Automated Reverse Engineering of Free Formed Objects Using Morse Theory

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Abstract

In this paper, a method for surface reconstruction by means of optimized NURBS (Non-Uniform Rational B-Splines) patches from complex quadrilateral bases on triangulated surfaces of arbitrary topology is proposed. To decompose the triangulated surface into quadrilateral patches, Morse theory and spectral mesh analysis are used. The quadrilateral regions obtained from this analysis is then regularized by computing the geodesic curves between each corner of the quadrilateral regions. These geodesics are then fitted by a B-splines curves creating a quadrilateral network on which a NURBS surface is fitted. The NURBS surfaces are then optimized using evolutive strategies to guaranty the best fit as well as C^1 continuity between the patches.

1. Introduction

The process of reverse engineering consist of recovering from a densely digitized 3-D object an approximated and compact surface representation that is directly compatible with advanced CAD systems such as CATIA V5, ProEng, and many more. Finding a useful and general method for performing this task has proven to be a nontrivial problem and there were until recently no real automated surface approximation software capable of doing this without human intervention, hence the flurry of commercial reverse engineering packages available in industry such as Polyworks, RapidForm, Geomagic, etc. Many of these software packages can perform some the reconstruction task automatically but many of them requires a significant amount of user inputs especially when ones deal with high-level representation such as CAD modelling.

This paper focuses on the automation of reverse engineering of free-formed objects using an approximation approach; where it is assumed that there is no *a priori* information of the surface topology or orientation available from the geometric sensors; only the three-dimensional coordinates of the points are available.

One can find in the literature many surface reconstruction algorithms that can convert a points-cloud into a surface representation. Unfortunately, many of these algorithms do not analytically describe the points cloud because they only use representations that approximate the surface by simple primitives, such as triangular meshes and voxel. Other algorithms use implicit functions such as radial basis functions to reconstruct the point cloud even though they are not an industry standard. On the other hand, NURBS surfaces are an industry standard, but have the inconvenience of not being able to represent complex surfaces easily without large overhead associated with them. This is why, it is necessary to develop a more robust methods which give on one hand a more high-level analytic description of the object surface and topology such as NURBS, and then efficiently handle great quantity of data, preserving the fine details of the surface being reconstructed.

This paper is organized as following: Section 2, describes the problem of reverse engineering. Section 3, a review of the pertinent literature in 3-D reconstruction. Section 4 describes the method for the adjustment of surfaces by means of optimized NURBS patches. Section 5 describes the experimental results of the proposed algorithm, and finally, in Section 6 a conclusion is presented.

2. The Problem of Reverse Engineering

Computer-aided geometric design and computer-aided manufacturing systems are used in numerous industries to design and create physical objects from digital models. Typically, the process consist of constructing complex objects by a combination of simple geometrical primitives. Many of these primitives are combined by boolean operations or by specifying a boundary representation where the topology and the geometry of the object are well known. However the reverse problem, which is of inferring a geometric model from an existing physical object digitized by a 3-D sensor, is a much harder problem as it is hill-posed. Even if we know the geometry of the object with the 3-sensor, unfortunately the topology of the object was lost. We refer to this problem as reverse-engineering. There are various properties of a 3D object that one may be interested in recovering, including its shape, its color, and its material properties, and most importantly a series of geometric primitives and its associated topology. This paper addresses the problem of recovering 3D shape by using NURBS surfaces defined topologically as a network of quadrilaterals curves over the surface. The specification of the problem to be solved can be stated as follows:

> "Given a set of sample points X assumed to lie on or near an unknown surface U, create a surface model S approximating U" [16].

In the general surface reconstruction problem, we consider that the points X are noisy. No structure or other information is assumed. The surface U -assumed to be a manifold- may have arbitrary topology, including boundaries, they contain sharp features such as creases and corners. Since the points X are noisy samples, we do not attempt to interpolate them, but instead find an approximating surface. Of course, a surface reconstruction procedure cannot guarantee recovering U exactly, since it is only given information about U through a finite set of noisy sample points. The reconstructed surface S should have the same topological type as U and be close to U.

