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Age Analysis of Correlated Information in Multi-Source Updating Systems with MAP Arrivals

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***Abstract*—Information freshness in cyber-physical systems is essential for real-time applications that involve perception and control. Many of these applications rely on data generated by multiple correlated sources. To capture the inherent dependency in status update packets, we quantify the age of correlated information (AoCI) in a multi-source queueing system where each source generates packets following a Markovian arrival process (MAP) and these packets are subsequently served with exponentially distributed service times. Using the matrix geometric approach, we derive the sojourn time distribution in this MAP/M/1 queueing system. Furthermore, we obtain closed-form expressions for the average and peak AoCI in terms of the system parameters, considering both single and multiple source scenarios. Numerical results show the impact of correlated arrivals with different distribution properties on the achievable AoCI.**

***Index Terms*—Age of information, MAP/M/1 queueing model, correlated sources, matrix geometric approach, renewal processes.**

I. Introduction

THE advent of wireless technologies such as 5G and beyond has enabled widespread adoption of networked sensors for time-sensitive applications. Conventional performance metrics like throughput and latency fail to quantify the freshness of received information due to varying wireless conditions, prompting the introduction of *age of information* (AoI) as a destination-centric metric [1], [2]. Assuming each status update packet contains measurement data and a timestamp, AoI is formally defined as $\Delta(t) = t - u(t)$, where $u(t)$ is the timestamp of the most recently received packet at the destination. Without fresher updates, $\Delta(t)$ increases linearly with time. Upon receiving a fresher status update, the destination updates $u(t)$, reducing AoI.

Inspired by a real-time multi-view image processing application, the *age of correlated information* (AoCI) was first introduced in [3], where a complete scene image is reconstructed by a remote monitor upon receiving overlapping field-of-view images from multiple cameras. This concept proves valuable in low-latency communication systems where the AoI at the destination changes only upon receiving the fresher (partial) status updates from correlated sources. In [4], an optimal scheduling policy that jointly minimizes the average AoI and the energy cost associated with two devices observing the same physical process (hence being correlated) was proposed. A multi-armed bandit algorithm was explored in [5] to minimize the average AoI in IoT networks with stochastically identical and non-identical multi-channel setups, in the presence of correlated information sources. It has been shown in [6] that in a non-collaborative game setup, the competition among sources in an M/M/1 sensing system can be influenced by the correlation of their content. To minimize AoCI through adaptive scheduling, the challenge of latent battery states was addressed in [7] by formulating the dynamic update procedure as a partially observable Markov decision process. In [8], novel scheduling policies were explored to optimize the weighted-sum of average AoI in scenarios with time-dependent correlations.

Given the ubiquity of data dependency that exists among multiple sources in practice, the analysis of AoCI through the lens of queueing theory under generally-distributed update arrival times becomes indispensable. Such an analytical framework is essential for understanding the interplay among the number of sources, their offered traffic, and the minimum achievable AoCI. However, no such analysis, particularly within the context of a multi-source queueing system, has been done yet.

In this letter, we investigate an updating system with correlated sources, where the arrivals of status updates follow a *Markovian arrival process* (MAP). This tractable subclass of Markov renewal process, which includes Poisson, Erlang-renewal, and PH-renewal processes, is typically used to capture correlations in packet inter-arrival times, reflecting various traffic patterns [9], [10]. Our main contributions are: (i) derivation of a closed-form expression for the sojourn time distribution of update packets in a MAP/M/1 model, (ii) derivation of closed-form expressions for the average and peak AoCI metrics in single and multiple-source scenarios, and (iii) numerical evaluation of the average AoCI under different arrival processes and varying system parameters.

II. System Model

Consider a system with a set of sources $\mathcal{N}$ ($|\mathcal{N}|=N$) shown in Fig. 1. These sources can be partitioned into multiple subsets such that each subset contains correlated sources producing real-time status information pertaining to a separate and independent physical process. These status packets are then transmitted to a receiver, where they are enqueued and processed in a *first-come-first-served* (FCFS) manner, with service times following an exponential distribution parameterized by μ. Packets from each source $i\in\mathcal{N}$ arrive based on a continuous-time MAP, with a mean arrival rate of $\lambda_i > 0$. The symmetric matrix $\mathbf{X} = (x_{i,j})\in\{0,1\}^{N\times N}$ encodes the correlation relationship between any ordered pair of sources (i,j). Specifically, $x_{i,j}=1$ indicates dependency between updates from i and j implying they are in

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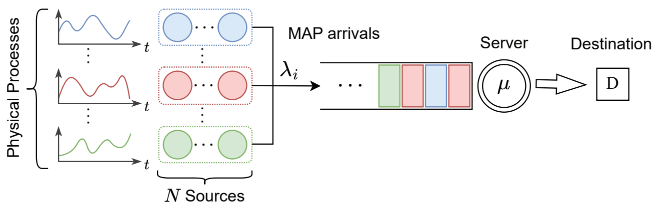

Fig. 1. Schematic of the MAP/M/1 system with MAP update arrivals from N sources, grouped by correlated data sensed from distinct physical processes.

