Search Overview

- Introduction to Search
- Blind Search Techniques
- Heuristic Search Techniques
- Game Playing search
  - Perfect play
  - Resource limits
  - α–β pruning
  - Games of chance
- Constraint Satisfaction Problems
- Stochastic Algorithms

Games vs. search problems

- "Unpredictable" opponent
  $\Rightarrow$ solution $\equiv$ contingency plan

- Time limits
  $\Rightarrow$ unlikely to find goal
  $\Rightarrow$ must approximate

- Plan of attack:
  - algorithm for perfect play
    [von Neumann, 1944]
  - finite horizon, approximate evaluation
    [Zuse, 1945; Shannon, 1950; Samuels, 1952–57]
  - pruning to reduce costs
    [McCarthy, 1956]

Types of games

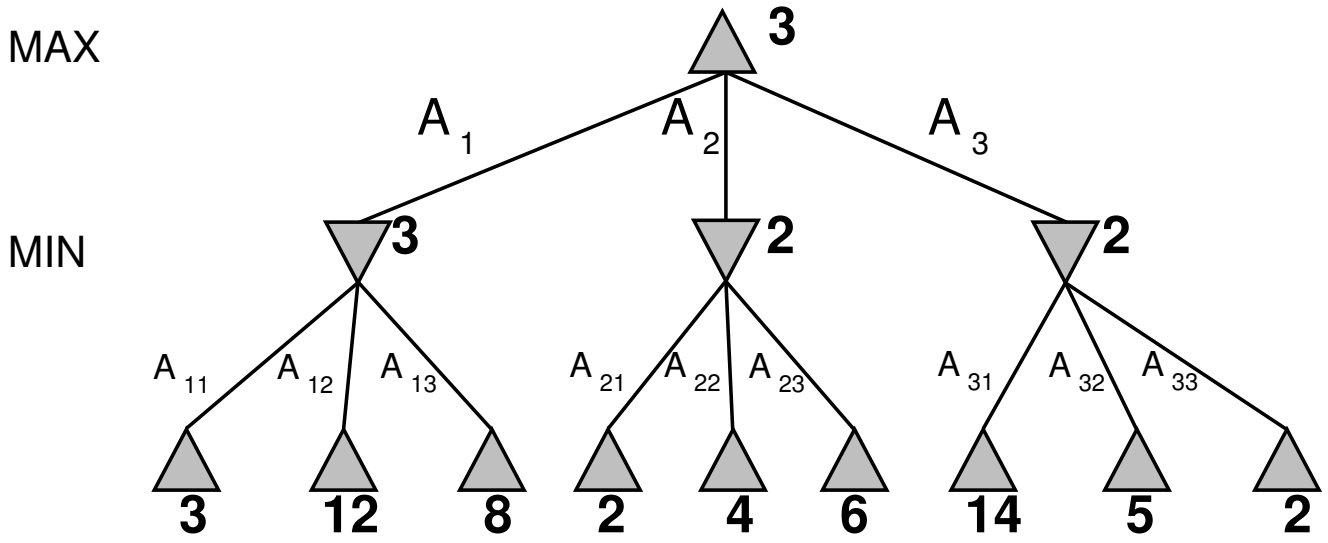
	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information		bridge, poker, scrabble nuclear war

Minimax

- Perfect play for
  deterministic, perfect-information games

Idea: choose move leading to position
with highest *minimax value*
$\equiv$ best achievable payoff against best play

- Eg, 2-ply game:



Minimax algorithm

function Minimax-Decision(*game*) **returns** *an operator*

    for each *op* **in** Operators[*game*] **do**
        Value[*op*] ← Minimax-Value(Apply(*op*, *game*), *game*)
    end
    return the *op* with the highest Value[*op*]

---

function Minimax-Value(*state*, *game*) **returns** *a utility value*

    if Terminal-Test[*game*](*state*) **then**
        return Utility[*game*](*state*)
    else if max is to move in *state* **then**
        return the highest Minimax-Value of Successors(*state*)
    else
        return the lowest Minimax-Value of Successors(*state*)

Properties of minimax

Complete: ??

Optimal: ??

Time complexity: ??

Space complexity: ??

Properties of minimax

Complete: Yes, if tree is finite

[chess has specific rules for this]

Optimal: Yes, against an optimal opponent.

Otherwise??