2.0.1. Morse Theory for Triangular Meshes. The application of the Morse theory to triangular meshes implies a discrete solution. The Laplacian equation is used to find a Morse function which describes the topology represented on the triangular mesh. In this sense, additional points of the feature of the surface might exist, which produce a basis domain which adequately represents the geometry of the topology itself and the original mesh. In this work, the application of the Morse theory to triangular meshes a more robust version is proposed finding a Morse function which can be appropriated to a certain number of critical points. The Morse theory relates the topology of a surface S with its differential structure specified by the critical points of a Morse function $h : S \rightarrow \mathbb{R}$ [20] and is related to mesh spectral analysis.

Spectral mesh analysis is performed by initially calculating the Laplacian at every vertices. The discrete Laplacian operator on piecewise linear functions over triangulated manifolds is given by:

$$\Delta f_i = \sum_{j \in N_i} W_{ij}(f_j - f_i) \quad (1)$$

where N_i is the set of vertices adjacent to vertex i and W_{ij} is a scalar weight assigned to the directed edge (i, j).

Representing the function $\mathbf{f}$, by the column vector of its values at all vertices $\mathbf{f} = [f_1, f_2, \ldots, f_n]^T$, one can reformulate the Laplacian as a matrix:

$$\Delta \mathbf{f} = -L\mathbf{f} \quad (2)$$

where the Laplacian matrix L is defined by:

$$L_{ij} = \begin{cases} \sum_k W_{ik} & \text{if } i = j, \\ -W_{ij} & \text{if } (i,j) \text{ is an edge of } S, \\ 0 & \text{in other case.} \end{cases} \quad (3)$$

where k is the number of neighbors of the vertex i. Eigenvalues $\lambda_1 = 0 \leq \lambda_2 \leq \ldots \leq \lambda_n$ of L form the *spectrum* of mesh S. Besides describing the square of the frequency and the corresponding eigenvectors $e_1, e_2, \ldots, e_n$ of L, define piecewise linear functions over S of progressively higher frequencies [24].

3. Literature Review

A wide gamut of algorithms for surface reconstruction have been proposed in the literature in recent years [4] [16] [3]. We can divide these methods into two main categories: interpolation methods and approximation methods.

Interpolation Methods This type of algorithms tries to obtain a piecewise linear manifold interpolating a sample data set P. These methods are appropriate for noise-free data sets.

Different approaches have been used, producing algorithms based on Delaunay triangulation. Edelsbrunner and Mucke [13] pioneered the algorithm based on Delaunay triangulation introducing an alpha-shape algorithm. This algorithm selects candidates Delaunay triangles based on the radius of the smallest empty circum-sphere. They also extended this notion to weighted alpha-shapes in which the data points can be assigned scalar weighs to cope with non-uniform samplings. For three dimensions, Amenta and Bern [1] gave an algorithm that selects a subset of the Delaunay triangles of P as the output surface. They defined a sampling condition under which their output is homeomorphic to the surface of the original geometric object. They also defined the concept of poles. Other strategies use advancing fronts algorithms. Bernardini *et al.* [5] pivoting ball algorithm is conceptually based on the alpha shape and consists of rolling a ball over the data set. It is appropriate for large data sets but is extremely dependent on sampling. Based on Delaunay triangulation, Boyer and Petijan [7] gave an incremental algorithm over a 3D Delaunay triangulation that is based on a regular interpolant. More recently, David Cohen-Steiner, and Frank Da [9] developed another incremental algorithm based on the Delaunay triangulation that produces satisfactory results when the models have sharp features, irregular sampling, and large data sets.

Approximation Methods Instead of constructing a piecewise linear manifold interpolating the sampling points, these methods construct a polynomial or an implicit manifold near the set of sample points.

