Iterative Budgeted Exponential Search

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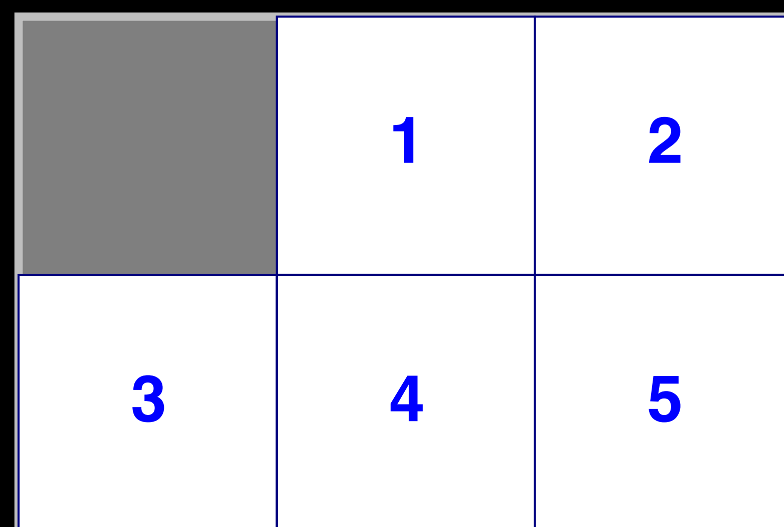
Talk jointly presented by Laurent Orseau and Nathan Sturtevant

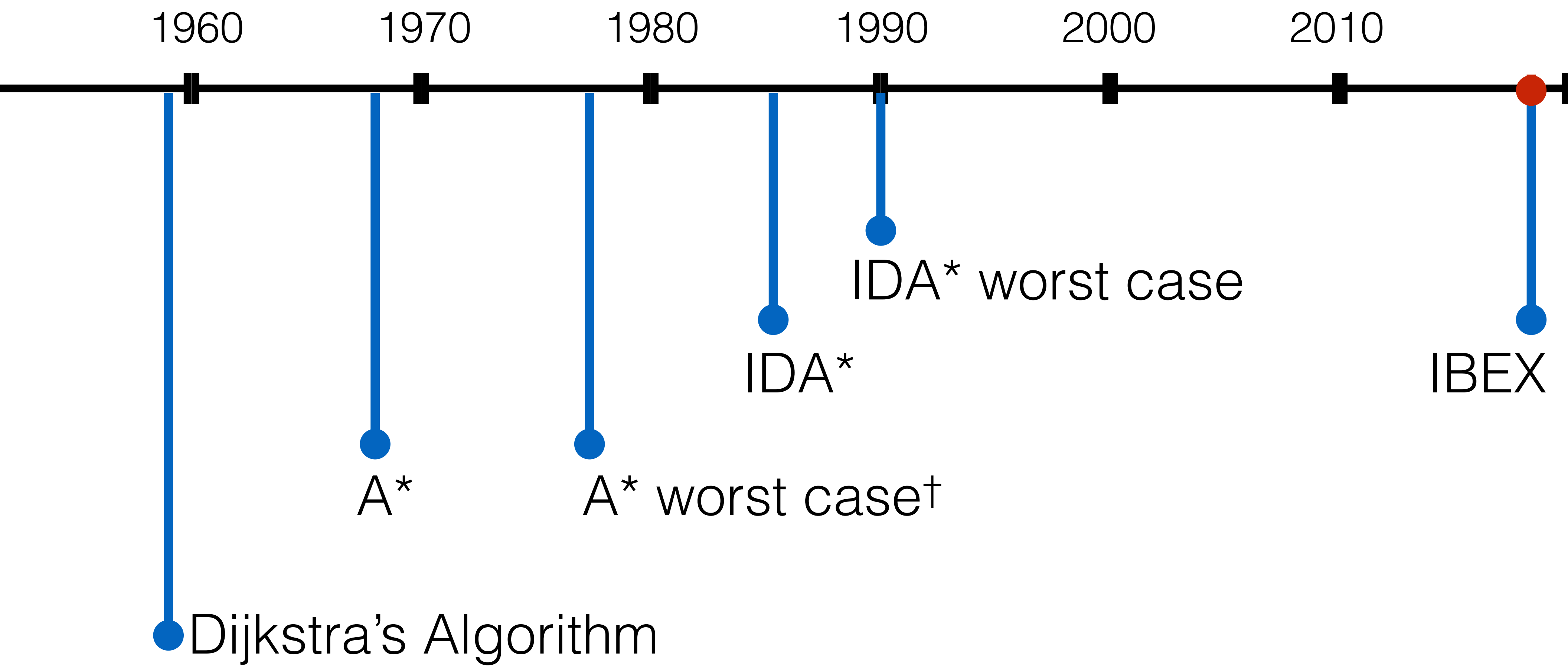
Start

h = 11

Goal



Shortest Path Algorithm History



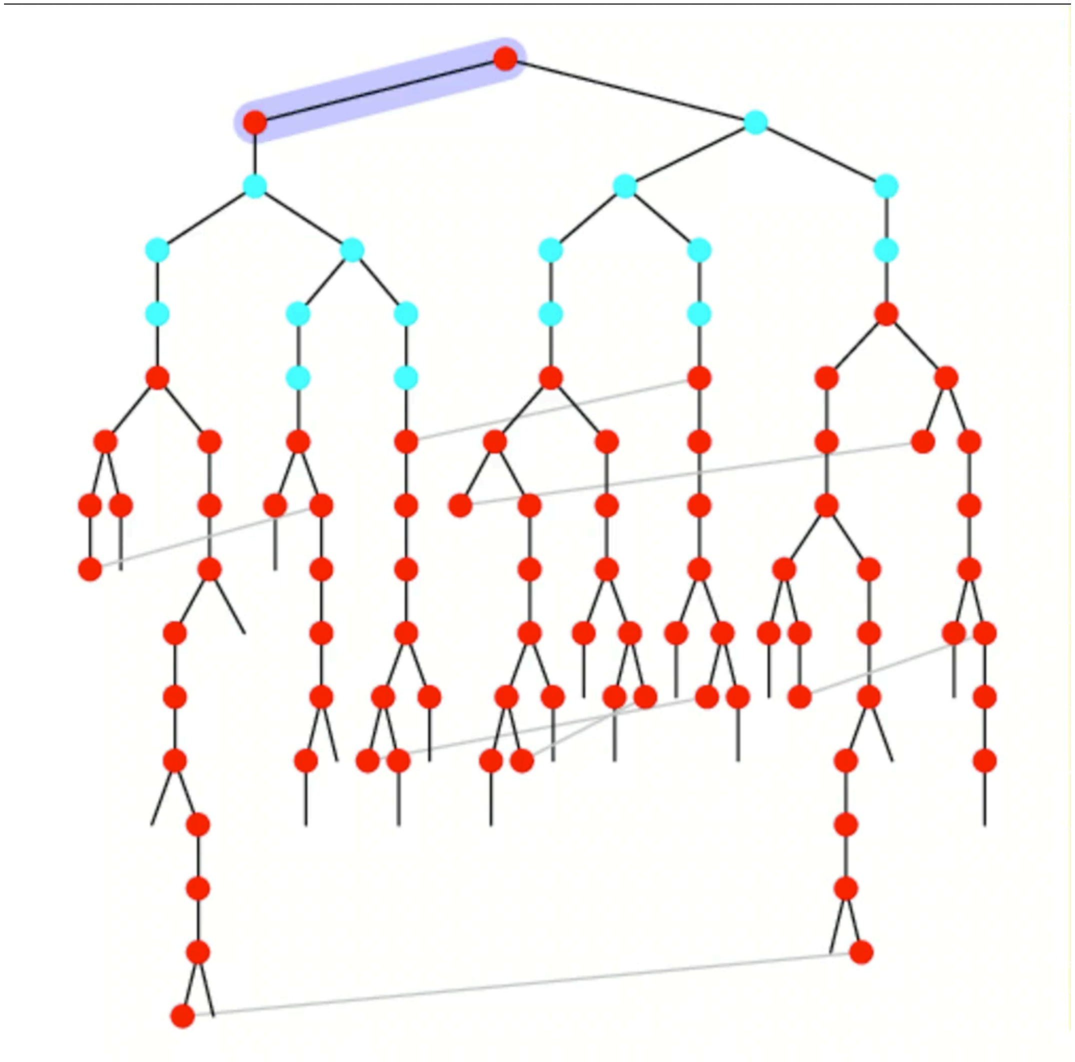
The N^2 problem

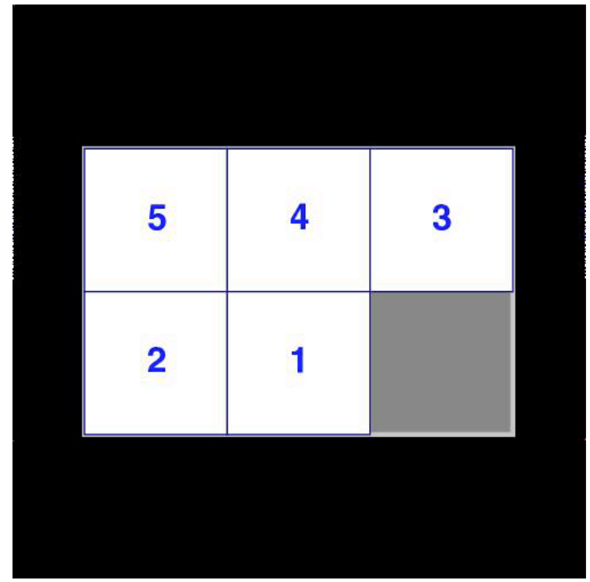
- Algorithms perform N^2 re-expansions of N states
- IDA*: N^2 expansions of $N = b^d$ states
- A*: 2^N expansions of N states
  - B & B': N^2 (Martelli 1977; Mero, 1984)
- Weighted A*: N^2 (Chen et al, 2019)

