Region Segmentation

- Connected components analysis often results in many small disjointed regions.

- A connection (or break) at a single pixel can split (or merge) entire regions.

- There are three basic approaches to segmentation:
  - Region Merging - recursively merge regions that are similar.

  ![Region Merging Diagram](image1)

  - Region Splitting - recursively divide regions that are heterogeneous.

  ![Region Splitting Diagram](image2)

  - Split and merge - iteratively split and merge regions to form the “best” segmentation.
Hierarchical Clustering

- Clustering refers to techniques for separating data samples into sets with distinct characteristics.

- Clustering methods are analogous to segmentation methods.
  - Agglomerative clustering - “bottom up” procedure for recursively merging clusters ⇒ analogous to region merging
  - Divisive clustering - “top down” procedure for recursively splitting clusters ⇒ analogous to region splitting
Image Regions and Partitions

- Let $R_m \subset S$ denote a region of the image where $m \in \mathcal{M}$.
- We say that $\{R_m | m \in \mathcal{M}\}$ partitions the image if
  
  \[
  \text{For all } m \neq k, \quad R_m \cap R_k = \emptyset \\
  \bigcup_{m \in \mathcal{M}} R_m = S
  \]

- Each region $R_m$ has features that characterize it.
Typical Region Features

- **Color**
  - Mean RGB value
  - 1-D color histograms in R, G, and B
  - 3-D color histogram in (R,G,B)

- **Texture**
  - Spatial autocorrelation
  - Joint probability distribution for neighboring pixels (e.g. the spatial co-occurrence matrix)
  - Wavelet transform coefficients

- **Shape**
  - Number of pixels
  - Width and height attributes
  - Boundary smoothness attributes
  - Adjacent region labels
Recursive Feature Computation

- Any two regions may be merged into a new region.
  \[ R_{new} = R_k \cup R_l \]

- Let \( f_n = f(R_n) \in \mathbb{R}^k \) be a \( k \) dimensional feature vector extracted from the region \( R_n \).

- Ideally, the features of merged regions may be computed without reference to the original pixels in the region.
  \[
  f(R_k \cup R_l) = f(R_k) \oplus f(R_l)
  \]
  \[
  f_{new} = f_k \oplus f_l
  \]

  here \( \oplus \) denotes some operation on the values of the two feature vectors.
Example of Recursive Feature Computation

Example: Let $f(R_k) = (N_k, \mu_k, c_k)$ where

$$N_k = |R_k|$$

$$\mu_k = \frac{1}{N_k} \sum_{s \in R_k} x_s$$

$$c_k = \frac{1}{N_k} \sum_{s \in R_k} s$$

We may compute the region features for $R_{new} = R_k \cup R_l$ using the recursions

$$N_{new} = N_k + N_l$$

$$\mu_{new} = \frac{N_k \mu_k + N_l \mu_l}{N_{new}}$$

$$c_{new} = \frac{N_k c_k + N_l c_l}{N_{new}}$$
Recursive Merging

- Define a distance function between regions. In general, this function has the form

\[ d_{k,l} = D(R_k, R_l) \geq 0 \]

- Ideally, \( D(R_k, R_l) \) is **only** a function of the feature vectors \( f_k \) and \( f_l \).

\[ d_{k,l} = D(f_k, f_l) \geq 0 \]

- Then merge regions with minimum distance.
Example of Merging Criteria

- Distance between color means
  \[ d_{k,l} = \frac{N_k}{N_{new}}|\mu_k - \mu_{new}|^2 + \frac{N_l}{N_{new}}|\mu_l - \mu_{new}|^2 \]

- Distance between region centers
  \[ d_{k,l} = \frac{N_k}{N_{new}}|c_k - c_{new}|^2 + \frac{N_l}{N_{new}}|c_l - c_{new}|^2 \]

- Distance formed by a weighted combination of the two
  \[ d_{k,l} = \alpha \left( \frac{N_k}{N_{new}}|\mu_k - \mu_{new}|^2 + \frac{N_l}{N_{new}}|\mu_l - \mu_{new}|^2 \right) \\
  + \beta \left( \frac{N_k}{N_{new}}|c_k - c_{new}|^2 + \frac{N_l}{N_{new}}|c_l - c_{new}|^2 \right) \]
Recursive Merging Algorithm

- Define a distance function between regions
  \[ d_{k,l} = D(f(R_k), f(R_l)) > 0 \]

Repeat until \(|\mathcal{M}| = 1\) {
  
  Determine the minimum distance regions
  \[ (k^*, l^*) = \arg \min_{k,l \in \mathcal{M}} \{d_{k,l}\} \]

  Merge the minimum distance regions
  \[ R_{k^*} \leftarrow R_{k^*} \cup R_{l^*} \]

  Remove unused region
  \[ \mathcal{M} \leftarrow \mathcal{M} - \{l^*\} \]
}

- This recursion generates a binary tree.
Merging Hierarchy and Order Identification

- Clustering can be terminated when the distance exceeds a threshold

\[ d_{k_*, l*} > \text{Threshold} \Rightarrow \text{Stop clustering} \]

- Different thresholds result in different numbers of clusters.