Programming in Prolog: List

- To form lists:
  - Special constant symbol: `[]`
  - Special function symbol: `cons`

- Like *Lisp*:

```
Lisp:    (cons 'a  (cons 'b  (cons 'c  () )))
Prolog:   cons( a,  cons( b,  cons( c, [] )))
```

(Lie)

Example: `append`

- *Prolog* uses *predicates*; not *functions*.
  Use

  `append(X,Y,Z)`

  which holds *iff* `Append` of `X` and `Y` is `Z`
  [not "`append(X,Y)`" returns "`Z`"]

- `append` is 3-place predicate:

```
1:  append(            [], Y, Y ).
2:  append( cons(E, X), Y, cons(E, Z) )
                          :- append( X,Y,Z ).
```

- Notice: If first arg is constant,
  ⇒ only 1 head will unify.

`Append` Goals

```
1:  append(            [], Y, Y ).
2:  append( cons(E, X), Y, cons(E, Z) )
                          :- append( X,Y,Z ).
```

- Goal:

  `append( [], cons(a,[]), V1)`

  succeeds with

  `V1 = cons(a,[])`

- Goal:

  `append( cons(a, cons(b,[])), cons(c, cons(d,[])), V2)`

  succeeds with

  `V2 = cons(a,cons(b,cons(c,cons(d,[]))))`

Computing `append` Values

```
append( cons(a, cons(b,[])), cons(c, cons(d,[])), V)
   (2)
     E1 = a
     X1 = cons(b, [])
     Y1 = cons(c, cons(d,[]))
     V  = cons(a, Z1)
append( cons(b,[]), cons(c, cons(d,[])), Z1 )
   (2)
     E2 = b
     X2 = []
     Y2 = cons(c, cons(d,[]))
     Z1 = cons(b, Z2)
append( [], cons(c, cons(d,[])), Z2 )
   (1)
     Y2 = cons(c, cons(d,[]))
     Z2 = cons(c, cons(d,[]))
success
```

⇒ V = cons(a, Z_1) = cons(a, cons(b, Z_2))
= cons(a, cons(b, cons(c, cons(d, []))))

Prolog's List Shorthand

- *Prolog*'s convention:

  `[s1, ···, sn | t]` is abbreviation for
  `cons(s1, cons(s2, ··· cons(sn, t) ···))`

  `[s1, ···, sn]` is abbreviation for
  `cons(s1, cons(s2, ··· cons(sn, []) ···))`
  (ie, for `[s1, ···, sn | [] ]`)

- Like *Lisp*:

  `(s1 ··· sn . s)` is evaluation of
  `(cons 's1 (cons 's2 ··· (cons 'sn 's) ···))`

  `(s1 ··· sn)` is evaluation of
  `(cons 's1 (cons 's2 ··· (cons 'sn () ) ···))`
  (ie, for `(s1 ··· sn . () ) )`

- *Lisp* ⇝ *Prolog*:
  - Change "( )" to "[ ]"
  - Add "," between args

  "(a + b * c)" ⇝ "[a, +, b, *, c]"

Alternate Description of `append`

Can use abbreviation:

```
| ?- [user].
| append( [], Y, Y ).
| append( [E | X], Y, [E | Z] ) :- append( X,Y,Z ).
| ^D user con...
yes
| ?- append( [a, b], [c, d], V ).
V = [a,b,c,d] _
yes
| ?-
```

This is SAME definition!
"Expands" into same def'n shown earlier!