A pioneering work was presented by Hoppe *et al.* [16]. They proposed an algorithm that locally estimates a signed distance function defined on $\mathbb{R}^3$ that returns the distance to the closest point in the manifold. The distance is negative at interior points to the manifold and positive at exterior points. They use an estimation of this distance function to the closest point in the input sample. The output surface is a polygonization of the zero set of the estimated distance function. Radial Basis Function (RBF) has been proposed by various authors: Savchenko [22], Carr *et al.* [8] and Turk and O'Brien [23]. Carr *et al.* [8] presented a polyharmonics radial basis function that can fit very large data sets consisting of millions of data points and arbitrary topology. In this method the holes are smoothly filled and the surface are smoothly extrapolated. Levin [17] developed a mesh independent method for smooth surface approximation and moving least-squares (MLS) by introducing a different paradigm based on a projection procedure. This is the closest work to our algorithm.

4. Approximation of Smooth Surfaces Using Morse Theory

One of the most important phases in the 3D reconstruction process is the the process of fitting high-level surface primitive such as NURBS from triangular meshes.

The majority of the literature on re-meshing methods, focuses on the problem of producing well formed triangular meshes (ideally Delaunay). However, the ability to produce quadrilateral meshes is of great importance as it is a key requirement to fit NURBS surface on a large 3–D mesh. Quadrilateral topology is the preferred primitives for modelling many objects and in many application domains. Many formulations of surface subdivision such as SPLINES and NURBS, require complex quadrilateral bases. Recently, methods to automatically quadrilateralize complex triangulated mesh have been developed such as the one proposed by Dong et al. [11]. These methods are quite complex, hard to implement, and have many heuristic components.

In this section, a method for the surface approximation by means of optimized NURBS patches from complex quadrilateral bases on triangulated surfaces of arbitrary topology is proposed. This process of quadrilateralization produces regions composed exclusively of smooth quadrilaterals. To decompose the triangulated surface into quadrilateral patches, Morse theory and spectral mesh analysis are used. The quadrilateral border joining the critical points are regularized by computing geodesic curves between each corner and then B-splines approximate those geodesics. Following the geodesic curves approximation a NURBS surface is then fitted by changing the NURBS's weight to represent the data inside the quadrilateral region. Such NURBS surfaces fitting is non-linear and an evolutive strategy optimization method is used to minimize the distance between the surface and the points inside the quadrilateral region. The optimization also take into account the smooth joint at the boundary to guarantee C^1 continuity. The proposed algorithm for surface approximation by means of optimized NURBS patches is proposed in Algorithm 1.

Algorithm 1: Method for fitting the surface by means of optimized NURBS patches.

```
Adjustment by means of optimized
NURBS patches();
begin
    1. Quadrilateralization of the triangular mesh;
    2. Regularization of triangular mesh;
    3. Fitting of optimized NURBS patches using
    evolutionary algorithm;
end
```

In the following section, each steps of the proposed algorithm is described.

4.1. Quadrilateralization of Triangular Mesh

One of the first step of our algorithm consist of converting a triangular representation into a network of quadrilateral that is a complete description of the object's geometry. This is necessary as the representation by means of NURBS patches requires building a regular base on which the NURBS surfaces sits. Because of the complex and diverse forms of free-formed objects, obtaining a quadrilateral description of the whole surface is not a trivial task. In order to solve this problem, we propose the following algorithm Algorithm 2.

Algorithm 2: Quadrilateralization method of a triangular mesh.

```
Quadrilateralization();
begin
    1. Critical points computation;
    2. Critical points interconnection;
end
```

4.1.1. Localizing Critical Points. Initially, the quadrilateral's vertices are obtained as a critical points-set of a Morse function. Morse's discrete theory guarantees that, without caring about topological complexity of the surface represented by triangular mesh, a complete quadrilateral description is obtained. That is to say, it is possible to completely divide objects' surfaces by means of rectangles. In this procedure, an equation system for the Laplacian matrix is solved by calculating a set of eigen-values and eigenvectors for each matrix (Equation 3) [19].

A Morse-Smale complex is obtained from the connection of a critical points-set which belongs to a field of the Laplacian matrix. The definition of a field of the matrix is obtained by selecting the set of vectors associated to a solution value of the equation. As Morse function represents a function in the mesh, each eigen-value describes the frequency square of each function. Thus, selecting each eigenvalue directly indicates the quantity of critical points which the function has. For higher frequency values, a higher number of critical points will be obtained. This permits representing each object with a variable number of surface patches. The eigen value computations assigns function values to every vertex of the mesh, which permits determining whether a vertex of the mesh is at critical points of the Morse function. In addition, according to a value set obtained as the neighborhood of the first ring of every vertex, it is possible to classify the critical points as maximum, minimum or "saddle points." Identification and classification of every critical point permits building the Morse-Smale complex.