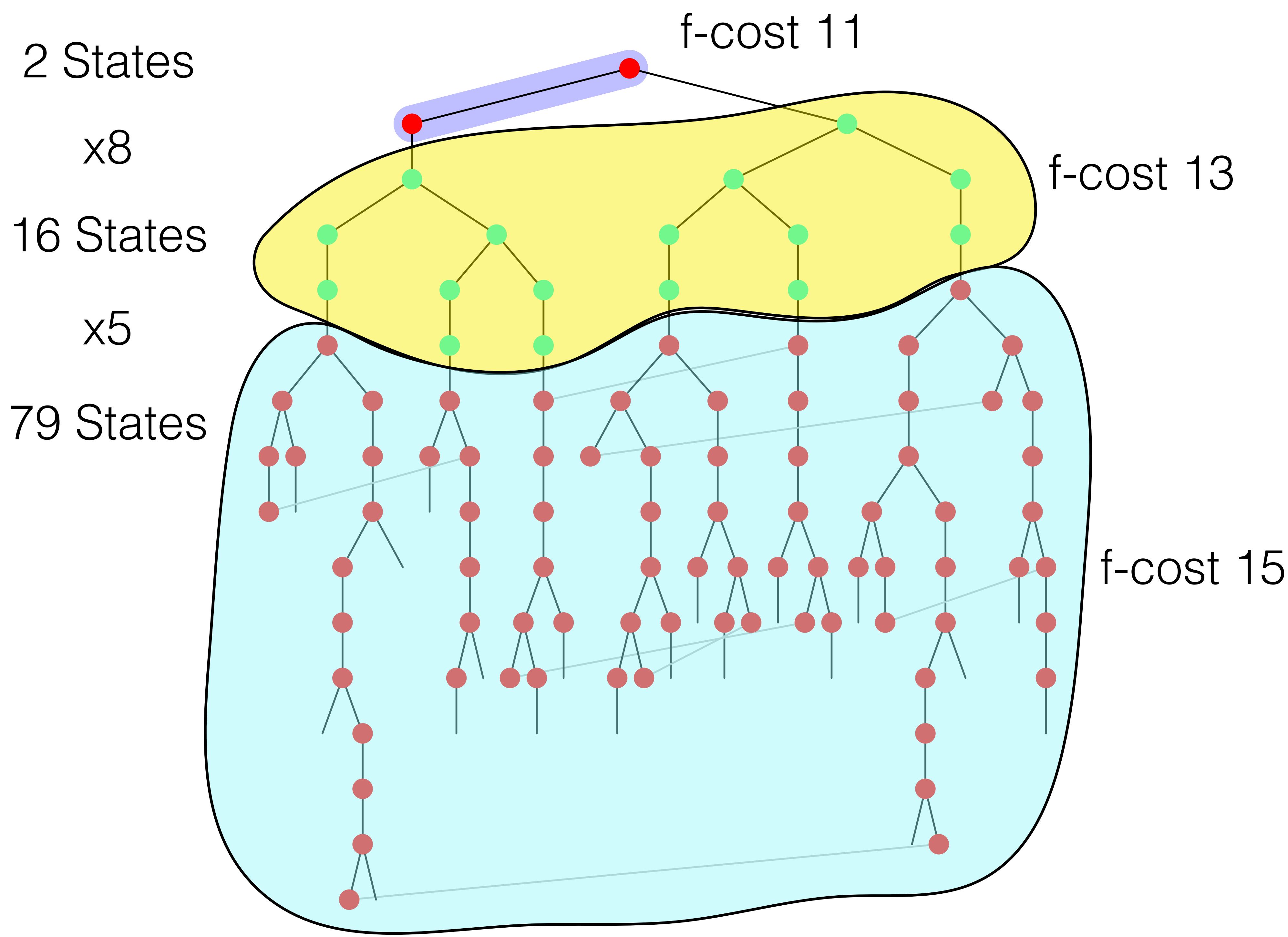
IDA* Refresher

- IDA* does *iterative deepening* search on *f*-costs
  - $f(n) = g(n) + h(n)$
- Next iteration *f*-cost:
  - Smallest unexplored from previous iteration
- If *f*-cost layers grow exponentially
  - IDA* is asymptotically optimal in node expansions