Computing `append` Values

```
append( [a,b], [c,d], V )
   |  (2)
   |     E1 = a
   |     X1 = [b]
   |     Y1 = [c,d]
   |     V  = [a | Z1]
append( [b], [c,d], Z1 )
   |  (2)
   |     E2 = b
   |     X2 = []
   |     Y2 = [c,d]
   |     Z1 = [b | Z2]
append( [], [c,d], Z2)
   |  (1)
   |     Y2 = [c,d]
   |     Z2 = [c,d]
success
```

⇒ V = [a | Z1] = [a | [b | Z2]]
  = [a | [b | [c,d]]] = [a,b,c,d]

Verifying `Append` Values

Does "`append( [a,b], [c] )`" = "`[a,b,c,d]`"?

```
append( [a,b], [c], [a,b,c,d] )
   |  (2)
   |     E1 = a
   |     X1 = [b]
   |     Y1 = [c]
   |     Z1 = [b,c,d]
append( [b], [c], [b,c,d] )
   |  (2)
   |     E2 = b
   |     X2 = []
   |     Y2 = [c]
   |     Z2 = [c,d]
append( [], [c], [c,d] )
   | No possible unification!
   X
```

Using append to extract SubList

Find W s.t. `append([a], W) = [a,b,c]`

```
append( [a], W, [a,b,c] )
   |  (2)
   |    E1 = a
   |    X1 = []
   |    Y1 = W
   |    Z1 = [b,c]
append( [], W, [b,c] )
   |  (1)
   |    W = [b,c]
success
```

$\Rightarrow$ W = Y_1 = [b,c].

Similarly,

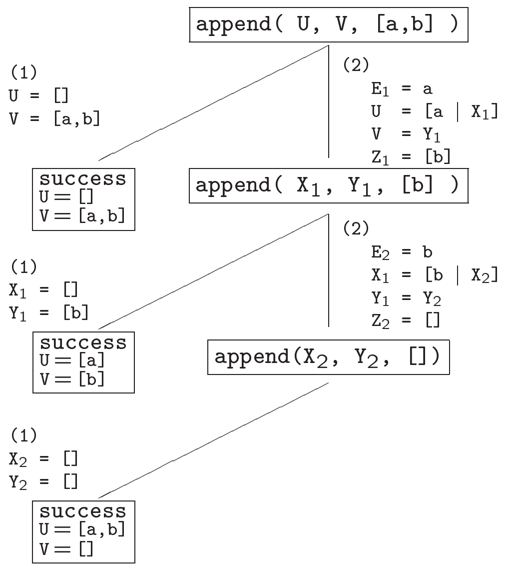
`append(D, [c,d], [a,b,c,c,d])`

succeeds with `D = [a,b,c]`.

Other uses of append

- `append([W,b,c], [d,e], [a,b,c,d,e])`
  succeeds with W = a
- `append([a,b,c], [d,e], [a,b,W,d,e])`
  succeeds with W = c
- `append([a,b,c], [d,e], [a,b | W])`
  succeeds with W = [c,d,e]
- `append([a,b,c], [d | W], [a,b,c,d,e,f])`
  succeeds with W = [e,f]
- `append([a,X,c], [Y,e], [a,b,Z,d,e])`
  succeeds with X = b, Y = d, Z = c.
- What about `append(U, V, [a,b])` ?

append(U, V, [a,b]) ?



append and Variables

- `append(A, B, [a,b,c,d,e])` succeeds with

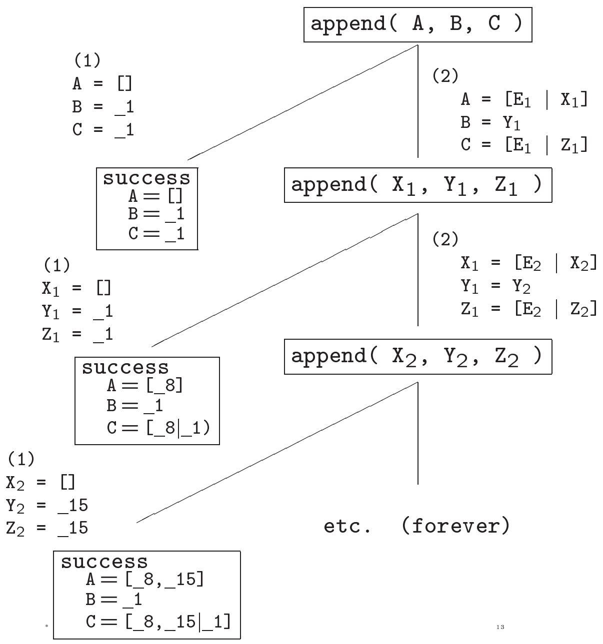
A = []	B = [a,b,c,d,e]
A = [a]	B = [b,c,d,e]
A = [a,b]	B = [c,d,e]
A = [a,b,c]	B = [d,e]
A = [a,b,c,d]	B = [e]
A = [a,b,c,d,e]	B = []