This method to obtain critical points was tested with smooth and irregular arbitrary topology of real objects, respectively (See Figure 1(a)). For each object, the critical points obtained for different harmonics configuration are used.

4.1.2. Critical Points Interconnection. Once critical points are obtained and classified, then they should be connected to form the quadrilateral base of the mesh. The connection of critical points is started by selecting a "saddle point" and by building two inclined ascending lines and two declined descending lines. Inclined lines are formed as a vertex set ending at a maximum critical point. In addition, a descending line is formed by a vertex path which ends at a minimum critical point. One can then join two paths if both are ascending or descending.

After calculating every paths, the triangulation of K surface is divided into quadrilateral regions which forms Morse-Smale complex cells [19]. Specifically, every quadrilateral of a triangle falls into a "saddle point" without ever crossing a path. The complete procedure is described in Algorithm 3:

Algorithm 3: Bulding method of MS cells.

```
Critical points interconnection();
begin
    Let T={F,E,V} M triangulation;
    Initialize Morse-Smale complex, M=0;
    Initialize the set of cells and paths, P=C=0;
    S=SaddlePointFinding(T);
    S=MultipleSaddlePointsDivission(T);
    SortByInclination(S);
    for every s ∈ S in ascending order do
        CalculeteAscedingPath(P);
    end
    while exists intact f ∈ F do
        GrowingRegion(f, p0, p1, p2, p3);
        CreateMorseCells(C, p0, p1, p2, p3);
    end
    M = MorseCellsConnection(C);
end
```

In Figure 1(b), it is observed how resulting quadrilateral patches are adequately formed, and they are directly obtained from intrinsic surface properties, adjusting to the objects' geometry.

4.2. Regularization of the Quadrilateral Border Curves

Because the surface needs to be fitted using NURBS patches, it is necessary to regularize the quadrilateral curves obtained from the mesh. The curves are regularized and fitted by b-splines using the following Algorithm 4.

One of the quadrilateral border is selected from the mesh, and later a border is selected from each quadrilateral border and its opposite. The initially selected border is random. The opposite order is searched as one which does not contain the vertices of the first one. If the first selected border has vertices A and B, it is required that the opposite border does not contain vertices A and B, but the remaining, B and C.

Later, B-splines are fitted on selected borders with a λ density, to guarantee the same points for both borders are chosen, regardless of the distance between them. In general, a B-spline does not interpolate every control point; therefore, they approximate curves which permit a local manipulation of the curve, and they require fewer calculations for coefficient determination.

Algorithm 4: Quadrilateral mesh regularization method..

```
Regularization();
```
begin
1. Quadrilateral selection;
2. Selection of a border of the selected quadrilateral and its opposite;
3. Regularization using B-splines with lambda density;
4. Regularized points match by means of geodetics FMM;
    4.1 Smoothing of geodetic with B-splines;
5. Points generating for every B-spline line with lambda density;

end

Having these points at selected borders, it is required to match them. This is done with FMM (*Fast Marching Method*). This algorithm is used to define a distance function from an origin point to the remainder or surface with a computational complexity of $O(n \times \log n)$. This method integrates a differential equation to obtain the geodetic shortest path by traversing the triangle vertices.

At the end of the regularization process, B-splines are fitted on geodetic curves and density λ points are generated at every curve which unite the border points of quadrilateral borders, to finally obtain the grid which is used to fit the NURBS surface.

4.3. Fitting of Optimized NURBS Patches Using an Evolutionary Algorithm

This section presents a method based on an evolutionary strategy (ES), to determine the weights of control points of a NURBS surface, without modifying the location of sampled points of the original surface. The main goal is to reduce the error between the NURBS surfaces and the data points inside the quadrilateral regions. In addition, the algorithm make sure that the C^1 continuity condition is preserved for all optimized NURBS patches. The proposed algorithm is described in Algorithm 5.

