengths.

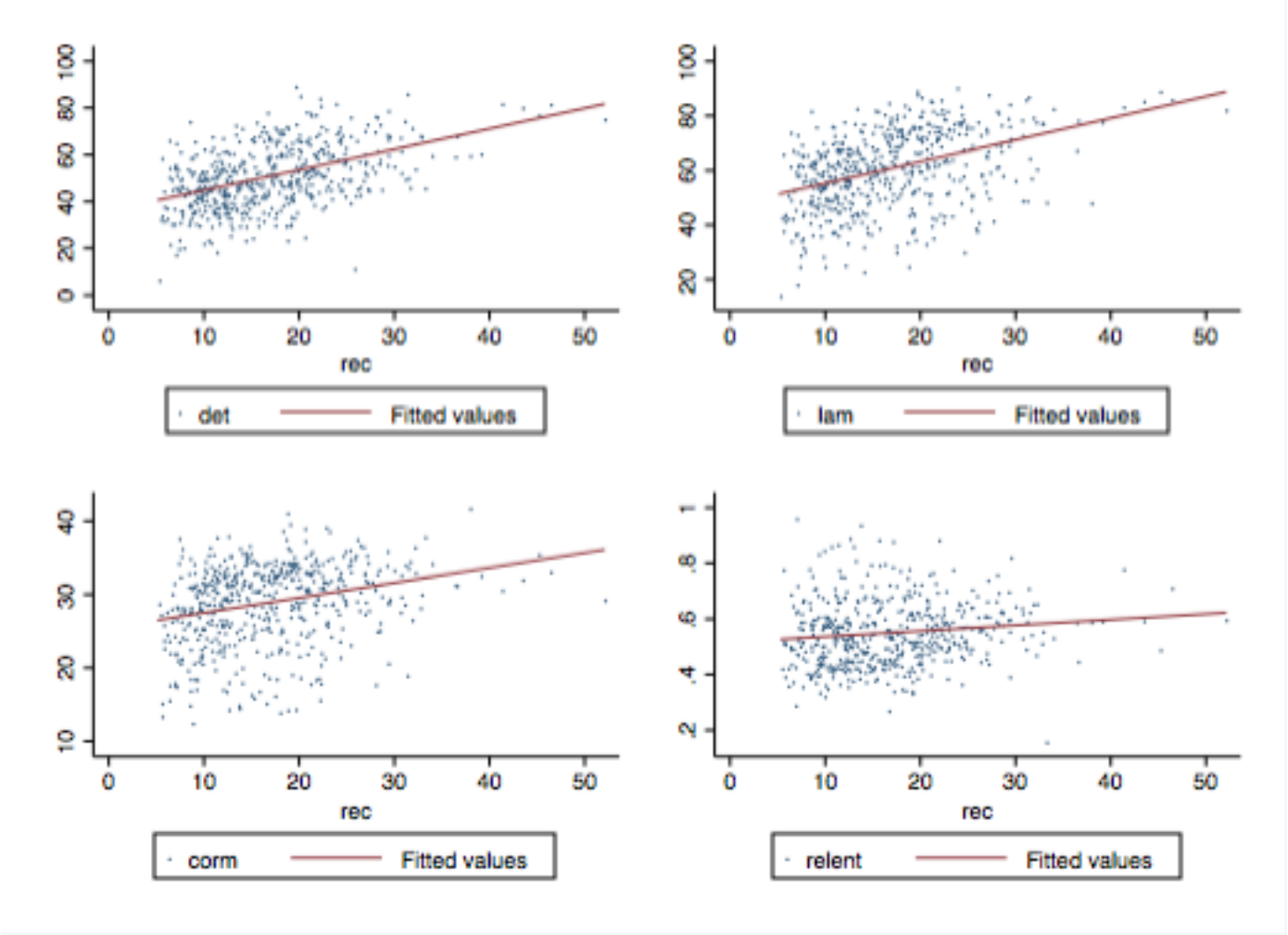
Histogram with bins P_{bin} each with $\#P_{bin}$ observations.