- What about `append(A, B, C)` ?

succeeds with	A=[]	B=_1	C=_1
then	A=[_8]	B=_1	C=[_8 | _1]
then	A=[_8,_15]	B=_1	C=[_8,_15| _1]

. . . forever . . .

append(A, B, C) ?



append Goals

```
| ?- [user].
| append( [], Y, Y ).
| append( [E | X], Y, [E | Z] ) :- append( X,Y,Z ).
| ^D user con...
yes
| ?- append( [a, b, c], [d, e], V ).
V = [a, b, c, d, e] _
yes
| ?- append( [a, X, c], [Y, e], [a, b, Z, d, e] ).
X = b
Y = d
Z = c ;
no                  % Says "no" as user asked for 2nd answer.
| ?- append( U, V, [a,b,c]).
U = []
V = [a,b,c] ;
U = [a]
V = [b,c] ;
U = [a,b]
V = [c] ;
U = [a,b,c]
V = [] ;
no
```


Other Uses of append

- `last` element of a list
  `last( [a,b,c], c )`
  `last( L, E ) :- append( Y, [E], L )`
- `member` of a list
  `member( c, [a,b,c] )`
  `member( E, L ) :- append( Y, [E | Z], L )`
- `prefix` at beginning of a list
  `prefix( [a,b], [a,b,c,d] )`
  `prefix( L1, L3 ) :- append( L1, L2, L3 )`
- `suffix` at end of list
  `suffix( [c,d], [a,b,c,d] )`
  `suffix( L2, L3 ) :- append( L1, L2, L3 )`
- `middle` of a list
  `middle( [a,b,c,d,e,f], [a,b], [e,f], [c,d] )`

```
middle( L, L1, L3, L2 ) :-  append( L1, L2, L12 ),
                            append( L12, L3, L ).
```


How to Solve Problems (Prolog)

- I could solve the BIG problem if...
  I could solve some specific subproblem

Eg: I could solve
`mortal(socrates)`
if I knew that
`man(socrates)`

Note just sufficiency – not necessity!

So
`mortal(socrates) :- man(socrates).`

- Generality: In fact. . .

`mortal(X) :- man(X).`

How to Solve Problems (con't)

Eg: I could solve

```
append( [E | X], Y, ??)
```

if I could solve

```
append( X, Y, Z)
```

Then answer would be

```
?? = [E | Z]
```

So

```
append( [E|X], Y, [E|Z] ) :- append( X, Y, Z ).
```

- One more thing...
  Some things are just true – "base case"

```
append( [], Y, Y ).
```


Other Examples of List Programming

- Equality of Lists

  ```
  equal(X,X).
  ```

  (Easy: unification does all of the work!)

  Note: built into *Prolog* as "="

  $\boxed{\texttt{=}(t_1,\ t_2)}$ succeeds iff t_1 and t_2 unifiable.

- List membership:

  member(X, [X | L]). *(5)*
  member(X, [Y | L]) :- member(X,L). *(6)*

  Can be used to *generate* elements of list:

  `member(X, [a,b,c])` succeeds with `X=a`
  then, if required, `X=b`
  then, if required, `X=c`.

  Useful for trying fixed set of possibilities.