Algorithm 5: Optimization and continuity method of NURBS patches method.

```
Adjustment by optimized NURBS
patches();
```
begin
1. Optimization of the NURBS patches;
    1.1. Multiple ES usage with deterministic replacement by inclusion;
    1.2. Application of ES to control weights of NURBS;
2. Union of NURBS patches with continuity C^1;
    2.1. Check continuity between axis;
    2.2. Check continuity at vertices;

end

4.3.1. Optimization of NURBS Parameters. A NURBS surface is completely determined by its control points $\mathbf{P}_{i,j}$. The main difficulty in fitting NURBS surface locally is in finding an adequate parametrization for the NURBS and the ability to automatically choose the number of control points and their positions. The NURBS's weight function $w_{i,j}$ determine the local influence degree of a point in surface topology. Generally, weights of control points for a NURBS surface are assigned in an homogeneous way and are set equal to 1, reducing NURBS to simple B-spline surface. The determination of NURBS control points and weights for arbitrarily curved surfaces adjustment is a complex non-linear problem.

The minimization problem can be expressed by:

$$\delta = \sum_{l=1}^{np} \left(Z_l - \frac{\sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) w_{i,j} \mathbf{P}_{i,j}}{\sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) w_{i,j}} \right)^2 \quad (4)$$

where $N_{i,p}(u)$ and $N_{j,q}(v)$ are B-spline basis functions of p degree and q in the parametrical directions u and v respectively, $w_{i,j}$ are the weights, $\mathbf{P}_{i,j}$ the control points,and np the number of control points. If the number of knots and their positions are fixed, same as the weights set and only the control points $\left(\{\{\mathbf{P}_{i,j}\}_{i=1}^{n}\}_{j=1}^{m} \in \mathbb{R}\right)$, they are considered during minimization of Equation 4, then we have a simpler linear mean square problem. If knots or the weights are considered unknown it will be necessary to solve a non-linear problem. In many applications, the optimal position of knots is not necessary. Hence, the knots location problem is solved by using a heuristic.

The optimization process is formally described as follows: Let $\mathbf{P}_t = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n\}$ a points-set in $\mathbb{R}^3$ sampled from the surface of a physical object, our problem consists of $E(s) = \frac{1}{n} \sum_{i=1}^{n} d^2(\mathbf{p}_i, S_i) < \delta$, where $d(\mathbf{p}_i, S_i)$ represents the Euclidian distance between a point of the set $\mathbf{P}_t$ of sampled points of the original surface S, and a point on the approximated surface S'. To get the configuration of surface S', E is minimized to a tolerance lower than the given δ. Minimization is performed using an evolution strategy $(\mu + \lambda)$ defined as follows:

- **Representation Criteria:** Representation is performed using of pairs of real vectors.
- **Treatment criteria of outlier individuals:** A filtering

of individuals is performed making outlier individuals ignored.

- **Genetic Operators**:
  - **Individual:** The initial values w_i, δ_i of ALELOS of every individual are uniformly distributed in interval [0.5, 1.5]. This range is chosen because it is where the initial values of ALELOS of individuals are given, which correspond to weights of control points and therefore should not initially set to zero.
  - **Mutation:** Individuals mutation will not be correlated with $n\ \sigma' s$ (mutation steps) as established in individual configuration, and it is performed as indicated in the following equations:

$$\sigma_i^{'} = \sigma_i e^{(c_0.N(0,1)+c_i.N_i(0,1))} \quad (5)$$

$$x_i^{'} = x_i + \sigma_i^{'}.N_i(0,1) \quad (6)$$

where $N(0,1)$ is a normal distribution with expected value 0 and variance 1, c_0, c_i are constants which control the size of the mutation step. This refers to the change in mutation step σ. Once the mutation step has been updated, the mutation of ALELOS of individuals is generated w_i.

- **Selection Criteria:** The best individuals in each generation are selected according to the result of the fitness function.