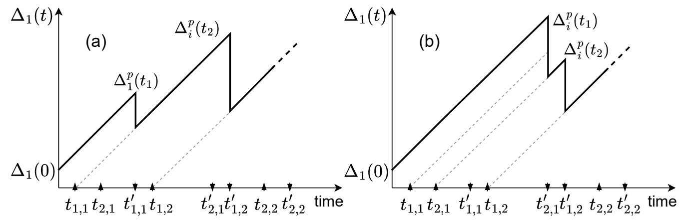

Fig. 2. Age evolution for source 1 in a two-source system with (a) uncorrelated and (b) correlated update packets generated at times $\{t_{1,1}, t_{2,1}\}$.

the same subset, and $x_{i,j} = 0$ indicates no dependency, thus updates from both sources are not required to reconstruct the real-time status of the respective processes.

A. Time-Average AoCI Definition

Let $t_{i,k}$ denote the generation timestamp of the k-th update from each $i \in \mathcal{N}$, and $t'_{i,k}$ be its arrival time at destination D. Fig. 2 illustrates the contrast between the ages under independent and correlated packets in a two-source system. For uncorrelated sources (Fig. 2a), the AoI of source i at D is given by $\Delta_i(t_k) = t'_{i,k} - t_{i,k}$, where $t_k \in [t'_{i,k}, t_{i,k+1})$ [3]. When the queue is empty at $t = 0$, the age is $\Delta_i(0)$. To quantify the AoI for any arbitrary pair of correlated sources (i, j), one must account for the temporal difference between the update received at the destination and the generation time of the first packet in the pair. As depicted in Fig. 2b, assuming $t_{i,k} < t_{j,k}$, the AoCI for source i at time t_k is computed as $\Delta_i(t_k) = t'_{j,k} - t_{i,k}$, where $t_k \in [t'_{j,k}, t_{i,k+1})$ and $t'_{j,k} > t'_{i,k}$. In either case, the status age at D increases linearly as packets arrive and the limiting average age for the updates from source i over the interval $(0, T)$ is given as [11], [12]:

$$\bar{\Delta}_i = \lim_{T\to\infty} \frac{1}{T}\int_0^T \Delta_i(t)\, dt = \left(\mathbb{E}[ZW] + \mathbb{E}[Z^2]/2\right) / \, \mathbb{E}[Z], \quad (1)$$

where Z denotes the inter-arrival time between consecutive updates from source i, and W represents the sojourn (waiting) time, which includes both the queueing time and the subsequent service time for such update packets.

B. Single-Source MAP/M/1 Model Formulation

Consider a two-dimensional stochastic process $\{N_p(t), A(t)\}$ on the state space $\Omega = \{(k, i); k \geq 0, 1 \leq i \leq m\}$ where $N_p(t)$ is the number of update packet arrivals from the single source in $(0, t]$ and $A(t)$ represents the phase of the arrival process. For any phase $i \in A(t)$, the Poisson arrival rate is $\gamma_i = \sum_{j=1}^{m} \gamma_{i,j}$. A state transition occurs in two cases: (i) with rate $\gamma_{i,j}$, there is a transition from state (k, i) to state $(k+1, j)$ due to an update arrival and possibly a phase shift, and (ii) with rate $\alpha_{i,j}$, there is a transition from state (k, i) to state (k, j) (where $i \neq j$) due to a phase shift. The phase process is a continuous-time Markov chain (CTMC) with an irreducible generator $\mathbf{D} = \mathbf{D_0} + \mathbf{D_1}$, where $\mathbf{D_0} = (d_{i,j}^{(0)})$ and $\mathbf{D_1} = (d_{i,j}^{(1)})$ are m-order matrices defined as [10]:

$$d_{i,j}^{(0)} = \begin{cases} -\gamma_i - \sum_{j\neq i} \alpha_{i,j}, & \text{if } j = i; \\ \alpha_{i,j}, & \text{if } j \neq i, \end{cases} \quad \text{for } 1 \leq i, j \leq m, \quad (2)$$

$$d_{i,j}^{(1)} = \gamma_{i,j}, \quad \text{for } 1 \leq i, j \leq m. \quad (3)$$