IDA* - Unit Costs



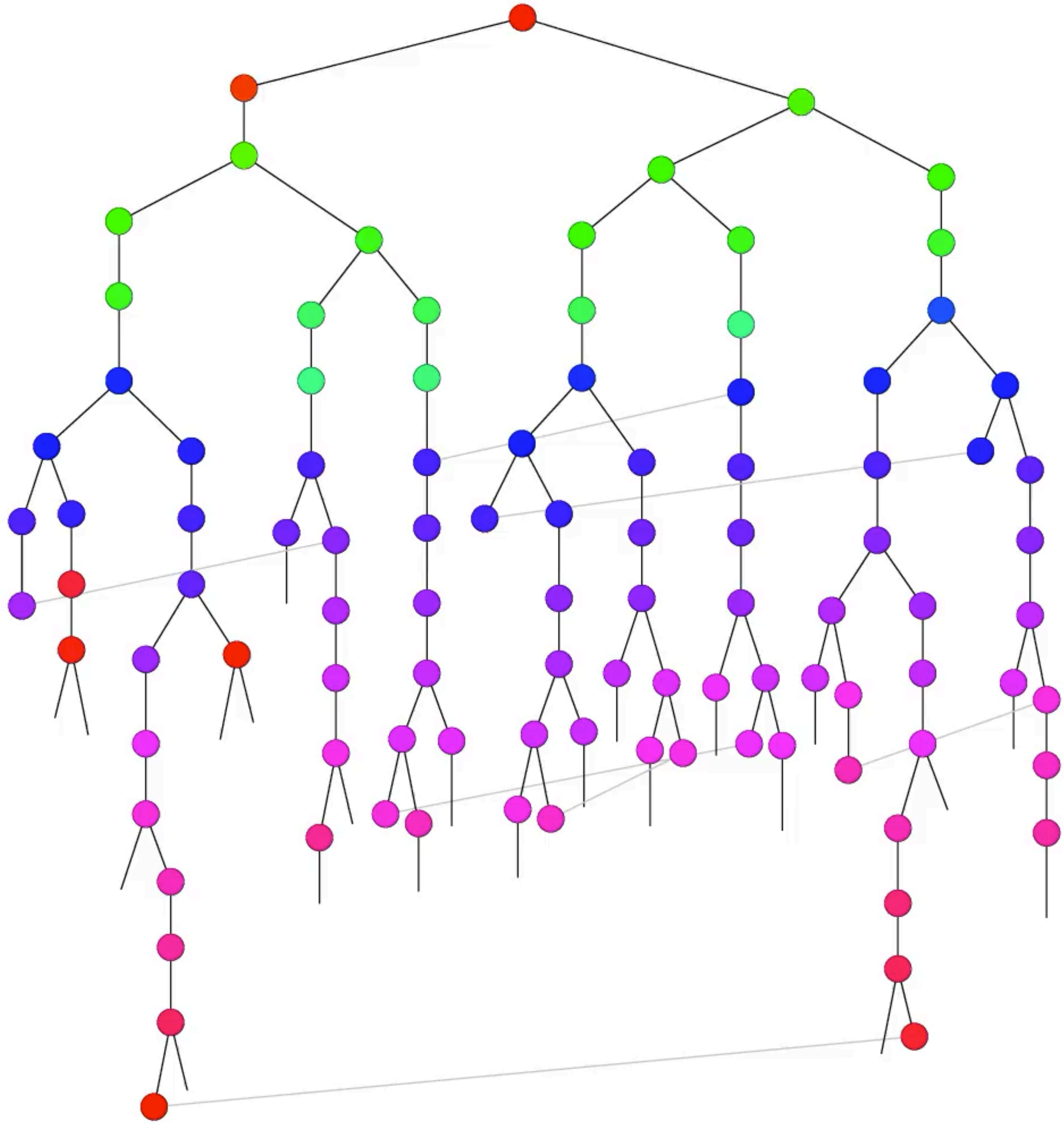


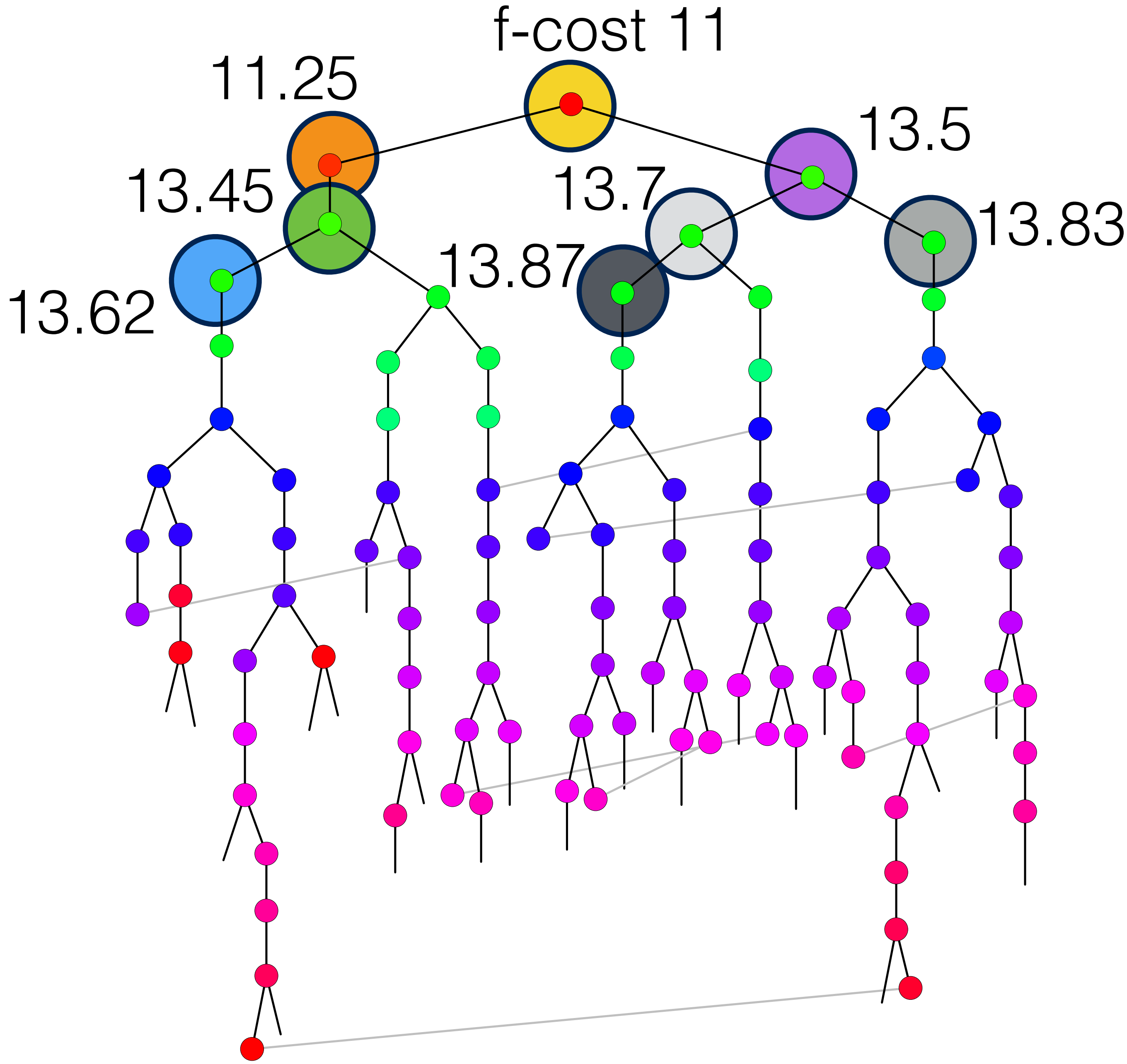


2 States
x8
16 States
x5
79 States
f-cost 11
f-cost 13
f-cost 15

IDA* Worst Case

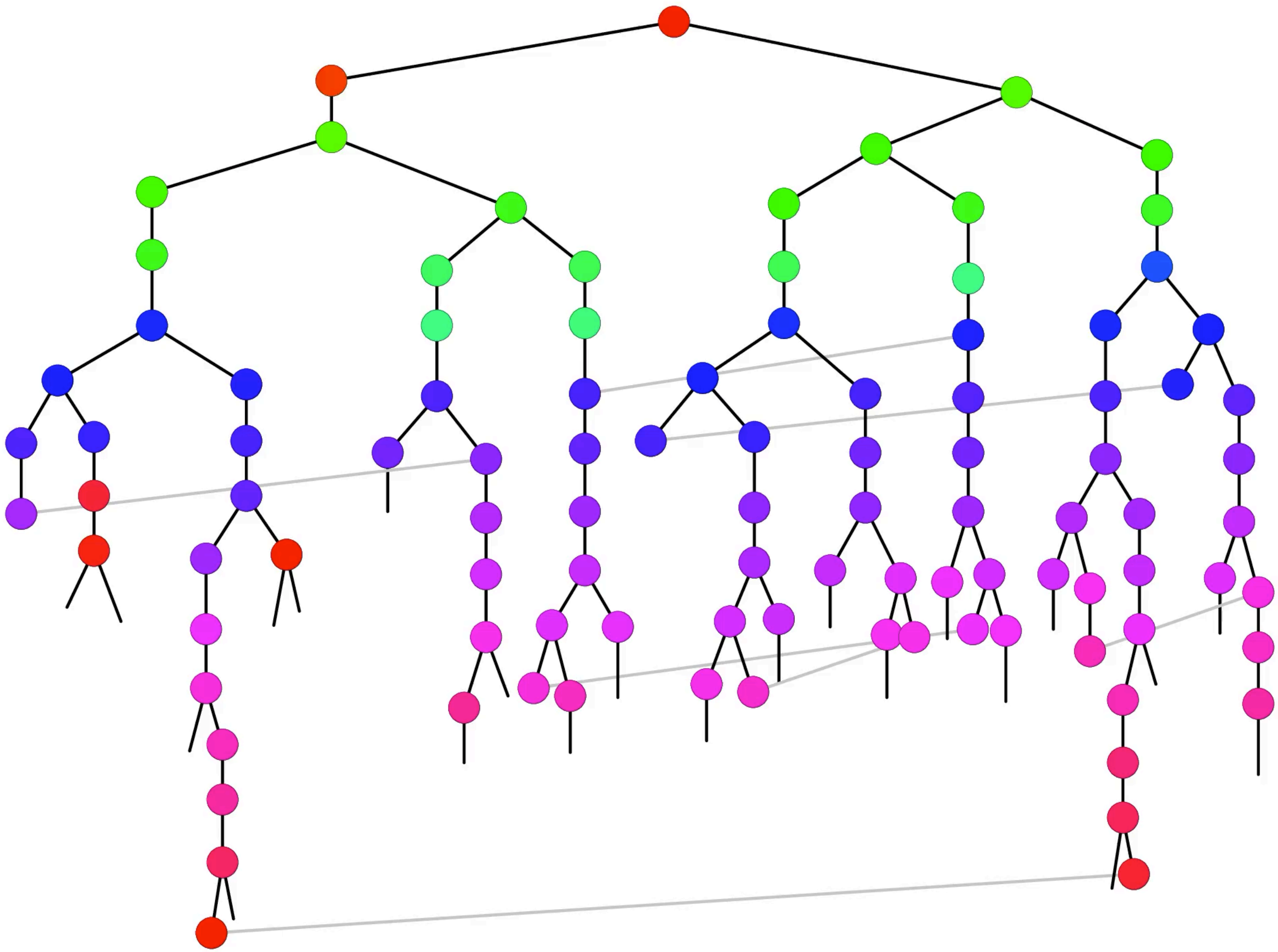
- *f*-cost layers grow exponentially
  - $1 + b + b^2 + b^3 + \ldots + b^d \approx b^d$
- What if *f*-cost layers grew linearly?
  - $1 + 2 + 3 + 4 + \ldots + b^d \approx (b^d)^2$
- Happens with non-unit edge costs:
  - STP: Cost of moving tile *t*: $\dfrac{t+2}{t+1}$




f-cost 11
11.25
13.5
13.45
13.7
13.83
13.87
13.62

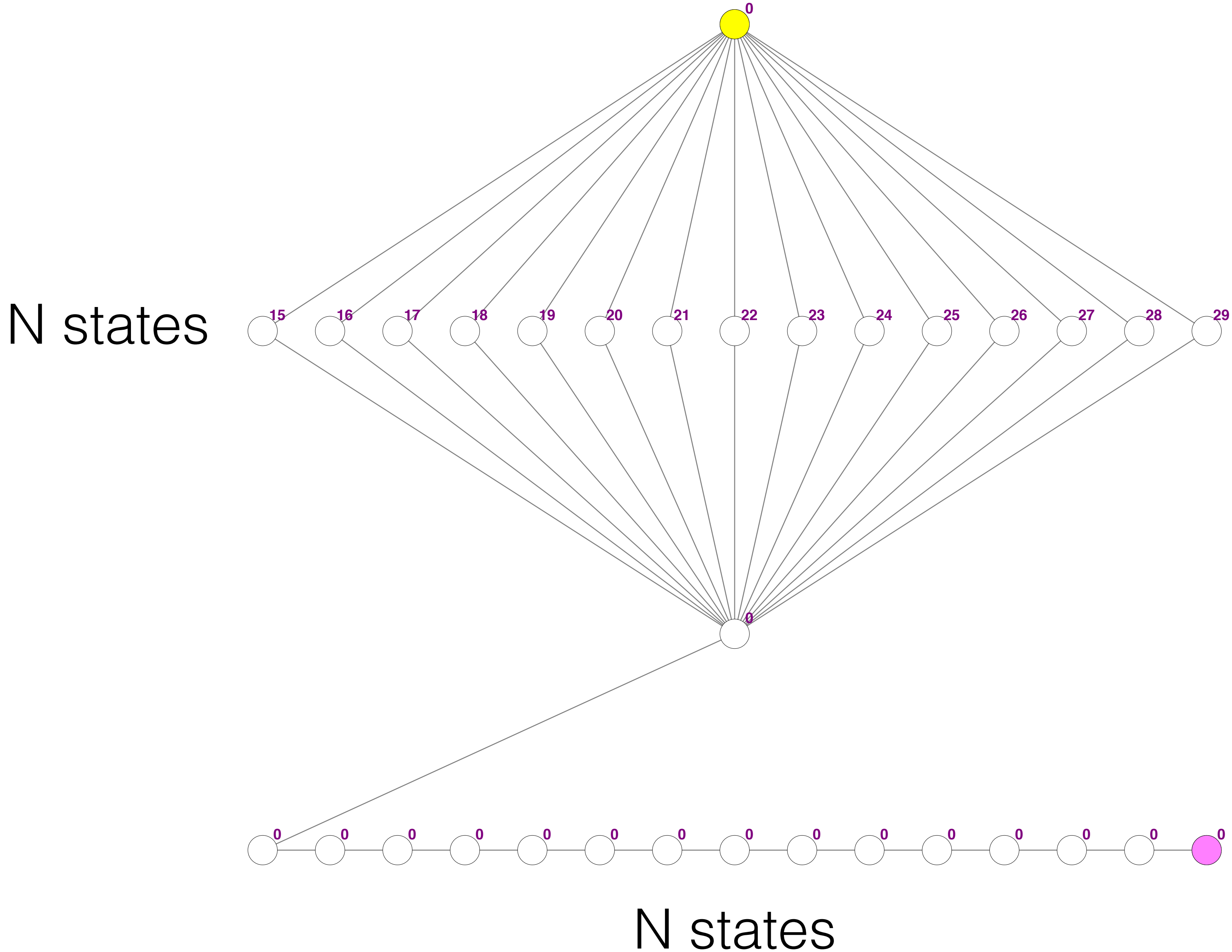
Previous Work

- Iterative Deepening
  - IDA* (Korf, 1985)
  - IDA^*_{CR} (Sarkar et al, 1990)
  - IDA^*_{IM} (Burns & Ruml, 2013)
  - EDA* (Sharon et al, 2014)