$$\text{entropy} = -\sum \#P_{\text{bin}} \cdot \log_2 P_{\text{bin}}$$

Relative entropy = Entropy / Number of different line lengths

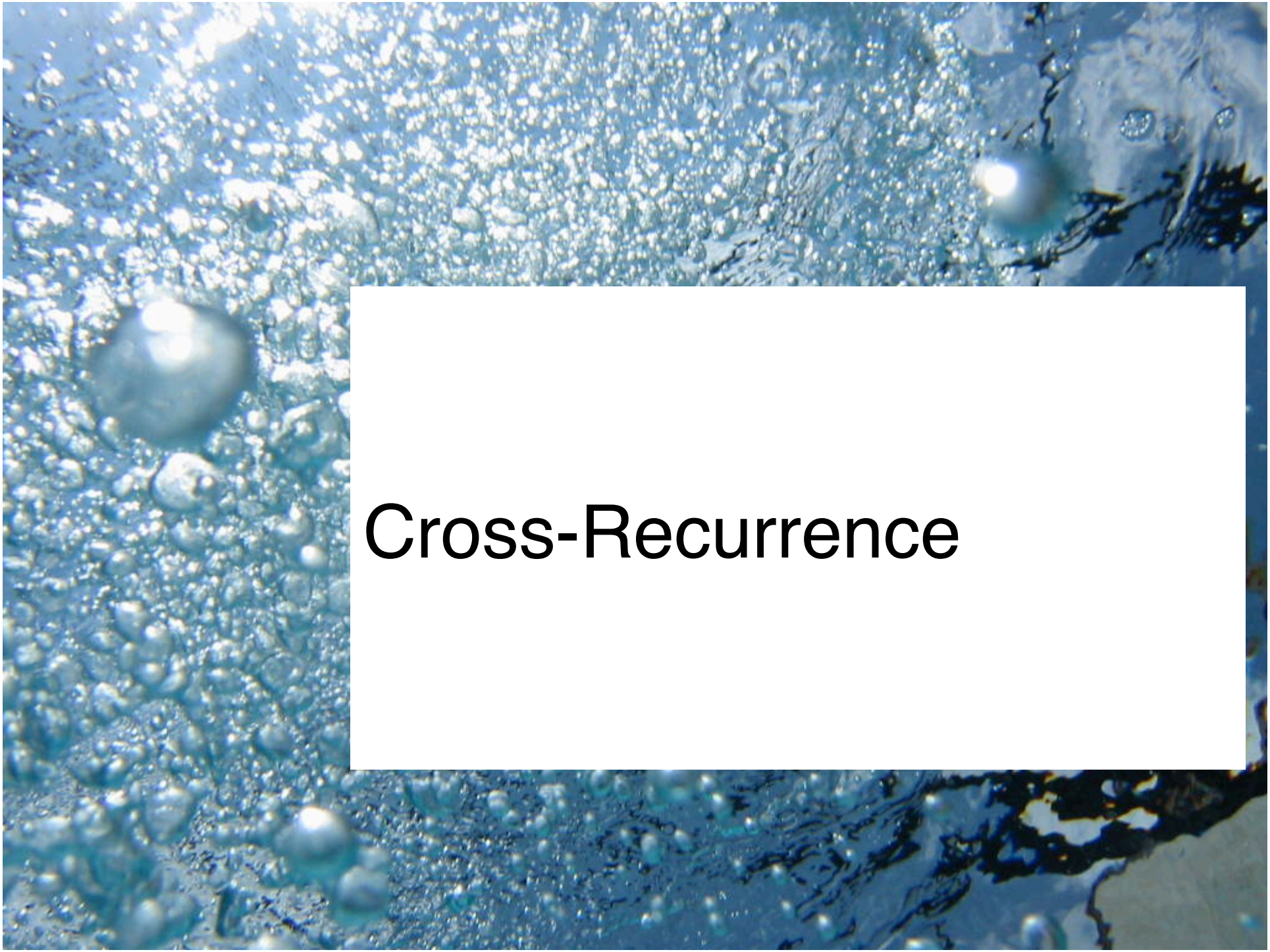

Statistical Properties

Correlations of RQA Measures

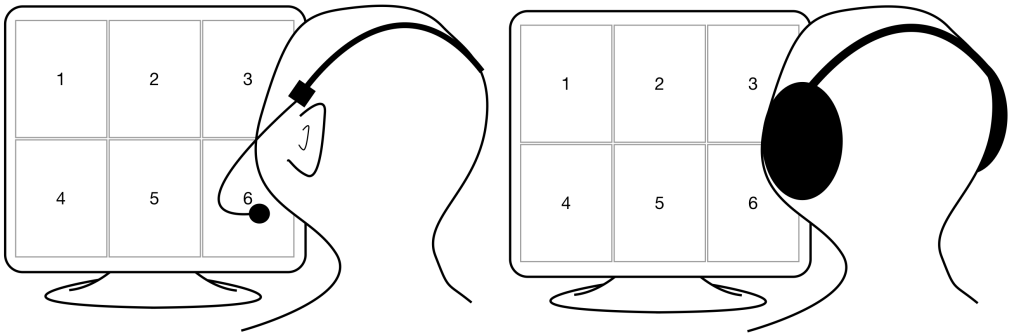


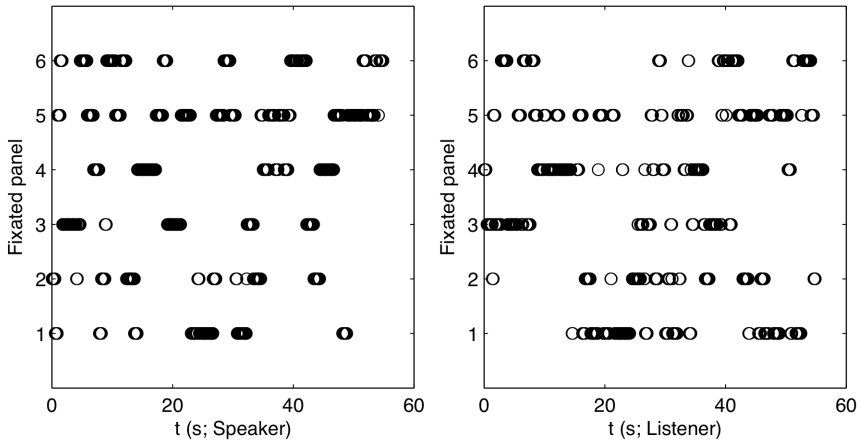
Correlations of RQA Measures

- While measures are descriptive, they are not independent
- Recommendation for statistical analyses:
  - In Anova design, use Manova
  - In Regression design, use Canonical Correlation