Matrix $\mathbf{D_0}$ contains negative diagonal elements and non-negative off-diagonal elements, reflecting phase transitions without packet arrivals, while $\mathbf{D_1}$ comprises non-negative elements corresponding to arrival rates in the m states. The infinitesimal generator matrix $\mathbf{Q}^*$ for the quasi-birth-death (QBD) process of this MAP/M/1 model exhibits the following structure, where $\mathbf{A_0} = \mathbf{D_1}$, $\mathbf{B_0} = \mathbf{D_0}$, $\mathbf{A_1} = \mathbf{D_0} - \mu\mathbf{I}$, $\mathbf{A_2} = \mu\mathbf{I}$, and $\mathbf{I}$ is the identity matrix:

$$\mathbf{Q}^* = \begin{bmatrix} \mathbf{B_0} & \mathbf{A_0} & \cdots & \cdots \\ \mathbf{A_2} & \mathbf{A_1} & \mathbf{A_0} & \cdots \\ \vdots & \ddots & \ddots & \vdots \end{bmatrix}. \quad (4)$$

Let $\mathbf{A} = \mathbf{A_0} + \mathbf{A_1} + \mathbf{A_2}$ be the stochastic matrix and $\mathbf{v}$ be the steady-state probability vector of the generator $\mathbf{A}$ satisfying:

$$\mathbf{v}\,\mathbf{A} = 0; \quad \mathbf{v}\,\mathbf{e} = 1, \quad (5)$$

where $\mathbf{e}$ is the unit column vector. The system is stable if and only if $\mathbf{v}\mathbf{A_0}\,\mathbf{e} < \mathbf{v}\mathbf{A_2}\,\mathbf{e}$. Moreover, the rate of update arrivals per unit time is given by $\lambda = \mathbf{v}\,\mathbf{D_1}\,\mathbf{e}$. Consequently, the invariant probability vector for $\mathbf{Q}^*$, represented as $\boldsymbol{\pi} = (\boldsymbol{\pi_0}, \boldsymbol{\pi_1}, \ldots, \boldsymbol{\pi_m})$, is the unique positive solution to the following system [13]:

$$\begin{cases} \boldsymbol{\pi_0}(\mathbf{B_0} + \mathbf{R}\mathbf{A_2}) = 0, \\ \boldsymbol{\pi_i} = \boldsymbol{\pi_0}\mathbf{R}^i, \quad i \geq 1, \\ \boldsymbol{\pi_0}(\mathbf{I} - \mathbf{R})^{-1}\mathbf{e} = 1, \end{cases} \quad (6)$$

where $\mathbf{R}$ is the minimal non-negative solution to the matrix equation $\mathbf{R}^2\mathbf{A_2} + \mathbf{R}\mathbf{A_1} + \mathbf{A_0} = 0$ and $\boldsymbol{\pi_i}$, for $i \geq 0$, is a stationary vector of dimension m.

C. Multi-Source MAP/M/1 Model Formulation

The traffic originating from N sources, with dependency matrix $\mathbf{X}$, can be effectively modeled using an extended MAP characterized by matrices $(\mathbf{D_0}, \mathbf{D_1}, \mathbf{D_2}, \ldots, \mathbf{D}_N)$, each of order m. Here, $\mathbf{D_0}$ governs transitions for no arrivals, while $\mathbf{D}_i$ governs transitions for update arrivals from source $i \in \mathcal{N}$. The associated CTMC has $\mathbf{D} = \mathbf{D_0} + \sum_{i=1}^{N} \mathbf{D}_i$ and the packet arrival rate per unit time from source i is $\lambda_i = \mathbf{v}\mathbf{D}_i\,\mathbf{e}$, with the total rate $\lambda = \sum_{i=1}^{N} \lambda_i$. The infinitesimal generator $\mathbf{Q}^*$ maintains a similar structure as in (4), with sub-matrices $\mathbf{A_0} = \sum_{i=1}^{N} \mathbf{D}_i$, $\mathbf{A_2} = \mu\mathbf{I}$, $\mathbf{A_1} = (\mathbf{D_0} - \mu)\mathbf{I}$, and $\mathbf{B_0} = \mathbf{D_0}$. Moreover, the stochastic matrix $\mathbf{A} = \mathbf{A_0} + \mathbf{A_1} + \mathbf{A_2}$ and $\mathbf{v}$ satisfy (5). Solving the same system as in (6), the steady-state probability vector $\boldsymbol{\pi}$ can be determined.

III. Average AoCI Analysis

Building on the structural findings from the previous section, we now derive the sojourn time distribution and average AoCI in closed form for both single and multiple source scenarios.

