Prolog Itself

- Based on $\boxed{\text{Refutation Resolution}}$
  which is be Sound, Complete

But. . .

- "Occurscheck"
  ⇒ not sound. . .

- Only deals with **Horn clauses**
  ≤ 1 positive literal per clause
  ⇒ not completely expressive
  no REASONING by Cases)

- Fixed **Search Strategy**: SLD
- Beyond Logic
  - Negation as Failure: /+
  - Search control: **!**
  - Operators: **assert**, **retract**, . . .

Prolog is Not Sound: Occurscheck

- Prolog does <u>not</u> insist on legal substitutions
  for efficiency reasons.

  $\boxed{\texttt{p(f(X), X)}}$ unifies w/ $\boxed{\texttt{P(Y, Y)}}$
  "succeeds" with `Y=f(f(f(f(f(f(f ...`

- "*OccursCheck*"

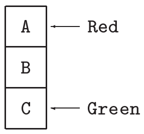
- Why?
  Unification w/o *OccursCheck* is $O(n)$
  (n = size of clause)

  Unification w/ *OccursCheck* is $O(n^2)$

But. . . ⇒ Too many clauses match
⇒ Too many conclusions reached
⇒ may conclude □ incorrectly
⇒ Not sound!

(In some Prolog's, user can reset OccursCheck)

Limitation of Horn Clauses



`(on A B)`
`(on B C)`
`(Red A)`
`(Green C)`
$\forall x. (\texttt{Red } x) \lor (\texttt{Green } x)$

Question: $\exists x, y.$ $(\texttt{on } x\ y)$ & $(\texttt{Red } x)$ & $(\texttt{Green } y)$

Answer: Yes

1. `B` is either `Red` or `Green`
2. If `B` is `Red`
   then `(x/B y/C)`
3. If `B` is `Green`
   then `(x/A y/B)`

- . . . not in Prolog.
  (Cannot express $\forall x. (\texttt{Red } x) \lor (\texttt{Green } x)$)

How to find a Proof?

- Problem
  - Easy to **recognize** a proof. . .
    Hard to **generate** a proof.
  - Many potential proofs;
    Most "paths" fail.
  - Goal: How to find a proof, efficiently?

- Good news:
  ⋆ Only one inference Rule: Resolution
  ⋆ Only considering Horn clauses
  ⇒ many "heuristics" **complete**

- Prolog's decisions. . .

General Situation

- Express $KB \cup \{\neg\sigma\}$,
  as set of clauses

- At any time, have "state":

f_1	$\cdots \vee A \vee \cdots \vee \neg B \vee \cdots$
f_2	$\cdots$
f_3	$\cdots \vee \neg A \vee \cdots$
f_4	$\cdots$
f_5	$\cdots \vee B \vee \cdots$
f_6	$\cdots$
f_7	$\cdots \vee \neg B \vee \cdots \vee Q \cdots$

- Should we smash...
  - f_1 with f_3 ?
  - f_1 with f_5 ?
  - f_5 with f_7 ?
  - ...??

Prolog's Decision

Q1: Which two clauses?

A: *Set-Of-Support (Backward)*
$\Rightarrow$ One clause is query, or descendant

View these as Goals/subGoals

- At any time, "Frontier" of SubGoals:
  Which one to work on?

A: Depth-first

Q2: Within SubGoal, `G` $= \{\ell_1, \ldots, \ell_k\}$,
which `literal` to work on?

A: *Ordered Resolution*
$\Rightarrow$ just ℓ_l

Q3: For given literal, ℓ_j in given SubGoal `G`
What to unify with?

A: Need to consider only $\left\{ \begin{array}{c} \text{fact} \\ \text{head of rule} \end{array} \right\}$ in KB

- Which rule/fact?

A: Chronological

Search Strategies

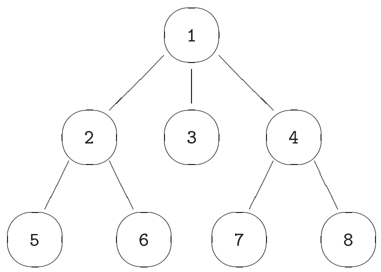
- Trying to find 1 solution to query
  (or determine that there are none)

- Which **SubGoal** to work on next?
  (Ignore order of literals within subgoal
  Ignore which fact/rule to use, ...)