Time complexity: $O(b^m)$

Space complexity: $O(bm)$

(depth-first exploration)

For chess:

$b \approx 35$, $m \approx 100$ for "reasonable" games

$\Rightarrow$ exact solution completely infeasible

Resource Limits

- Chess has $\approx 10^{40}$ pos'ns;
  $10^{10^{50}}$ possible games

  `http://mathworld.wolfram.com/Chess.html`

- Suppose we have 10 seconds/move.

  If explore 10^9 nodes/second
  $\Rightarrow \underline{10^{10}}$ nodes per move

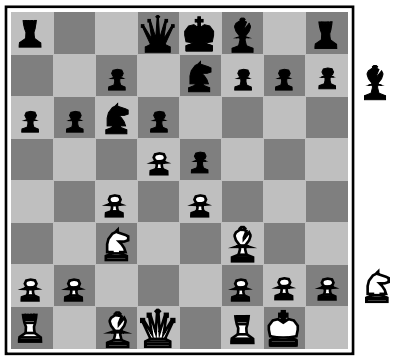
  Not NEARLY enough!

- Standard approach:

  - *cutoff test*
    eg, depth limit
    (perhaps add *quiescence search*)

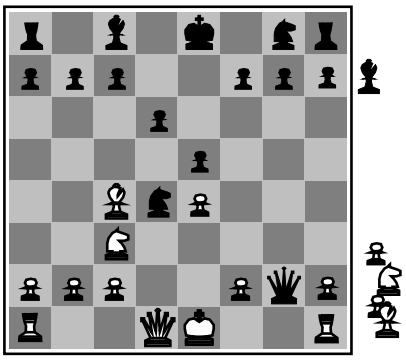
  - *evaluation function*
    = *estimated* desirability of position

Evaluation Functions



Black to move

White slightly better



White to move

Black winning

- Typically *linear* weighted sum of features

$$Eval(s) \quad = \quad w_1\, f_1(s) + w_2\, f_2(s) + \ldots + w_n\, f_n(s)$$

Eg, chess:

Approximation ... $w_1 = 9$

$f_1(s)$ = #WhiteQueens − #BlackQueens

$w_2 = 5$

$f_2(s)$ = #WhiteRooks − #BlackRooks

...

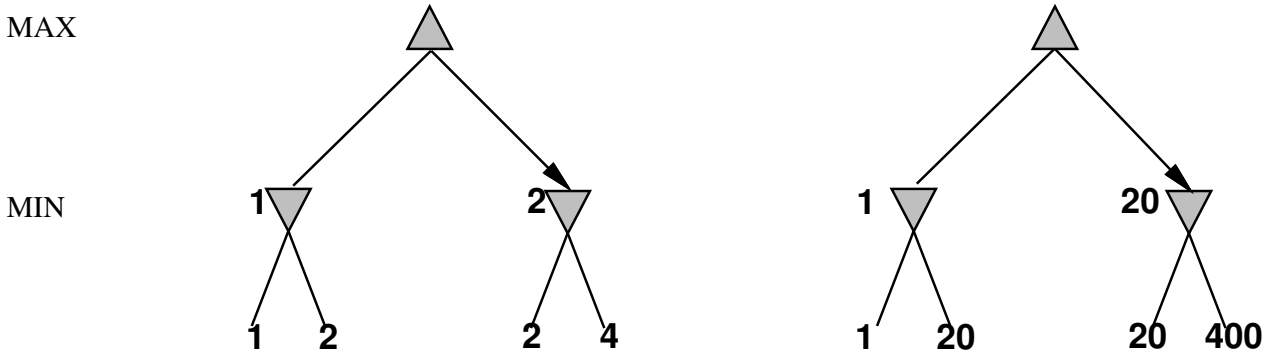
$w_5 = 0.3$

$f_5(s)$ = White'sControlOfCenter

...

- Which features $f_i(\cdot)$?
  What values for w_i?
  $\Rightarrow$ Machine Learning!

Digression: Exact values don't matter



- Behaviour is preserved under
  any *monotonic* transformation of Eval

- Only the order matters:
  payoff in deterministic games acts as
  ordinal utility function

Cutting off search

- MinimaxCutoff $\equiv$ MinimaxValue except
  1. Terminal? is replaced by Cutoff?
  2. Utility is replaced by Eval

- Does it work in practice?