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A. AoCI Derivation for Single-Source Model

To obtain the sojourn time, we define L as the number of update packets in the system just before the arrival of a new packet, and S_k as the exponential service time of the k-th packet. The packet currently in service has a residual service time instead of an exponential service time with a mean of $1/\mu$. As a result, S_k are identically distributed independent random variables and the sojourn time random variable (W) includes the service time of the packet in service. Hence, W follows the Erlang-$(L+1)$ distribution with mean $1/\mu$, i.e., $W=\sum_{k=1}^{L+1} S_k$. Conditioning on L and using the independence between S_k and L, the density of the sojourn time distribution can be expressed in terms of the following matrix exponential function:

$$f_W(t)=\sum_{n=0}^{\infty} f_{W|L=n}(t)\Pr(L=n)=\sum_{n=0}^{\infty}\mu\frac{(\mu t)^n}{n!}e^{-(\mu t)}(\boldsymbol{\pi_0}\mathbf{R}^n\mathbf{e}) = \boldsymbol{\pi_0}\,\mu\, e^{-\mu(\mathbf{I}-\mathbf{R})t}\,\mathbf{e}\,. \tag{7}$$

The cumulative distribution function (CDF) and probability density function (PDF) of the inter-arrival time (Z) for a packet in a MAP/M/1 queue are respectively given as [14]:

$$\begin{cases} F_Z(t)=\mathbf{v}[\mathbf{I}-e^{\mathbf{D_0}t}](-\mathbf{D_0})^{-1}\mathbf{D_1}\mathbf{e}; & \text{for } t\geq 0,\\ f_Z(t)=\mathbf{v}\,e^{\mathbf{D_0}t}\,\mathbf{D_1}\,\mathbf{e}; & \text{for } t\geq 0.\end{cases} \tag{8}$$

Using (8), the moments for Z are obtained to be:

$$\begin{cases} \mathbb{E}[Z]=\int_{-\infty}^{\infty} z f_Z(t)\,dt=\dfrac{\mathbf{v}\mathbf{D_1}(-\mathbf{D_0})^{-1}\,\mathbf{e}}{\mathbf{v}\mathbf{D_1}\,\mathbf{e}}=\dfrac{1}{\lambda},\\ \mathbb{E}[Z^2]=\int_{-\infty}^{\infty} z^2 f_Z(t)\,dt=\dfrac{2\,\mathbf{v}(-\mathbf{D_0})^{-1}\,\mathbf{e}}{\lambda}\,.\end{cases} \tag{9}$$

Considering the service times as i.i.d. exponential random variables with an average of $1/\mu$, the sojourn time for update packet k is $W_k{=}W_k^Q+S_k$, where W_k^Q and S_k denote the waiting and service times for packet k, respectively. If $W_k^Q{=}0$ then packet $k-1$ was served before packet k was generated. At the arrival of the k-th packet, $k-1$ packets are in queue or being served, making $W_k^Q{=}(W_{k-1}-Z_k)^+$. Thus, the conditional expected waiting time for the k-th packet, given $Z_k{=}x$, is:

$$\begin{aligned}\mathbb{E}[W_k^Q|Z_k=x]&=\mathbb{E}[(W_{k-1}-x)^+|Z_k=x]=\mathbb{E}\left[(W-x)^+\right]\\ &=\int_x^{\infty}(t-x)f_W(t)dt\\ &=\frac{1}{\mu}\boldsymbol{\pi_0}e^{-\mu(\mathbf{I}-\mathbf{R})x}(\mathbf{I}-\mathbf{R})^{-2}\,\mathbf{e}\,.\end{aligned} \tag{10}$$

From (9), we derive the expression for $\mathbb{E}[Z_kW_k]$, under the assumption that the service time S_k and Z_k are independent:

$$\begin{aligned}\mathbb{E}\left[Z_kW_k\right]&=\mathbb{E}[Z_k(W_k^Q+S_k)]=\mathbb{E}[Z_kW_k^Q+Z_kS_k]\\ &=\mathbb{E}[Z_kW_k^Q]+\mathbb{E}[Z_k]\,\mathbb{E}[S_k].\end{aligned} \tag{11}$$

Theorem 1. *The closed-form expression for $\mathbb{E}[Z_kW_k^Q]$ is:*

$$\mathbb{E}[Z_kW_k^Q]=\frac{1}{\mu^2}\boldsymbol{\pi_0}\mathbf{R}\mathbf{v}\mathbf{D_1}(\mathbf{I}-\mathbf{R}-\mathbf{D_0})^{-2}(\mathbf{I}-\mathbf{R})^{-2}\,\mathbf{e}\,. \tag{12}$$