- **Replacement criteria:** In ES, the replacement criteria is always deterministic, which means that μ or λ best members are chosen. In this case, the replacement by inclusion was used, in which the μ descendants are joined with the λ best members and are taken for the new population.

- **Recombination operator:** Two types of recombination are applied whether object variables w_i or strategy parameters σ_i are being recombined. For object variables, an intermediate global recombination is used:

$$b_i^{'} = \frac{1}{\rho}\sum_{k=1}^{\rho} b_{k,i} \quad (7)$$

where $b_i^{'}$ is the new value of ALELO i, and ρ is the number of individuals within the population.

For strategy parameters, an intermediate local recombination is used:

$$b_i^{'} = u_i b_{k_1,i} + (1-u_i) b_{k_2,i} \quad (8)$$

where $b_i^{'}$ is the new value of the ALELO i, and u_i is a real number which is distributed uniformly within the interval $[0,1]$.

4.3.2. NURBS Patches Continuity. Continuity in regular cases (4 patches joined at one of the vertex) is a solved problem [12]. However, in neighborhoods where the neighbors' number is different from 4 ($v \geqslant 3 \rightarrow v \neq 4$), continuity must be adjusted to guarantee a soft transition of the implicit surface function between patches of the partition. Although diverse continuity schemes between parametric functions exist, two of these approaches have been emphasized and they have become an industry standard. Continuity C^0 shows that a vertex continuity between two neighboring patches must exist. This kind of continuity only guarantees that spaces or holes at the assembling limit between two parametric surfaces does not exists. C^1 shows that continuity in normals between two neighboring patches must exist. This kind of continuity guarantees a soft transition between patches, offering a correct graphical representation.

In this paper, C^1 continuity between NURBS patches is guaranteed, using Peters continuity model [21] which guarantees continuity of normals between bi-cubical spline functions. Peters proposes a regular and general model of bi-cubical NURBS functions with regular nodes vectors and the same number of control points at both of the parametric directions. In our algorithm, Peter's model was adapted by choosing generalizing NURBS functions, with the same control points number at both of the parametric directions, bi-cubic basis functions and regular expansions in their node vectors.

Continuity Along the Quadrilateral Boundaries: To guarantee C^1 continuity between the boundaries of neighboring patches, extreme control points which affect the continuity between patches must be found. Due to data ordering within the proposed parametrization schema, two adjacent patches will have the same number of control points at the common axis, regardless of their disposition. To adjust continuity between axes, control points are calculated on the analyzed boundary, to make it co-lineal with neighboring control points on adjacent patches.

Equation 9 illustrates the new position for a control point at given P_{eje} axis, where P_A^{vec} is the neighbor point to P_{eje} at patch B. The new control point P_{eje} is the medium point between the two adjacent control points P_A^{vec} y P_B^{vec} which guarantees that control points on the axis and their adjacent neighbors at each patch is co-linear.

$$P_{eje} = \frac{P_A^{vec} + P_B^{vec}}{2} \quad (9)$$

Continuity at Quadrilateral Vertices: Continuity at vertices of quadrilateral regions is guaranteed by making sure that every adjacent control points at each vertices is co-planar.

Under the continuity criteria proposed by Peters, continuity at quadrilateral vertices are generalized, that is to say, the adjustment process is the same regardless of the number of patches which can be found at a given vertex. We have $\pi^T P = 0$, where π is a given plane and P is a point on the plane. If the system of equations is over-determined with more than four points, the equation which best adjusts a given point-set can be found.

Equation 10 represent the over-determined system where $P = [P_1, P_2, \ldots, P_n]^T$ with $n \geq 4$ are control points at the vertices. The equation is solved using Singular Values Decomposition SVD, with the last column of matrix P the equation of the plane which is adjusted to points-set P in the quadratic mean square error sense. [15].

$$\begin{bmatrix} P_x^1 & P_y^1 & P_z^1 & 1 \\ P_x^2 & P_y^2 & P_z^2 & 1 \\ \ldots & \ldots & \ldots & 1 \\ P_x^n & P_y^n & P_z^n & 1 \end{bmatrix} \begin{bmatrix} \pi_x \\ \pi_y \\ \pi_z \\ \pi_z \end{bmatrix} = 0 \qquad (10)$$