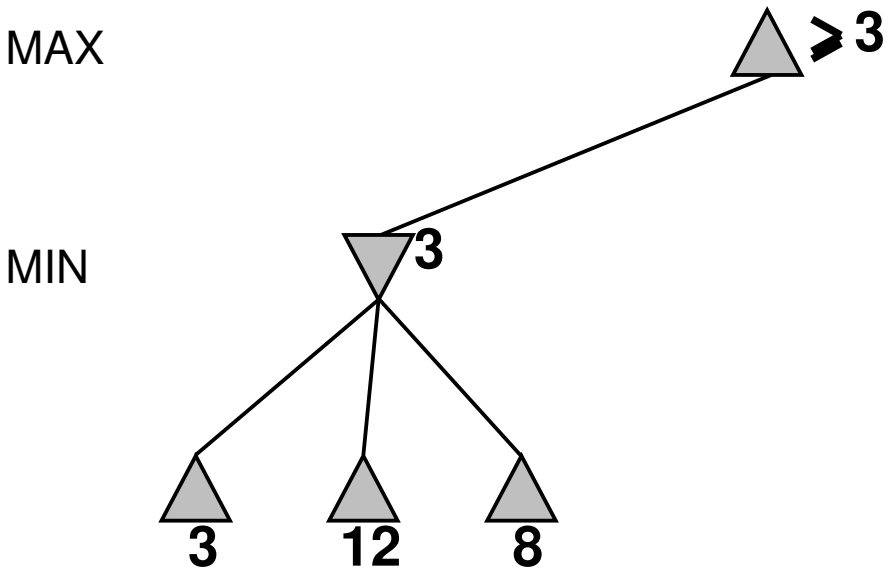
$$b^m = 10^6, \quad b = 35 \quad \Rightarrow \quad m = 4$$

  4-ply lookahead is a hopeless chess player!

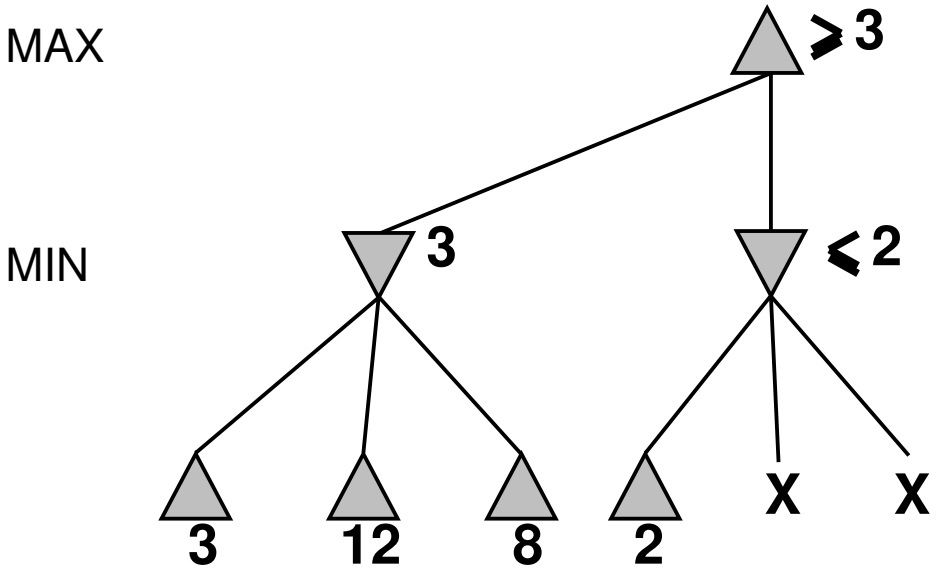
- 4-ply $\approx$ human novice
  8-ply $\approx$ typical PC, human master
  12-ply $\approx$ Deep Blue, Kasparov

- to do better . . .