Proof. Using the PDF of the inter-arrival time given in (8) and (10), $\mathbb{E}[Z_kW_k^Q]$ can be reformulated as:

$$\begin{aligned}\mathbb{E}[Z_kW_k^Q]&=\int_0^{\infty} t\,\mathbb{E}[W_i^Q|Z_i=t]f_{Z_i}(t)\,dt\\ &=\int_0^{\infty} t\left[\frac{1}{\mu}\boldsymbol{\pi_0}e^{-\mu(\mathbf{I}-\mathbf{R})t}(\mathbf{I}-\mathbf{R})^{-2}\mathbf{e}\right]\!\left(\mathbf{v}e^{\mathbf{D_0}t}\,\mathbf{D_1}\mathbf{e}\right)dt,\end{aligned}$$

which after integration yields (12). □

Applying Theorem 1, we now articulate the average and peak AoCI $(\bar{\Delta}^p)$ for the single-source scenario in Theorem 2 below.

Theorem 2. *The closed-form expressions for the average and peak AoCI for a single-source MAP/M/1 model are given as:*

$$\begin{cases}\bar{\Delta}=\dfrac{\lambda}{\mu^2}\boldsymbol{\pi_0}\mathbf{R}\mathbf{v}\mathbf{D_1}(\mathbf{I}-\mathbf{R}-\mathbf{D_0})^{-2}(\mathbf{I}-\mathbf{R})^{-2}\mathbf{e}+\dfrac{1}{\mu}+\mathbf{v}(-\mathbf{D_0})^{-1}\mathbf{e},\\ \bar{\Delta}^p=\dfrac{1}{\lambda}\left[1+\boldsymbol{\pi_0}\mathbf{R}(\mathbf{I}-\mathbf{R})^{-2}\,\mathbf{e}\right].\end{cases} \tag{13}$$

Proof. Upon substituting (11) in (1), we obtain:

$$\bar{\Delta}=\lambda(\mathbb{E}[Z_kW_k^Q]+\mathbb{E}[Z_k]\,\mathbb{E}[S_k]+\mathbb{E}[Z_k^2]/2).$$

Then using (9) and (12), along with $\mathbb{E}[S_k]{=}1/\mu$, leads to the closed-form expression for $\bar{\Delta}$. Peak AoCI is the maximum value of the age achieved before the latest update delivery. From (4) and Little's law, the average waiting time is computed as follows, where N_p denotes the number of packets in the system:

$$\mathbb{E}[W_k]=\frac{\mathbb{E}[N_p]}{\lambda}=\frac{\sum_{n=1}^{\infty} n\boldsymbol{\pi_n}}{\lambda}=\frac{\boldsymbol{\pi_0}\mathbf{R}(\mathbf{I}-\mathbf{R})^{-2}\,\mathbf{e}}{\lambda}. \tag{14}$$

Summation of $\mathbb{E}[Z_k]$ from (9) and $\mathbb{E}[W_k]$ from (14) yields $\bar{\Delta}^p$, which completes the proof. □

B. AoCI Derivation for Multi-Source Model

To calculate the sojourn time under correlated sources, we define $S_{i,k}$ as the exponential service time of the k-th packet from source i. For any two sources $i,j\in\mathcal{N}$, $x_{i,j}$ essentially signifies if the current update at i contains information dependent on updates from j during $S_{i,k}$. Given this dependency matrix $\mathbf{X}$ and the fact that $S_{i,k}$'s are i.i.d., the sojourn time (W_i) includes the service time of the update from i. Consequently, W_i conforms to the Erlang-$(L+1)$ distribution with a mean of $1/(\mu\sum_{j=1}^{N} x_{j,i})$:

$$W_i=\sum_{k=1}^{L+1}\sum_{j=1}^{N} S_{i,k}\,x_{j,i}\,,\quad i\in\mathcal{N}, \tag{15}$$

and the conditional density of W_i given L is derived to be:

$$\begin{aligned}f_{W_i}(t)&=\sum_{n=0}^{\infty}\sum_{j=1}^{N}\mu\frac{(\mu\sum_{j=1}^{N} x_{i,j}t)^n}{n!}e^{-(\mu\sum_{j=1}^{N} x_{i,j}t)}(\boldsymbol{\pi_0}\mathbf{R}^n\,\mathbf{e})\\ &=\boldsymbol{\pi_0}\,\mu\sum_{j=1}^{N} x_{i,j}\,e^{-\mu\sum_{j=1}^{N} x_{i,j}(\mathbf{I}-\mathbf{R})t}\,\mathbf{e}\,.\end{aligned} \tag{16}$$