A* (B/B')

- A* with a consistent heuristic is optimal*
- With an inconsistent heuristic A* might do 2^N expansions of N states
- B and B' immediately propagate shorter paths
  - Will do at most N^2 expansions of N states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N states
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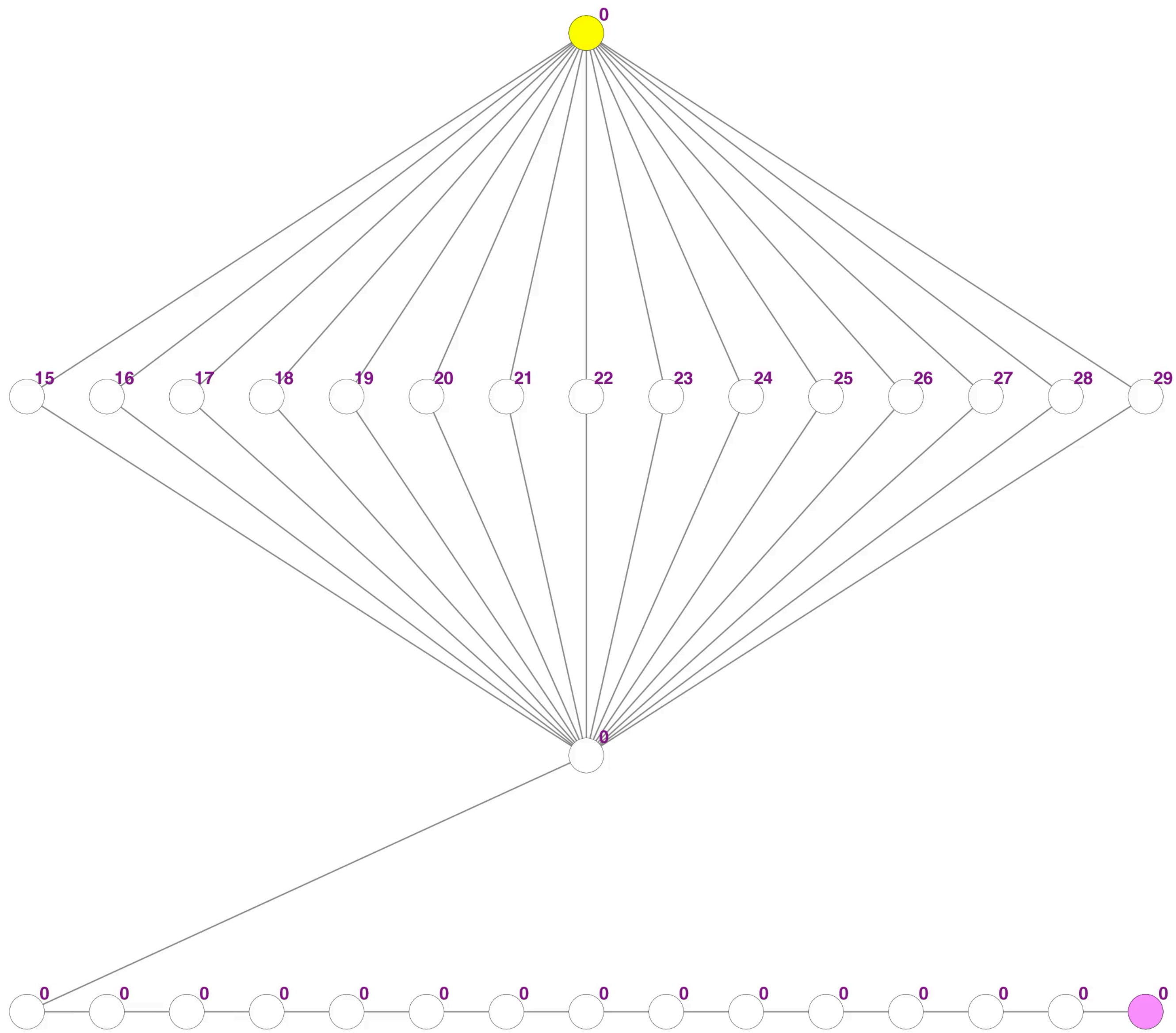
N states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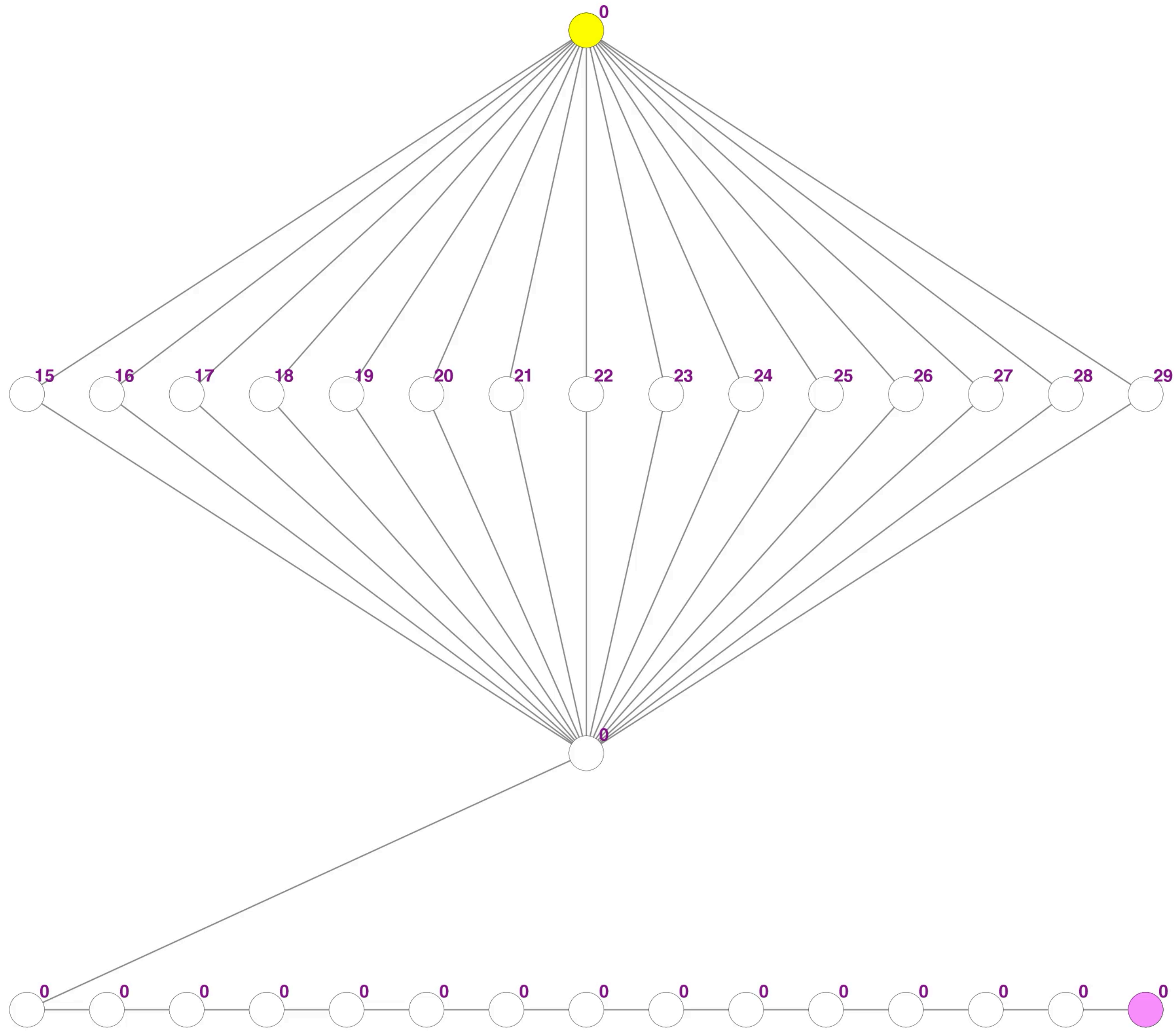
Previous Work

- Inconsistent Heuristics
  - B/B’ (Martelli 1977; Mero, 1984)
  - Delay (Sturtevant et al, 2008)
  - BPMX (Felner et al, 2011)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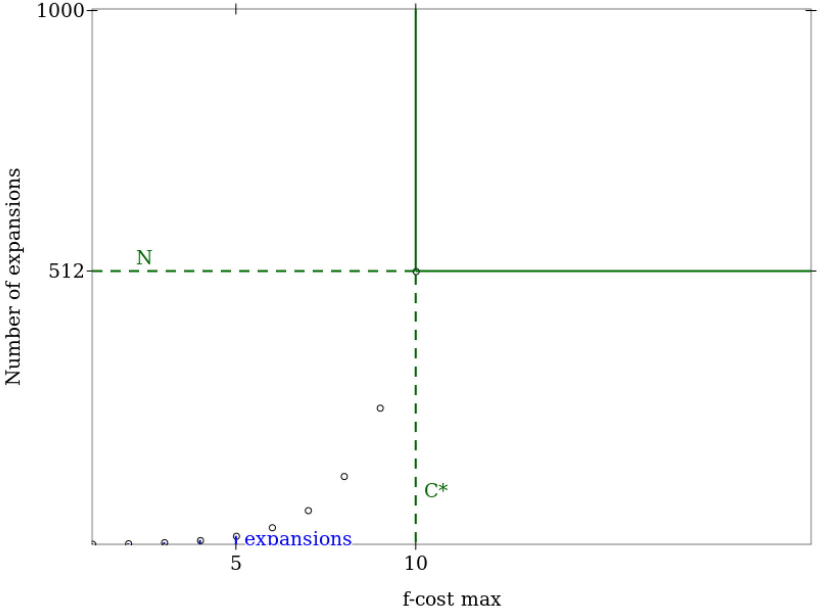
Demos