Cross-Recurrence

Cross-Recurrence

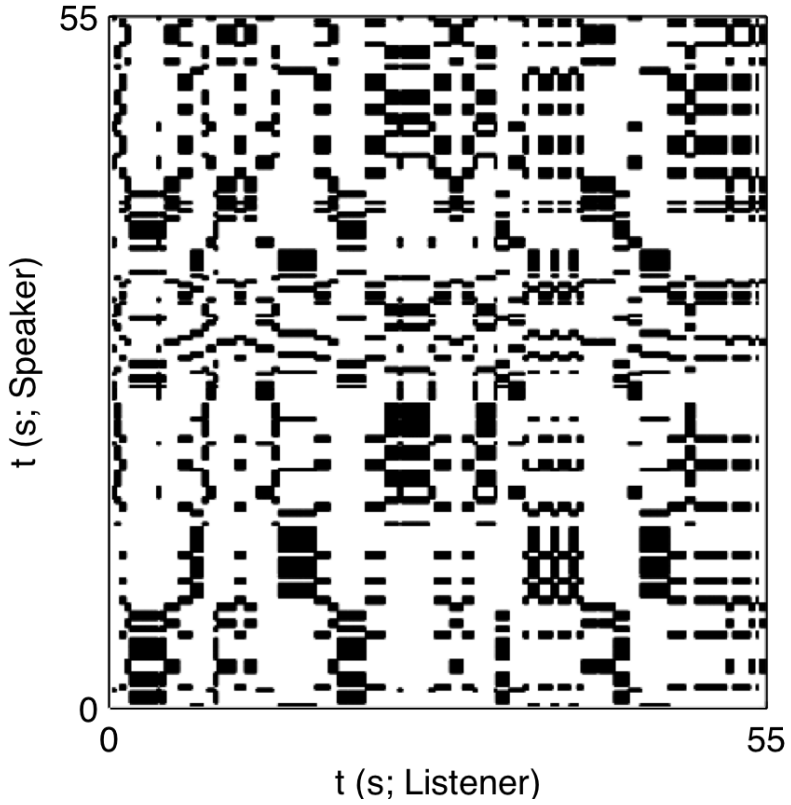




Dale, Warlaumont & Richardson, 2010

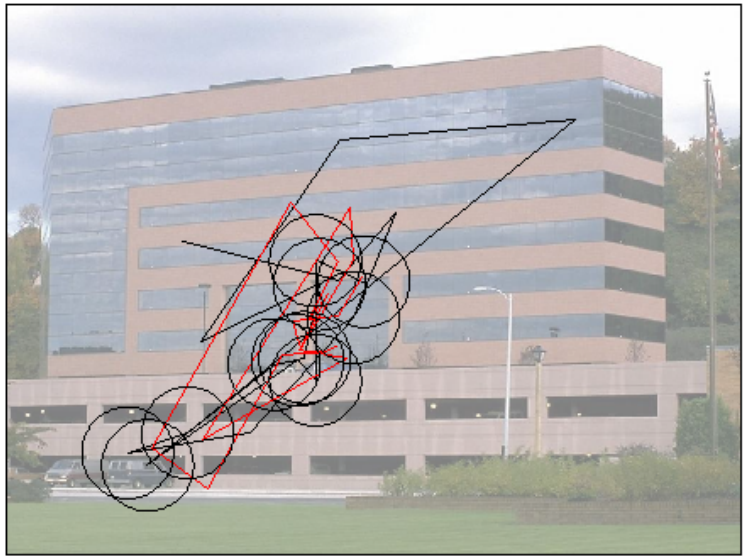
Left person is speaking for approximately 1 minute about a 2×3 grid displaying television characters while the right person listens in.

Cross-Recurrence Matrix



Dale, Warlaumont & Richardson, 2010

Repeated Viewing

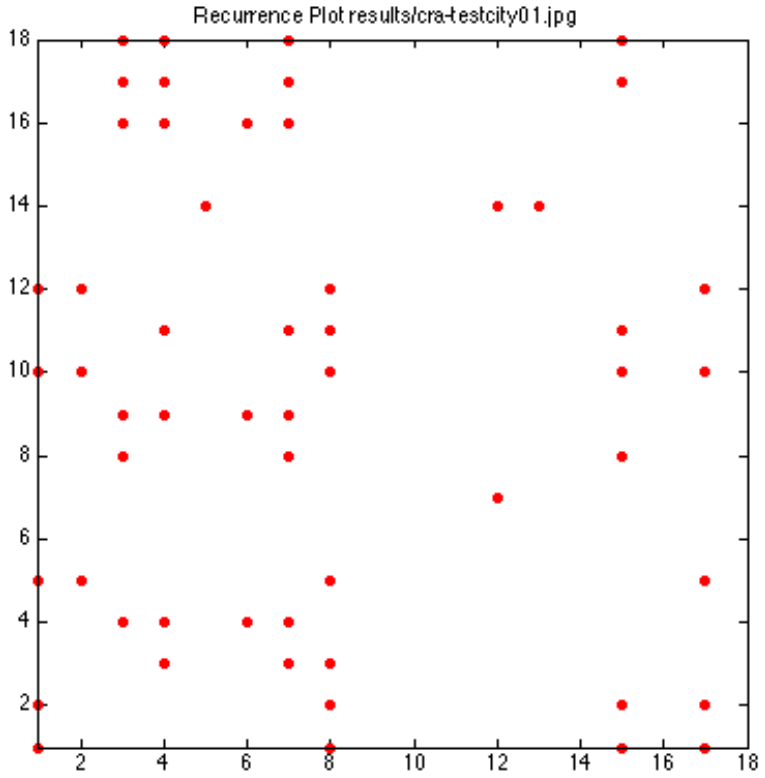


In this study, participants look at many images, each for about 10 seconds, and later see the same image again, among many others.

Black: First scanpath
Red: Second scanpath

Circles indicate a cross-recurrence

Cross-Recurrence Matrix



Differences to simple recurrence matrix:

- No line of incidence along the diagonal
- Non-symmetric

Cross-Recurrence Measures

- Measures are defined over the whole recurrence matrix since it is not symmetric

- Cross-recurrence $R_{ij}=1$ if fixations F_{1i} in first scanpath and F_{2j} in second scanpath are close to each other; otherwise $R_{ij}=0$.

Cross-Recurrence Measures

- Recurrence
  - Percentage of cross-recurrent fixations
- Determinism
  - Percentage of identical scanpath segments (of a given minimal length) in the two scanpaths
- Laminarity
  - Percentage of scanpath segments in one scanpath centered at a fixation point of the other scanpath

Cross-Recurrence Measures

- Corm (center of recurrence mass)
  - Zero if scanpaths mostly aligned in time
  - Positive if scanpath 1 leading scanpath 2
  - Negative if scanpath 2 leading scanpath 1
- Entropy
  - Complexity of the relation between scanpath 1 and 2