Moreover, the inter-arrival time of an update packet from source i (Y_i) has the following moments:

$$\mathbb{E}[Y_i]=\frac{\mathbf{v}\mathbf{D}_i(-\mathbf{D_0})^{-1}\mathbf{e}}{\mathbf{v}\mathbf{D}_i\,\mathbf{e}}=\frac{1}{\lambda_i},\quad \mathbb{E}[Y_i^2]=\frac{2\,\mathbf{v}(-\mathbf{D_0})^{-1}\mathbf{e}}{\lambda_i}. \tag{17}$$

By applying (15), (16), and (17), the closed-form expressions for $\bar{\Delta}_i$ and $\bar{\Delta}_i^p$ for any arbitrary source i are derived to be:

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$$\begin{cases} \bar{\Delta}_i = \dfrac{\lambda_i \boldsymbol{\pi}_\mathbf{0}\, \mathbf{R}\, \mathbf{v}\mathbf{D}_i(\mathbf{I}-\mathbf{R}-\mathbf{D_0})^{-2}(\mathbf{I}-\mathbf{R})^{-2}\,\mathbf{e}}{\left(\mu \sum_{j=1}^{N} x_{j,i}\right)^2} \\ \qquad + \dfrac{1}{\mu \sum_{j=1}^{N} x_{ji}} + \mathbf{v}(-\mathbf{D_0})^{-1}\mathbf{e}, \\ \bar{\Delta}_i^p = \dfrac{1}{\lambda_i}\left[1 + \boldsymbol{\pi}_\mathbf{0}\, \mathbf{R}(\mathbf{I}-\mathbf{R})^{-2}\,\mathbf{e})\right]. \end{cases} \tag{18}$$

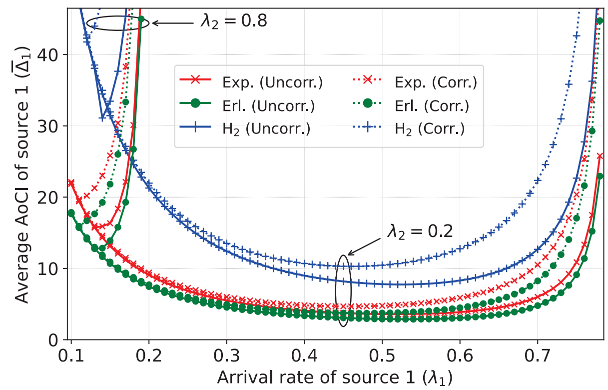
IV. Numerical Results and Discussions

We validate our results for the average AoCI and show the impact of correlated sources under varying model parameters. For the arrival process, we examine the following five distinct sets of values for $\mathbf{D_0}$ and $\mathbf{D_1}$, as considered in [9], [15] and [16]:

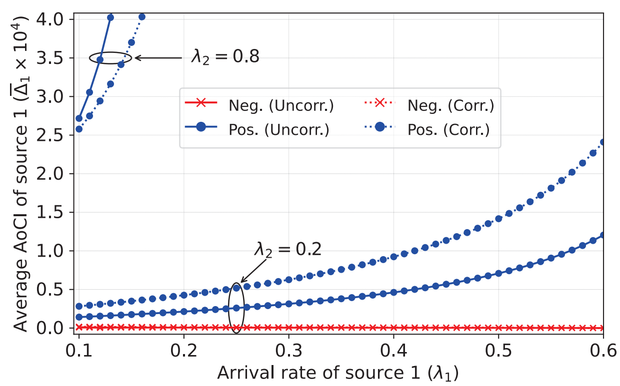
- **Exp.**: The classical Poisson process with parameter λ, such that $\mathbf{D_0} = [-\lambda]$ and $\mathbf{D_1} = [\lambda]$.
- **Erl.**: The Erlang inter-arrival time distribution of order 2 with parameter λ, such that $\mathbf{D_0} = [-\lambda, \lambda; 0, -\lambda]$ and $\mathbf{D_1} = [0, 0; \lambda, 0]$.
- **H_2**: The combination of two exponential distributions (hyperexponential), such that $\mathbf{D_0} = [-1.9\lambda, 0; 0, -0.19\lambda]$ and $\mathbf{D_1} = [1.71\lambda, 0.19\lambda; 0.171\lambda, 0.019\lambda]$, with mixing probability $p = 0.9$.
- **Neg.**: A MAP exhibiting negative correlation between consecutive inter-arrival times, with $\mathbf{D_0} = \lambda[-1.0024, 1.0024, 0; 0, -1.0024, 0; 0, 0, -225.797]$ and $\mathbf{D_1} = \lambda[0, 0, 0; 0.01002, 0, 0.9924; 223.539, 0, 2.258]$.
- **Pos.**: A MAP with positive correlation between consecutive inter-arrival times, where $\mathbf{D_0} = \lambda[-1.0024, 1.0024, 0; 0, -1.0024, 0; 0, 0, -225.797]$ and $\mathbf{D_1} = \lambda[0, 0, 0; 0.9924, 0, 0.01002; 2.258, 0, 223.539]$.