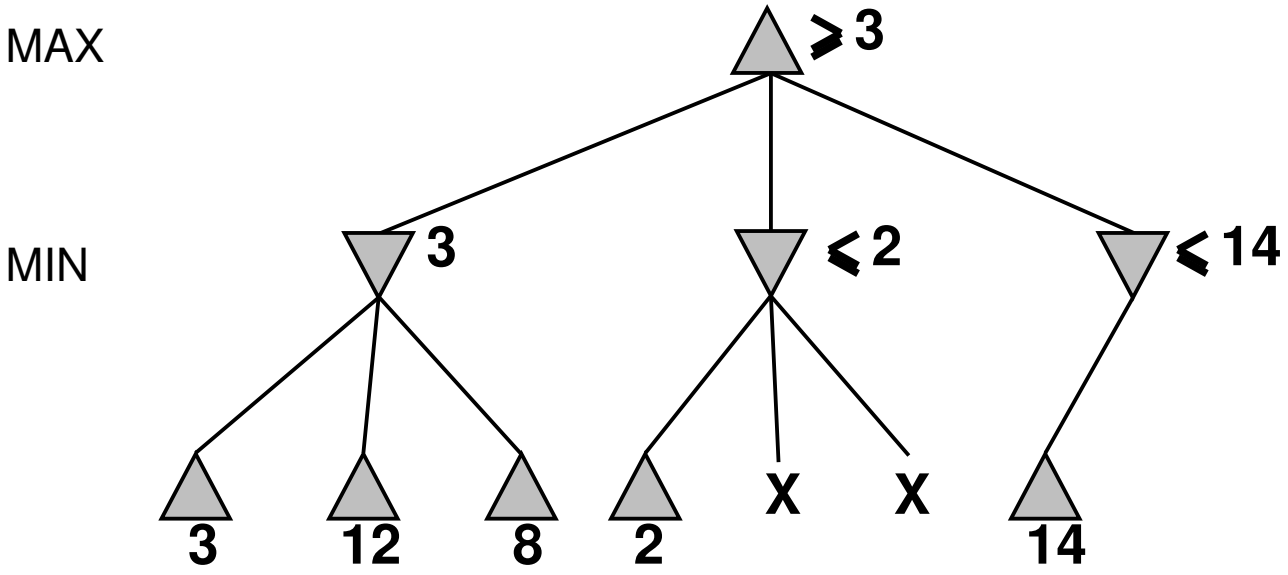
α–β pruning example



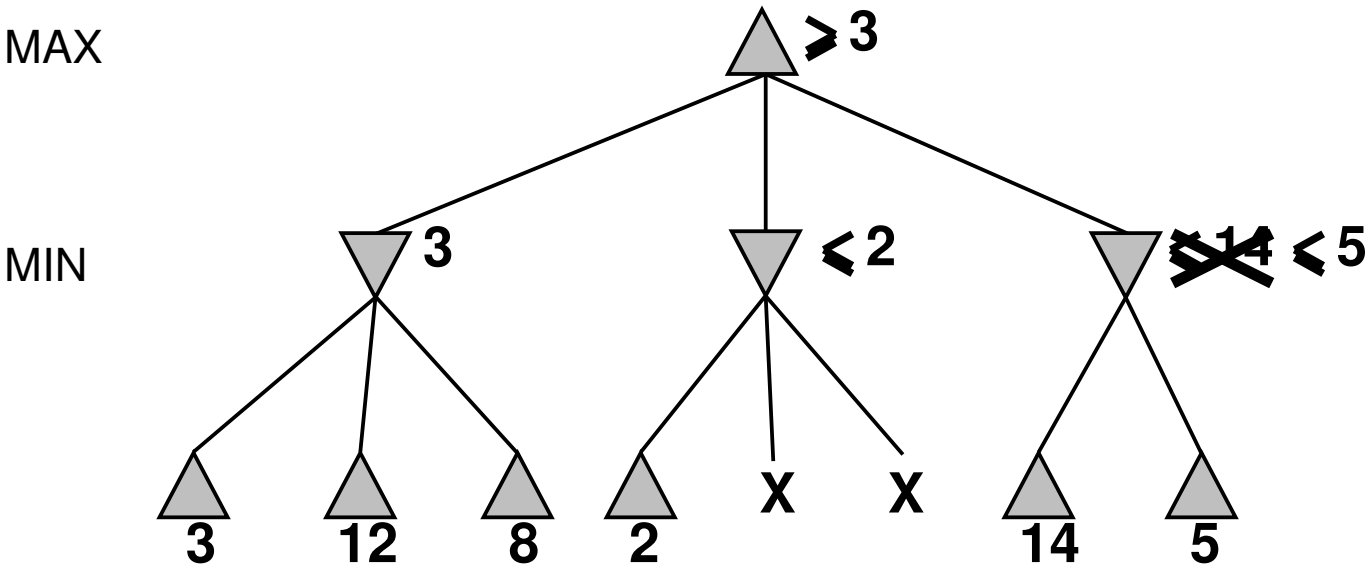
α–β pruning example



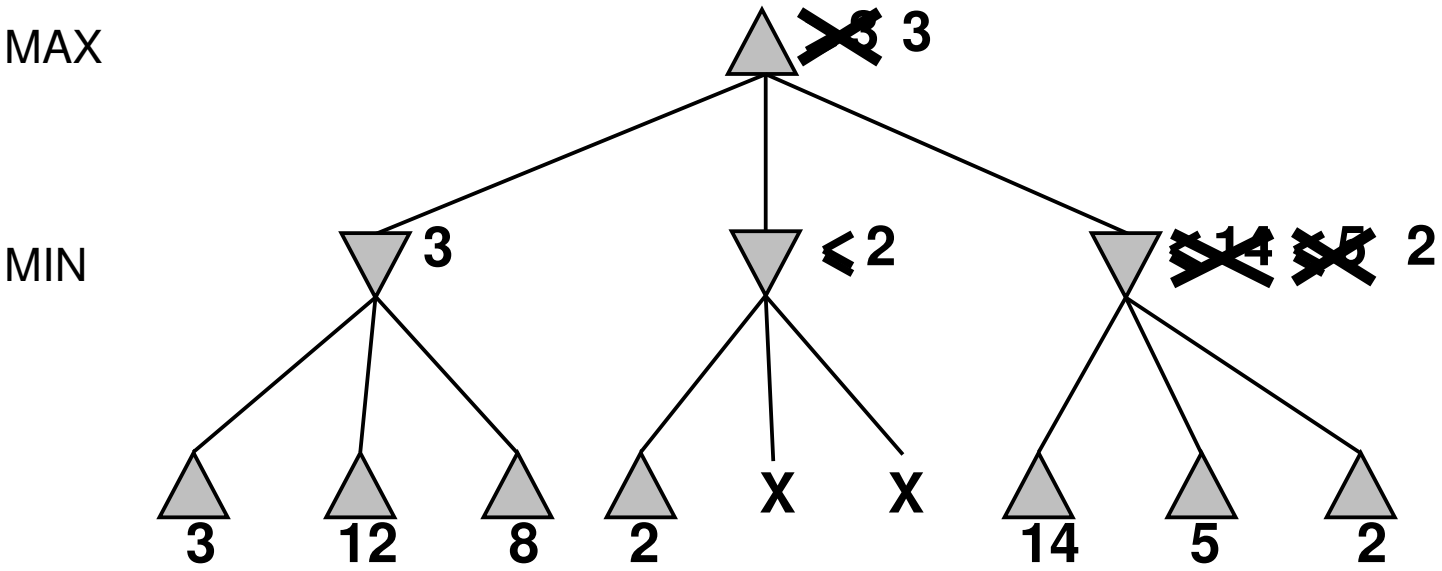
α–β pruning example



α–β pruning example



α–β pruning example



Properties of α–β

- Pruning *does* <u>*not*</u> affect final result

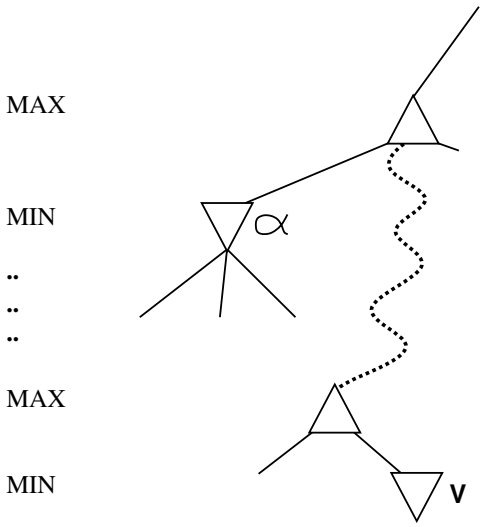
- Good move ordering improves effectiveness of pruning

- With "perfect ordering":
  time complexity = $O(b^{m/2})$
  $\Rightarrow$ *doubles* depth of search
  $\Rightarrow$ can easily reach depth 8
  $\Rightarrow$ play good chess!

- Shows value of "*metareasoning*":

  Reasoning about which computations are relevant

Why is it called α–β?