- All demos shown here runnable online:
  - https://www.movingai.com/SAS/BTS/

- Also running on my iPad; can give detailed demo

IDA* — Good Case



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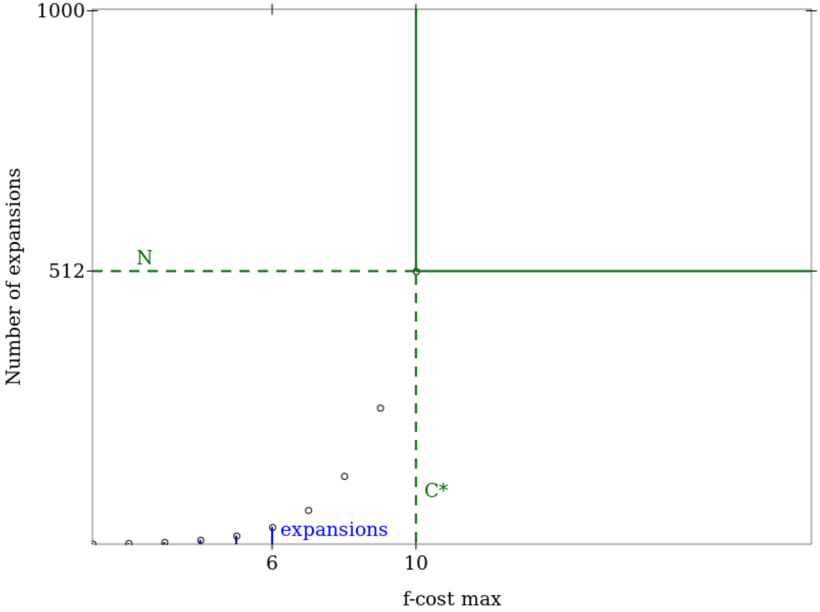

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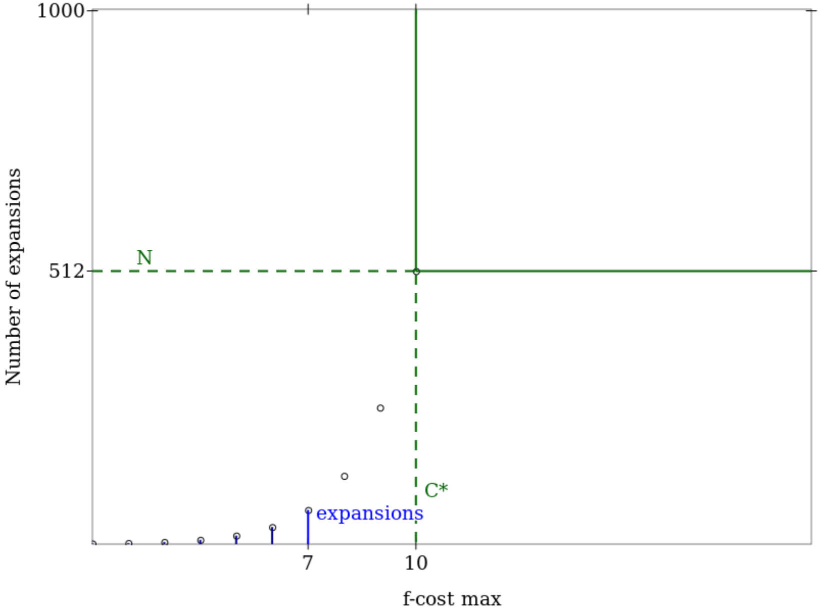

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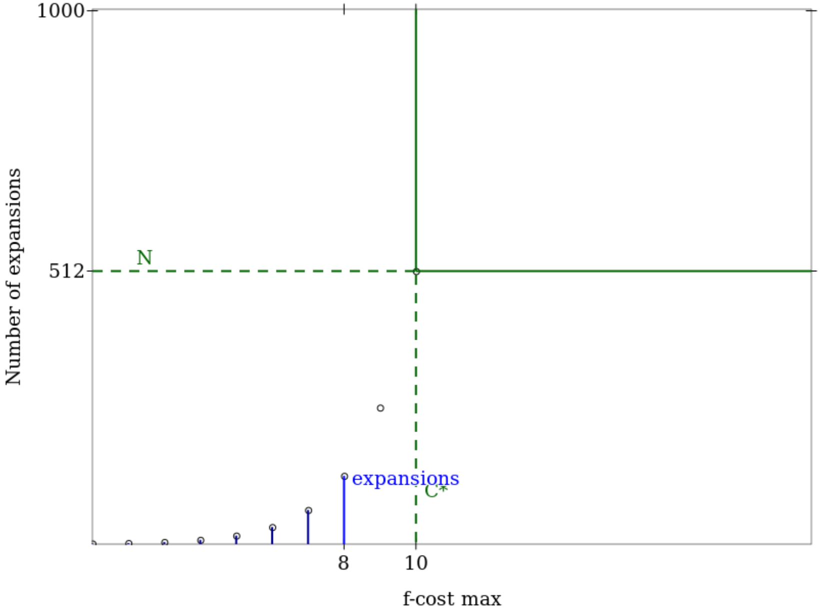

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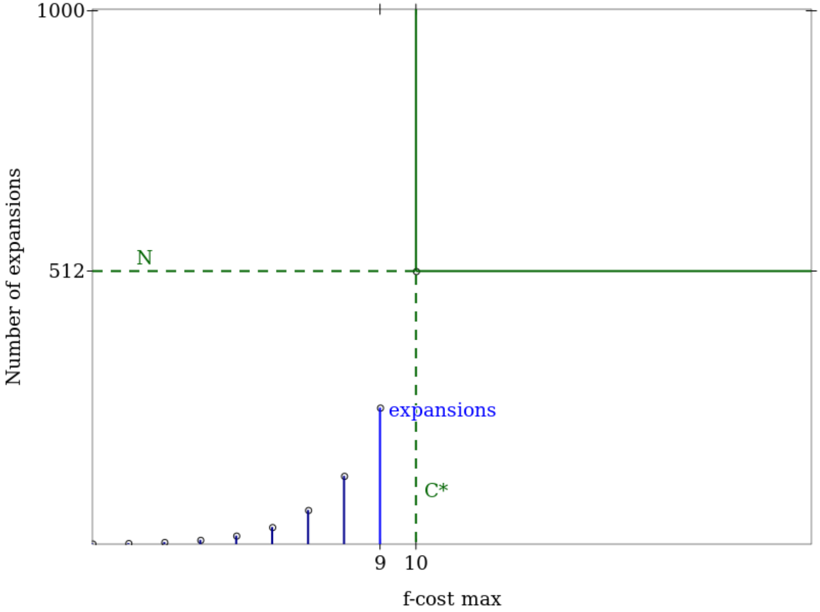

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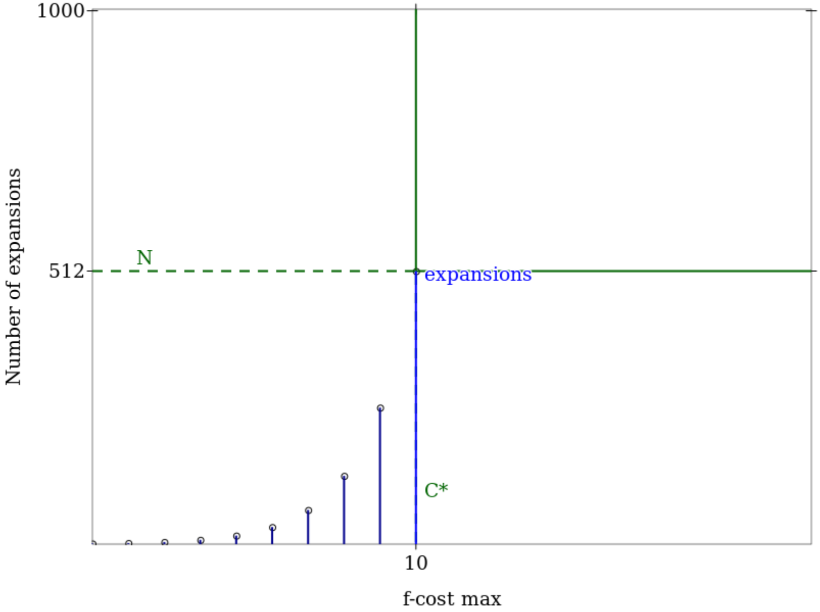
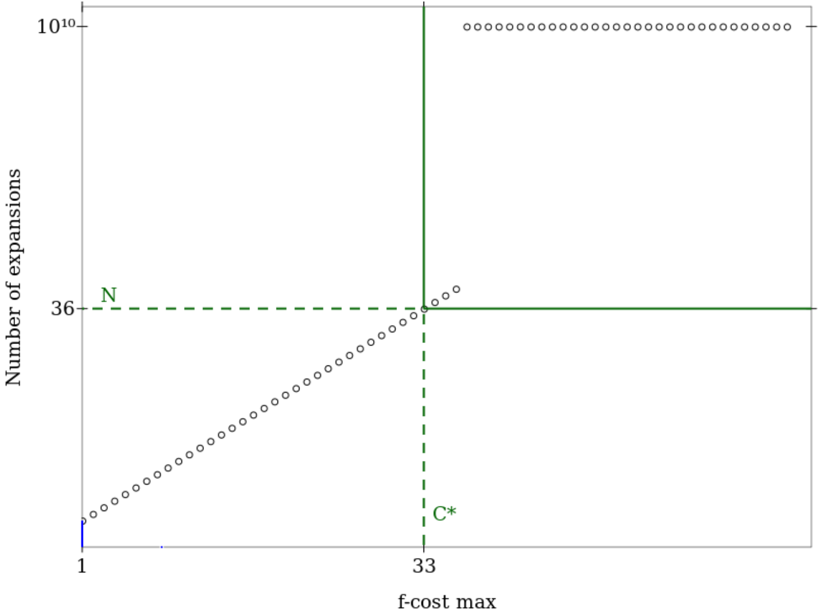