At λ=1, the five arrival processes exhibit standard deviations of 1, 0.5, 2.245, 1.4095, and 1.4095, respectively. The corresponding 1-lag correlation coefficients for successive inter-arrival times are 0, 0, 0, -0.4889, and 0.4889.

Fig. 3 shows the impact of data dependency in a two-source system by comparing the average AoCI of source 1 under both uncorrelated ($x_{1,2}$=0) and correlated ($x_{1,2}$=1) scenarios across various inter-arrival time distributions, with μ=1 and λ_2=$\{0.2, 0.8\}$. As evident in Fig. 3a, Erl. yields the lowest average age among the three renewal distributions, with λ_1^*=0.53 for uncorrelated and λ_1^*=0.47 for correlated sources, attributed to its low variability in inter-arrival times. Moreover, H_2, with a coefficient of variation greater than 1, consistently yields higher AoCI values. To account for traffic models with statistically significant correlations across large time scales, Fig. 3b contrasts Neg. and Pos., characterized by dependent packet inter-arrival times with the same variance. Under Neg. arrivals and for λ_2=0.2, the average AoCI decreases to a minimum of 0.95 at λ_1^*=0.6 when the two sources generate correlated updates, marking a 2.7% increase compared to scenarios where the sources update independently. This is due to the formation of clusters of short and long inter-arrival times in Pos. arrivals, resulting in higher AoCI, while Neg. arrivals exhibit the opposite trend. That is to say, Pos. arrivals tend to increase AoCI due to short inter-arrival times being followed by another short one, leading to abrupt spikes, whereas Neg. arrivals show a reduction in average AoCI, as short intervals are typically followed by longer ones.



(a) Comparison of $\bar{\Delta}_1$ among the three renewal processes.



(b) Comparison of $\bar{\Delta}_1$ between Neg. and Pos. processes.

Fig. 3. The average AoCI of source 1 against λ_1 in a two-source system, where λ_2=$\{0.2, 0.8\}$ and μ=1, illustrating the impact of correlated sources under (a) independent and (b) dependent inter-arrival time distributions.

Fig. 4 compares the peak AoCI for the same two-source model, highlighting the maximum AoCI achievable within the stable range of λ_1 (i.e., $\sum_{i \in \mathcal{N}} \lambda_i < \mu$) for low and high traffic rates from source 2. Both Fig. 4a and Fig. 4b show a substantial increase in the peak AoCI when $x_{1,2}$=1 for Exp. and H_2 distributions under λ_2=0.8. Specifically, in Fig. 4a, these peak age values reach 1235 and 3682, respectively (not shown). In Fig. 4b, with Pos. arrivals, the minimum peak AoCI reaches 3388 at λ_1^*=0.1 for λ_2=0.2, signifying a 98% increase compared to the uncorrelated scenario. Similar to Fig. 3b, $\bar{\Delta}_1^p$ becomes almost negligible for higher λ_1 rates.

Fig. 5 illustrates the impact of data dependency for larger N values, comparing average AoCI for N=$\{4, 6\}$. This comparison is based on matrix $\mathbf{X}$, where sources $\{1, 2\}$, $\{2, 4, 6\}$, and $\{3, 4\}$ are interdependent, while 5 is independent of the others. As seen in Fig. 5a, the minimum average age for source 1 increases by nearly 49% as N goes from 4 to 6 in the Erl. distribution. In line with findings from Fig. 3a, the H_2 distribution shows the largest increase, approximately 60.9%, for the same rise in N. An evident trend emerges as N increases, showing a corresponding rise in dependency between source pairs. Hence, the arrival rates conducive to achieving the minimum AoCI also decreases. For instance, in Erl., increasing N from 4 to 6 reduces λ_1^* from 0.25 to 0.22. In Fig. 5b, a slight increase in the minimum achievable AoCI for Neg. is observed as N increases. This is consistent with the trend observed for i.i.d. arrivals within the system stability range. For Pos., however, this increase is about