- Def'n of *Level*:
  - Initial Goal is at level 0.
  - Given (sub)goal at level i,
    its subgoals are at level $i + 1$.
    (Whether by matching fact,
    or backward chaining using rule)

(If each (sub)goal has 2 (sub)subgoals,
then level j has 2^j members.)

Example of Breadth-First Search Tree



Search Strategy I: Breadth-First

- Generate all (sub)goals at level i
  before "reducing" to level $i+1$.
  - Start with Goal, γ_0, at level 0.
  - Generate ALL of γ's subgoals, $\{\gamma_1^k\}_k$
  - Generate ALL subgoals of each γ_1^k;
    producing $\{\gamma_2^l\}_l$.
  - Generate ALL of γ_2^l's subgoals, $\{\gamma_3^m\}_m$.
    ...

- Stop when subgoal is `success`,
  or when no more reductions possible.

Breadth-First Search: Summary

+ Advantages:
  If there is a proof, guaranteed to find it.
  (in fact: finds SHORTEST proof)

+ DisAdvantages:
  Needs to store entire level
  (grows exponentially)

Search Strategy II: Depth-First

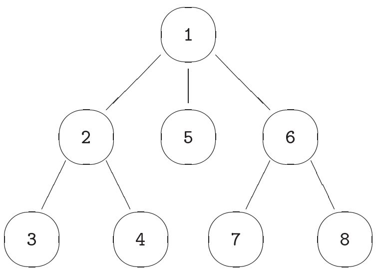
- Reduce *left-most* branch
  if `success`, then done
  if $\times$ then
  backtrack and reduce next left-most

- Until all branches done.

Given Goal, γ_0, find first solution (returning bindings).

Let *CPath* $\leftarrow$ $< \gamma_0 >$.
L: If *CPath*= $<>$ then FAIL. Otherwise:
Let *CNode* $\leftarrow$ `First`(*CPath*);
Let *CPath* $\leftarrow$ `Rest`(*CPath*).
If *CNode* = `success`, Return bindings.
Otherwise, If *CNode* = $\times$, then *Go to* L: .
Otherwise, Let $<\gamma_i>$ = Reductions of *CNode*.
Let *CPath* $\leftarrow$ *Append*($<\gamma_i>$, *CPath*)
Go to L: .

Example of Depth-First



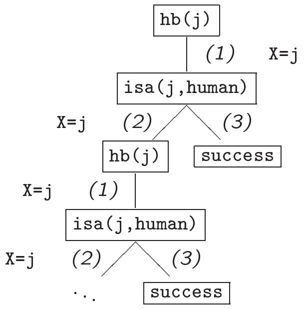
Depth-First Search: Summary

\+ Advantages:
Does NOT need to store entire level
just current brach
(grows one node at a time)

\+ DisAdvantages:
Does not find all proofs!
(Recall Ancestry example)

Infinite Loop, Prolog Style

$$KB_4 = \begin{cases} (1) & \text{hb(X) :- isa(X,human).} \\ (2) & \text{isa(X,human) :- hb(X).} \\ (3) & \text{isa(j,human).} \end{cases}$$



Which Rule/Fact?

- For each (sub)goal:
  Prolog always re-starts at beginning of DB
  [NOT where it left off.]
  uses *first* which unifies!

- To avoid Infinite Loop:
  Exchange *(2)* and *(3)*

Prolog's Decisions

- To select subgoal:
  Depth First
- To select literal in subgoal:
  Left to Right (only $i = 1$)
- To find KB's propositions
  Chronological (oldest first)
  (both atomic formulae and rules)
- Called SLD Resolution
  Linear resolution with
  Selection procedure for
  Definite clauses
  but in french

So: Prolog can miss some proofs,
but is relatively fast, and
requires relatively little storage