IDA* — Bad Case



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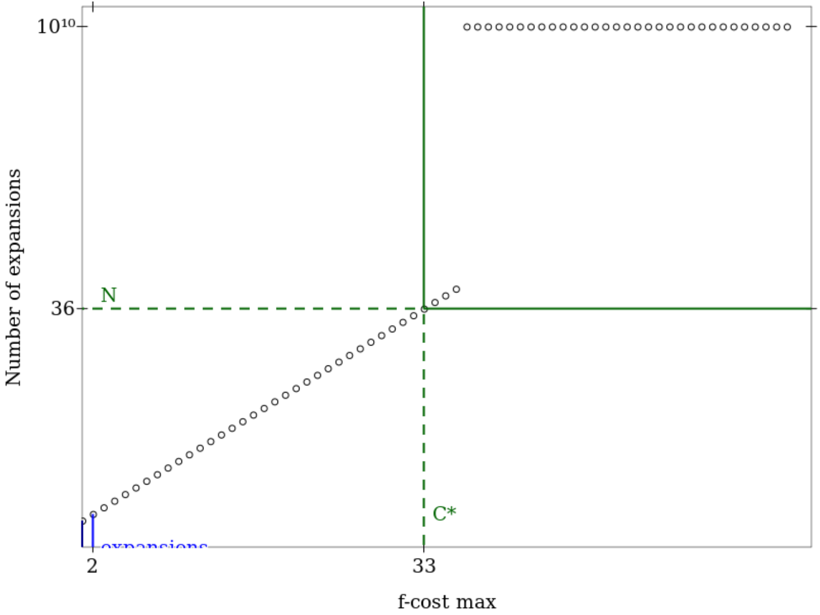

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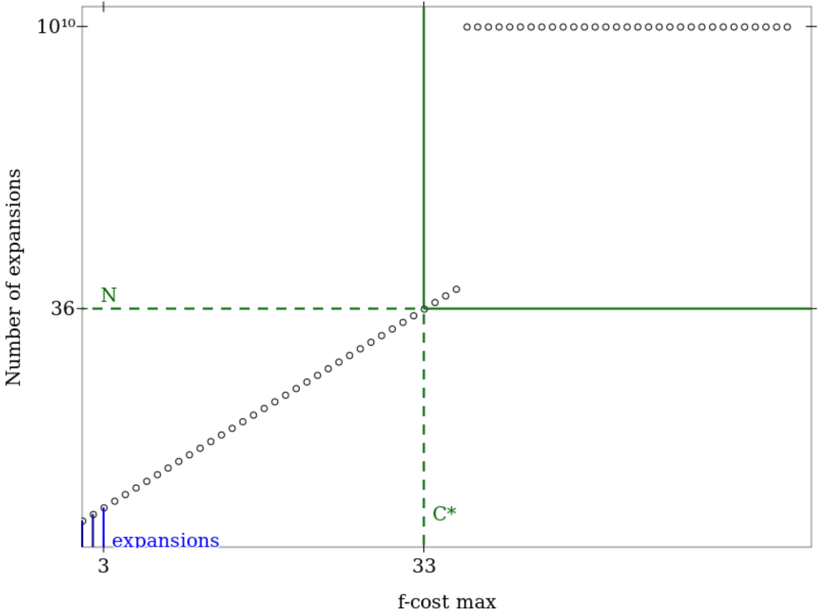
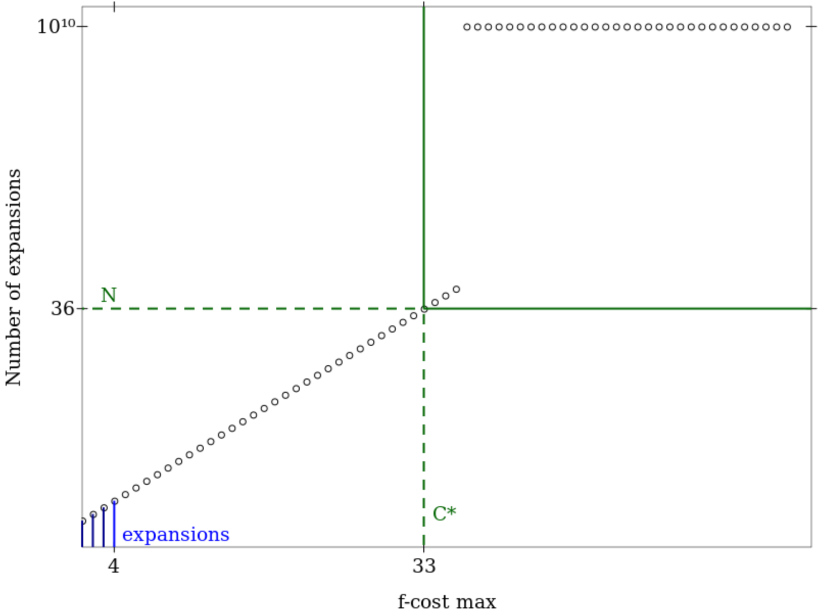


IDA* — Bad Case



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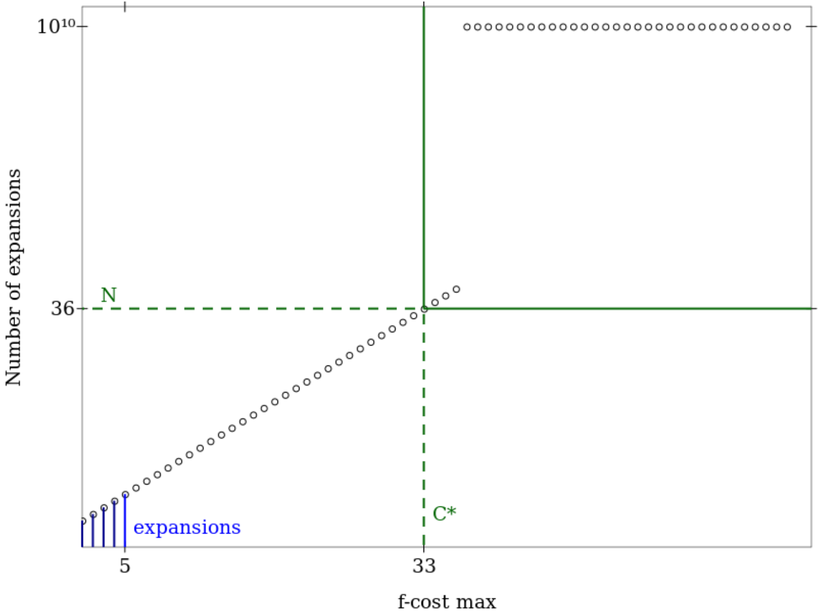

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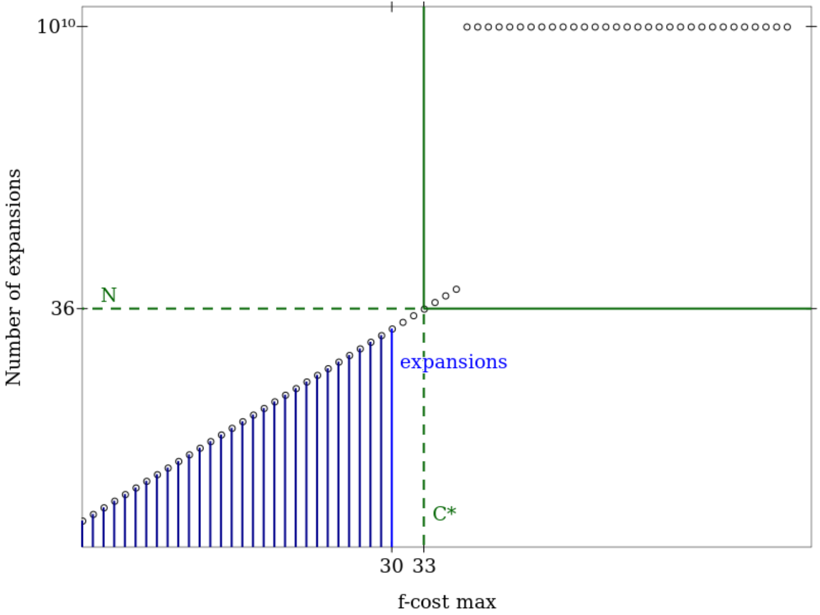

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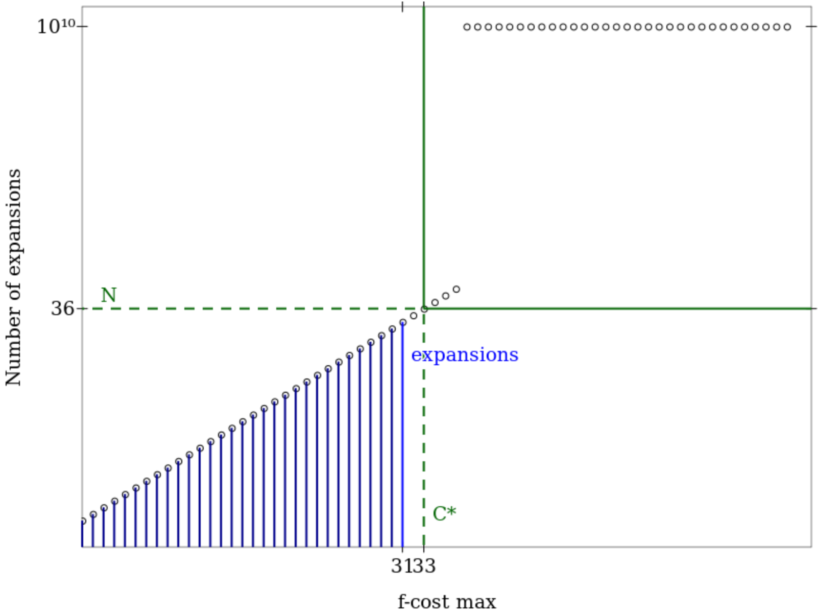