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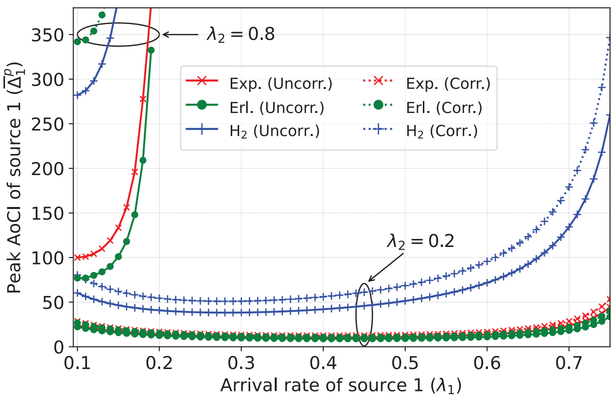
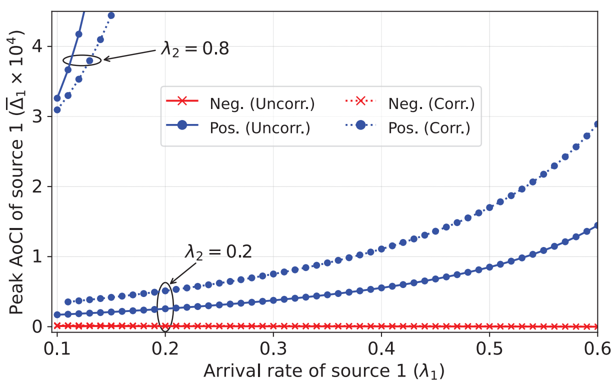
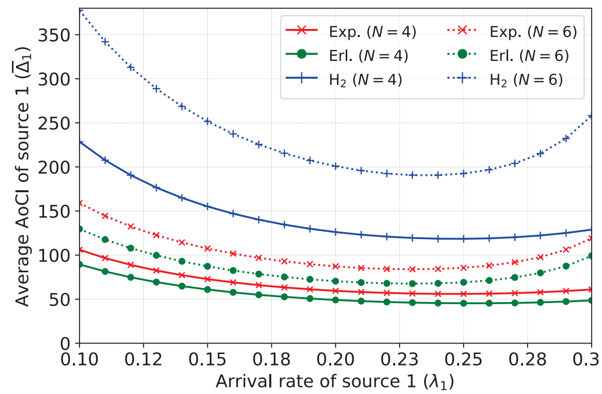


(a) Comparison of $\bar{\Delta}_1^p$ among the three renewal processes.

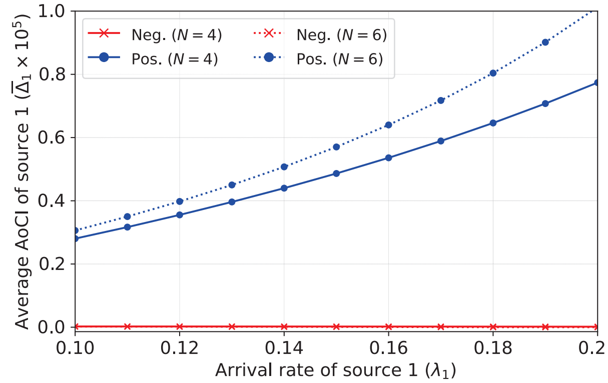


(b) Comparison of $\bar{\Delta}_1^p$ between Neg. and Pos. processes.

Fig. 4. The peak AoCI of source 1 against λ_1 in a two-source system, where λ_2=$\{0.2, 0.8\}$ and μ=1, illustrating the impact of correlated sources under (a) independent and (b) dependent inter-arrival time distributions.



(a) Comparing $\bar{\Delta}_1$ across three renewal processes with varying N.



(b) Comparing $\bar{\Delta}_1$ between Neg. and Pos. processes for varying N.

Fig. 5. The average AoCI of source 1 against λ_1 for $|\mathcal{N}| = \{4, 6\}$, where $\forall i \in \mathcal{N}\backslash 1, \lambda_i = 0.3/(i-1)$ and μ=1 under (a) independent and (b) dependent inter-arrival time distributions. Here, $x_{1,2} = x_{2,4} = x_{2,6} = x_{3,4} = 1$.

9.2% which occurs at a relatively lower value of λ_1=0.1 as compared to all other distributions.

V. Conclusion

In this article, we examined the average and peak AoCI for a multi-source MAP/M/1 queueing system with correlated packet arrivals and data correlation among sources, deriving the closed-form stationary and sojourn time distributions using the matrix geometric approach. We validated the analytical results and compared the minimum achievable average AoCI across correlated sources, considering updates with different inter-arrival time distributions. A potential future direction is the analysis of AoCI under time-varying data dependency between sources in mobile and energy-harvesting updating systems.

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