Variants of `member`

- Alternative Definition #1

  member(X, [X | _]). *(1)*
  member(X, [_ |L]) :- member(X,L). *(2)*

  "_" is variable, unifies with ANYTHING

  Don't care about its value

  Each occurrance can be different.

- Alternative Definition #2

  nmember(X, [_ |L]) :- nmember(X,L). *(1)*
  nmember(X, [X | _]). *(2)*

  (Same information, different order.)

  Observe:

  – `member(a,[a,b,c,d,e])` requires 1 step
  – `nmember(a,[a,b,c,d,e])` requires 6 steps

  – `member(c,[a,b,c,d,e])` requires 3 steps
  – `nmember(c,[a,b,c,d,e])` requires 6 steps

Length of Proofs – `member`

$$KB_1 = \left\{ \begin{array}{ll} \texttt{member(X, [X | \_]).} & (1) \\ \texttt{member(X, [\_ |L]) :- member(X,L).} & (2) \end{array} \right\}$$

```
member( c, [a,b,c,d,e] )
        | (2)
        |   { X1/c, _/a, L1/[b,c,d,e] }
member( c, [b,c,d,e] )
        | (2)
        |   { X2/c, _/b, L2/[c,d,e] }
member( c, [c,d,e] )
        \   (1)
         \ { X3/c, _/[d,e] }
        success
```

Length of Proofs – nmember

$$KB_2 = \left\{ \begin{array}{ll} \texttt{nmember(X, [\_ |L]) :- nmember(X,L).} & \texttt{(1')} \\ \texttt{nmember(X, [X | \_]).} & \texttt{(2')} \end{array} \right\}$$

```
nmember( c, [a,b,c,d,e] )
        (1')
{ X1/c, _/a, L1/[b,c,d,e] }
nmember( c, [b,c,d,e] )
        (1')
{ X2/c, _/b, L2/[c,d,e] }
nmember( c, [c,d,e] )
        (1')                                  (2')
{ X3/c, _/c, L3/[d,e] }             { X3/c, _/[d,e] }
nmember( c, [d,e] )                 success
        (1')
{ X4/c, _/d, L4/[e] }
nmember( c, [] )
        (1')
{ X5/c, _/e, L5/[] }
nmember( c, [] )

FAIL
```


Big Example: Sorting

- `sort(X,Y)` holds iff
  - `Y` is a permuation of `X`
  - `Y` is in ascending order
- Perhaps:

  `sort(X,Y) :- permute(X,Y), ordered(Y).`
- *Terrible: $n!$ permutations of n-element* `X`*!*

`sort( [3,1,2], [1,2,3] )` is ok, but

`sort( [3,1,2], Y )`

⇝ `permute( [3,1,2], Y ), ordered(Y)`

```
⇝ Y / [3,1,2]
  Y / [3,2,1]
  Y / [2,3,1]
  Y / [2,1,3]
  Y / [1,3,2]
  Y / [1,2,3]
```


Quick Sort

Algorithm

Choose item in list as "pivot".
Put all items < pivot into L1.
Put all items > pivot into L2.
Sort L1, Sort L2.
Append the results (L1', pivot, L2')

In *Prolog*:

```
qsort( [], [] )           { Base case }
qsort( [Pivot|Rest], Sorted) :-
       part( Pivot, Rest, L1, L2 ),
       qsort(L1, Lessers), qsort(L2, Greaters),
       append(Lessers, [Pivot|Greaters], Sorted).

          { Now to split the list,
          into elements < and > the Pivot value }

part(Pivot, [], [], [] )

part(Pivot, [Item|Rest], [Item|L1], L2 ) :-
       lessThan(Item,Pivot), part(Pivot, Rest, L1, L2).

part(Pivot, [Item|Rest], L1, [Item|L2] ) :-
       notLessThan(Item,Pivot), part(Pivot, Rest, L1, L2).
```