Continuity is adjusted by projecting control points P onto the plane given by Equation 10:

$$\pi = n_1(x - x_o) + n_2(y - y_o) + n_3(z - z_o) \qquad (11)$$

where $N = [n_1, n_2, n_3]$ is the plane's normal and $P_0 = [x_0, y_0, z_0]$ is a point on the plane. The projection $P_I = [x_I, y_I, z_I]$ of a point $P = [P_x, P_y, P_z]$ on the plane is given by:

$$x_I = P_x + n_1 t_I \quad y_I = P_y + n_2 t_I \quad z_I = P_z + n_3 t_I \quad (12)$$

where t is the parametric value of the straight line which passes through point P in the direction of the plane's normal $\mathbf{N}$.

Using Equation 12, it is possible to project control points on the given plane, which guarantee the continuity of normals at vertices of the quadrilateral partition, ensuring that every adjacent control points are co-planar.

5. Experimental Results

In this section, the results of the methodology of three-dimensional reconstruction of free-form objects proposed in this paper are shown. To validate the functionality of the proposed algorithm for surface fitting by means of NURBS patches, the results for a free-form object (see Figure 1) are presented.

Tests were performed using a 3.0 GHz dual Opteron processor computer, with 1.0 GB RAM, running Microsoft Windows XP operating system. The methods were implemented using C++ and MATLAB. The data used were obtained with Kreon range scanners, available at the Advanced Man-Machine Laboratory – Department of Computing Science, University of Alberta, Canada.

The partial results obtained at each one of the intermediate stages of the proposed algorithms in this paper are shown in Figure 1. This object called Mask is composed of 18 range images, corresponding to 84068 points. The final model is composed of 105 optimized NURBS surfaces patches with an adjustment error of 1.02×10^{-3} (See Figure 1(f)). The reconstruction of the object took an average time of 32 minutes.

6. Conclusion

The methodology proposed in this thesis for the automation of reverse engineering of free-form three-dimensional objects has a wide application domain, allowing adjustment of surfaces regardless of topological complexity of the original objects.

A novel method for fitting triangular mesh using optimized NURBS patches has been proposed. This method is topologically robust and guarantees that the complex base be always quadrilateral creating a network of surfaces which is compatible with most commercial CAD systems.

In the proposed algorithm, the NURBS patches are optimized using multiple evolutionary strategies to estimate the optimal NURBS parameters. The resulting NURBS are then joined, guaranteing C^1 continuity. The formulation of C^1 continuity presented in this paper can be generalized, because it can be used to approximate regular and irregular neighborhoods which present model processes regardless of partitioning and parametrization.

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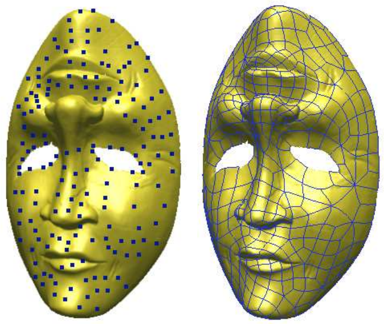
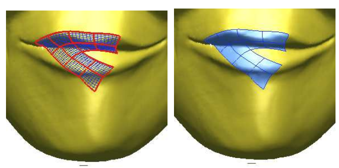
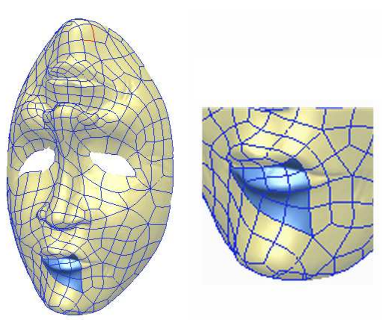


(a) Critical points. (b) Quadrilateralized object.



(c) Regularization. (d) NURBS patches optimization.



(e) Union of NURBS patches.

(f) Fitted surface by the optimized NURBS patches.

Figure 1. Reconstruction of a complex curve surface using a network of NURBS patches.

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