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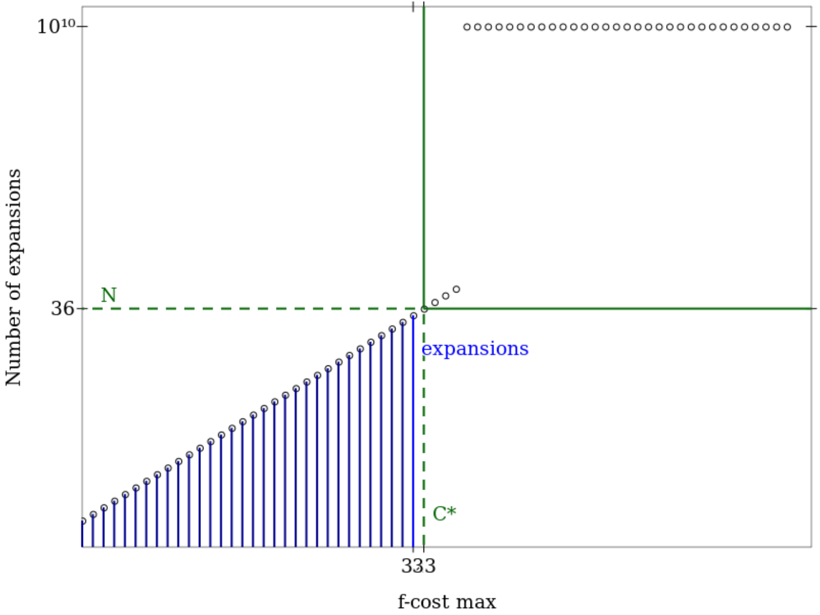

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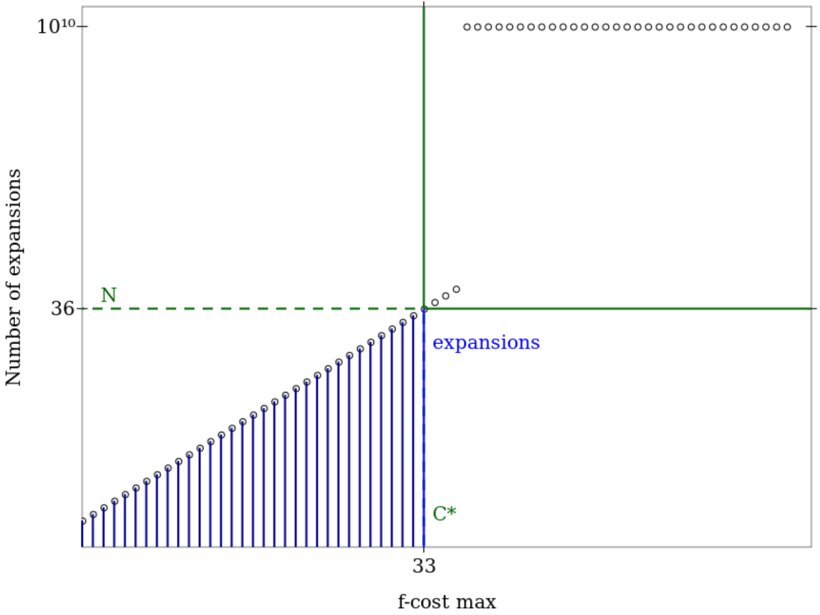
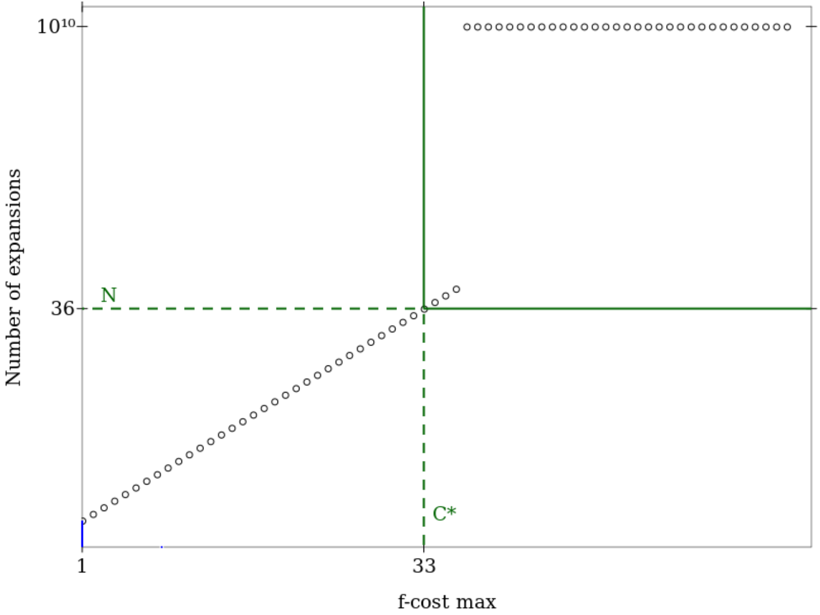


EDA*



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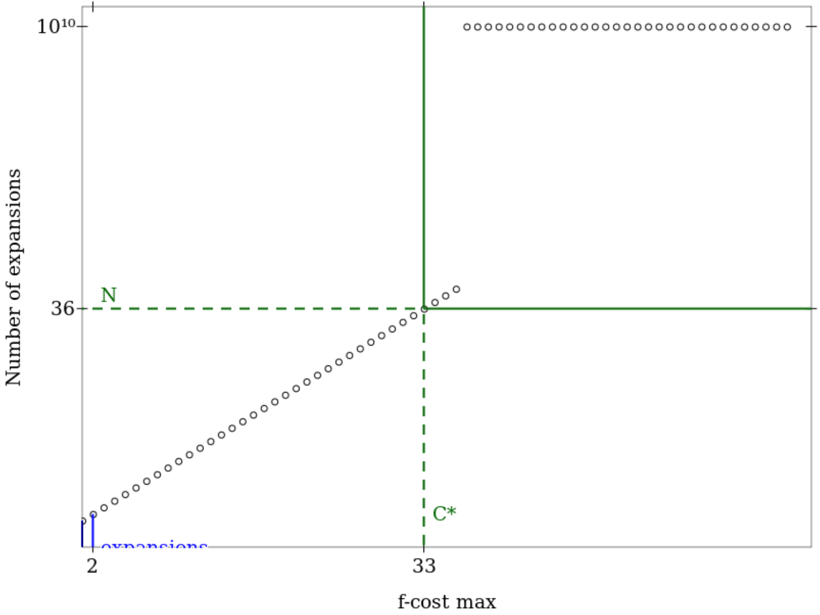

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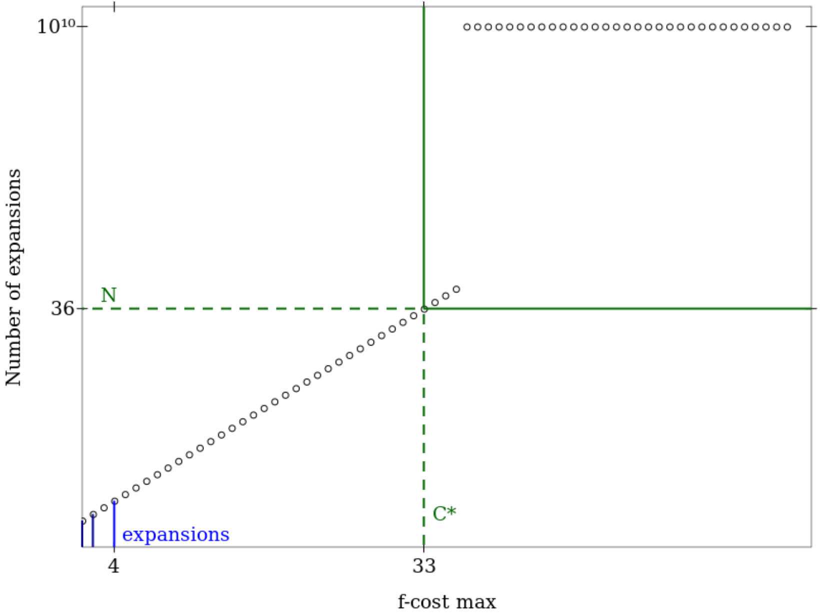