- α = best value (to max) found so far, off current path

- If V is worse than α, max will avoid it $\Rightarrow$ prune that branch

- Define β similarly for min

The α–β algorithm

- Basically Minimax + keep track of α, β + prune

function Max-Value(*state*, *game*, α, β) **returns** the minimax value of *state*
  inputs: *state*, current state in game
    game, game description
    α, the best score for max along the path to *state*
    β, the best score for min along the path to *state*

  if Cutoff-Test(*state*) **then return** Eval(*state*)
  for each *s* **in** Successors(*state*) **do**
    $\alpha \leftarrow$ Max(α, Min-Value(*s*, *game*, α, β))
    if $\alpha \geq \beta$ **then return** β
  end
  return α

---

function Min-Value(*state*, *game*, α, β) **returns** the minimax value of *state*

  if Cutoff-Test(*state*) **then return** Eval(*state*)
  for each *s* **in** Successors(*state*) **do**
    $\beta \leftarrow$ Min(β, Max-Value(*s*, *game*, α, β))
    if $\beta \leq \alpha$ **then return** α
  end
  return β

Deterministic games in practice

Checkers: *Chinook* ended 40-yr-reign of human world champion Marion Tinsley [1994].

- Endgame database for *perfect play* for *all* positions involving ≤ 8 pieces on board. . .

  . . . 443,748,401,247 positions!

Chess: *Deep Blue* defeated human world champion Gary Kasparov in 6-game match [1997].

- 200 million positions/sec
- very sophisticated evaluation
- undisclosed methods for extending some lines of search, up to 40 ply!

Othello: human champions refuse to compete against computers,

who are too good!

Go: human champions refuse to compete against computers,

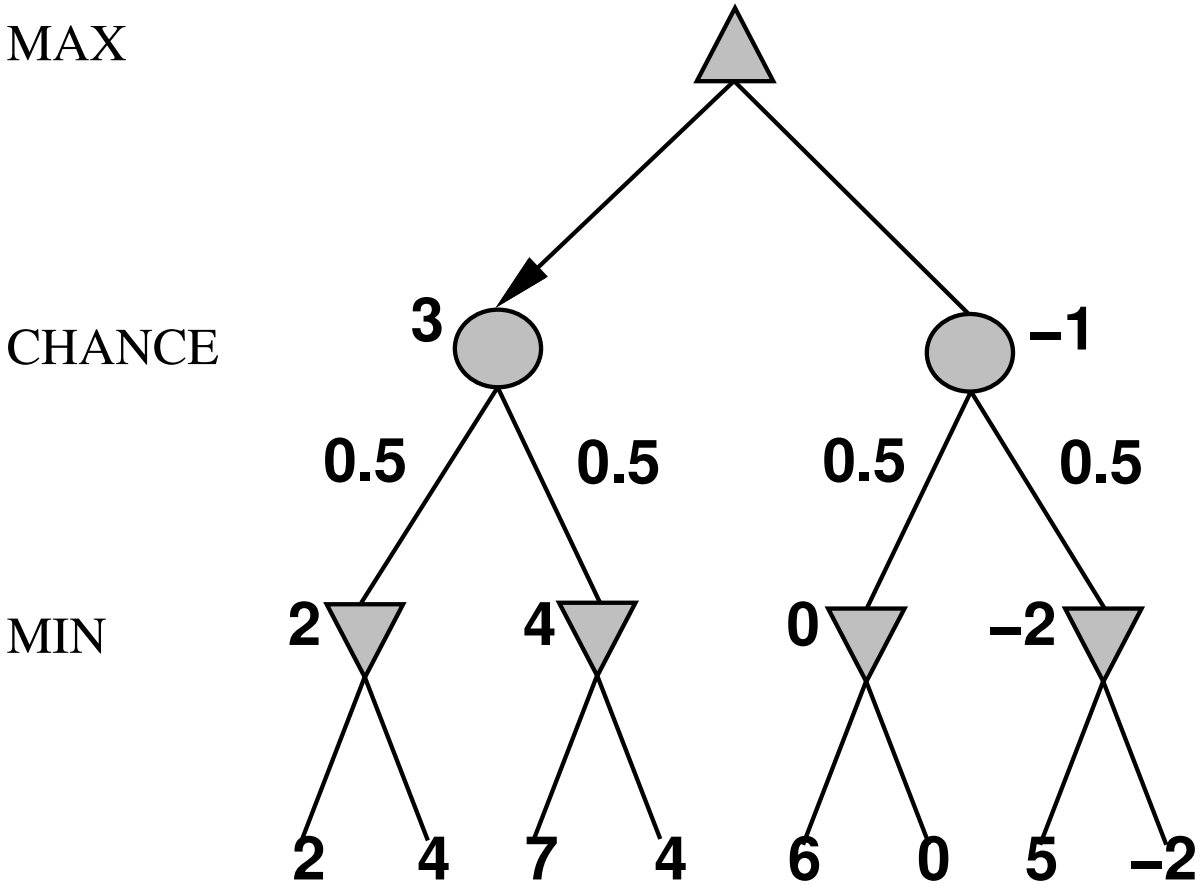
who are too bad!

$b > 300 \quad \Rightarrow$ most programs use *pattern knowledge bases* to suggest plausible moves

Nondeterministic games

- Backgammon: dice rolls determine legal moves

- Simplified example with coin-flipping instead of dice-rolling:



Algorithm for nondeterministic games

- Expectiminimax gives perfect play

  Just like Minimax, but also handles chance nodes:

  ...
  if *state* is chance node **then**
    return average of ExpectiMinimax-Value
        of Successors(*state*)
  ...

- A version of α–β pruning is possible
  (needs bounded leaf values)

Nondeterministic games in practice

- Dice rolls increase b:
  21 possible rolls with 2 dice

  Backgammon $\approx$ 20 legal moves
  (6,000 with 1-1 roll)

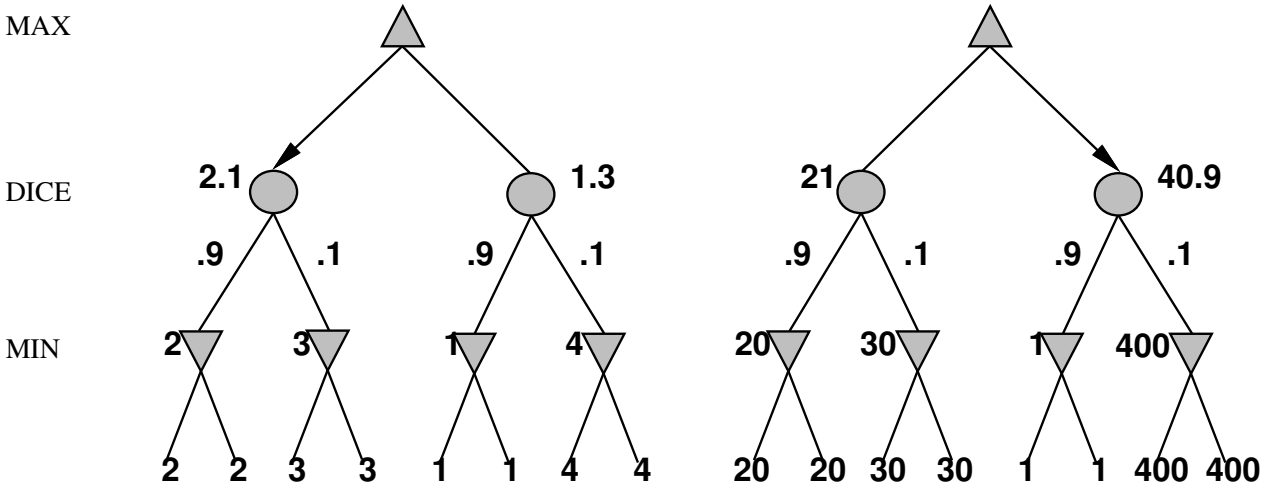
  depth 4 $\Rightarrow$ $20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$

- As depth increases,
  probability of reaching given node shrinks
  $\Rightarrow$ value of lookahead is diminished

- α–β pruning is much less effective

- TDGammon uses
  depth-2 search + very good Eval
  $\approx$ world-champion level

Digression: Exact values DO matter



- Behaviour is preserved only by
  positive linear transformation of Eval

$\Rightarrow$ Eval should be proportional to
expected payoff

Summary

- Games are fun to work on!
  . . . but dangerous. . .

- Illustrate several important points about AI
  - perfection is unattainable
    $\Rightarrow$ must approximate
  - good idea to
    think about what to think about
  - uncertainty constrains assignment of values to states

- Games are to AI
  as
  grand prix racing is to automobile design