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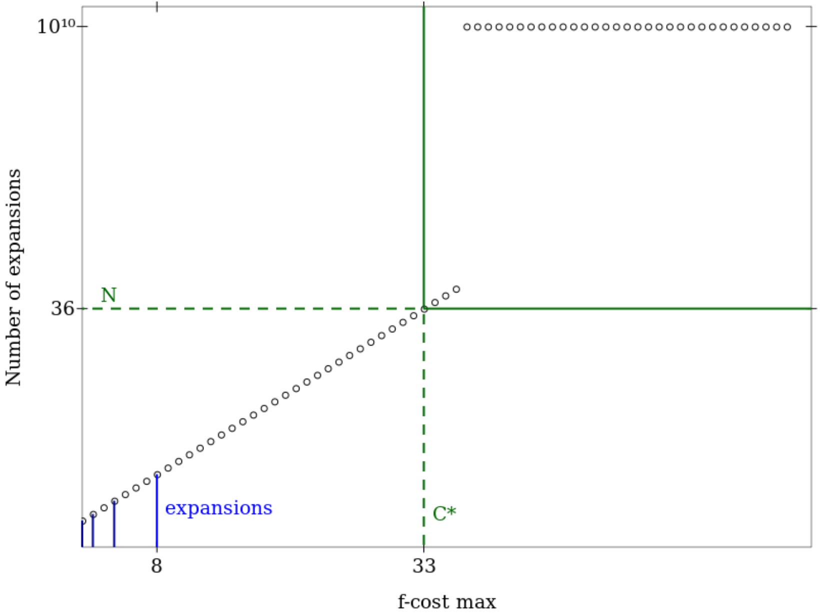
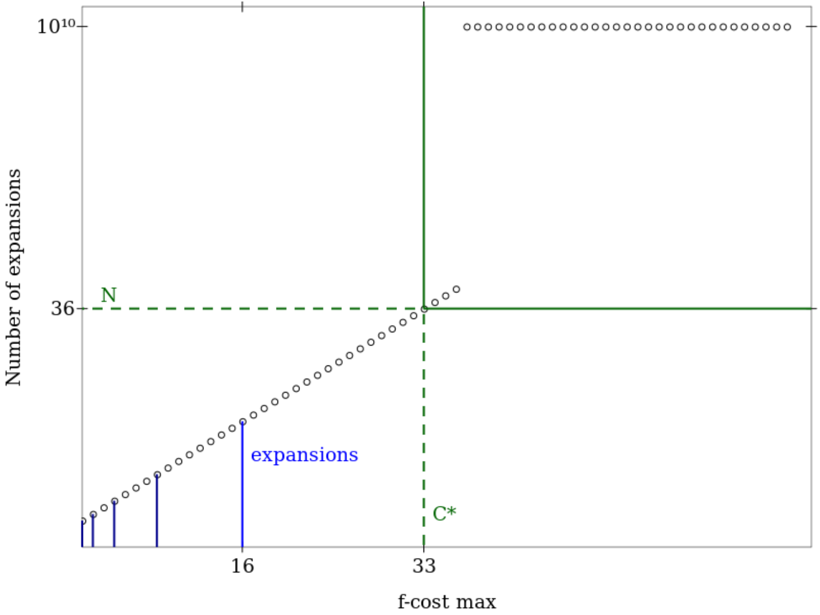


EDA*



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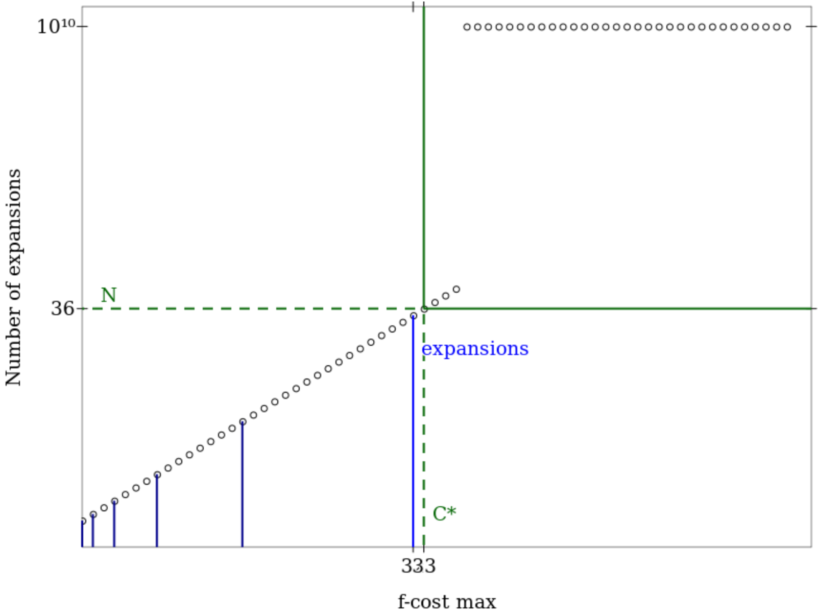
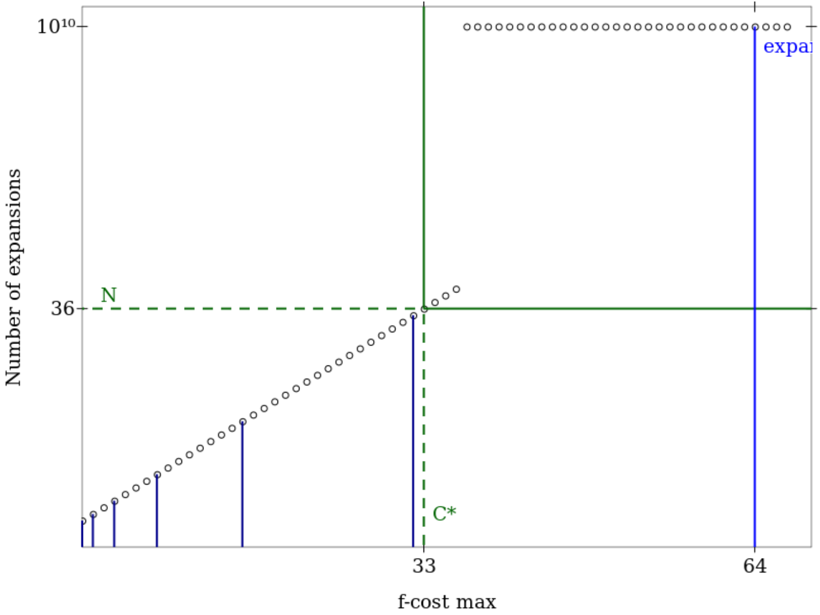


EDA*



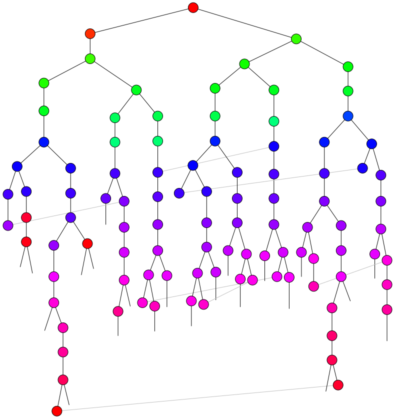
IBEX (main idea)

```
For budget = 2, 4, 8, 16, ...:
    cost_bound = Oracle(budget)
    search within cost_bound and within budget
```

Oracle(budget) returns largest cost for which budget is sufficient.

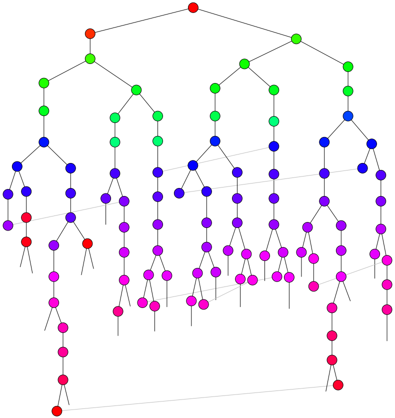
IBEX (main idea): With an oracle

budget	f-cost	expansions
2		



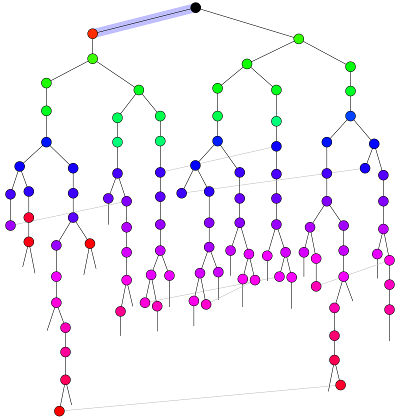
IBEX (main idea): With an oracle

budget	f-cost	expansions
2	11.2	



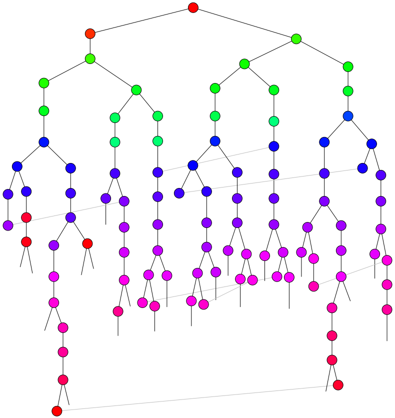
IBEX (main idea): With an oracle

budget	f-cost	expansions
2	11.2	2



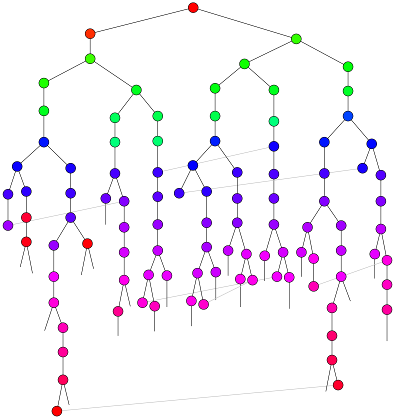
IBEX (main idea): With an oracle

budget	f-cost	expansions
2	11.2	2
4		



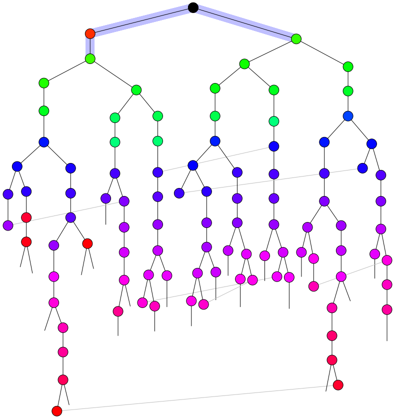
IBEX (main idea): With an oracle

budget	f-cost	expansions
2	11.2	2
4	13.5	



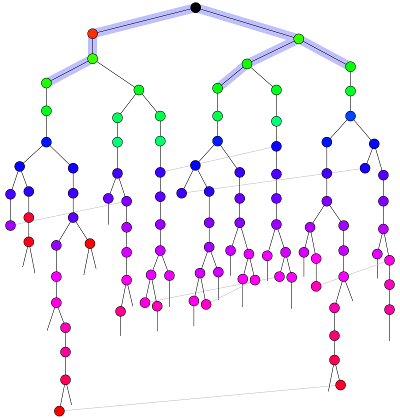
IBEX (main idea): With an oracle

budget	f-cost	expansions
2	11.2	2
4	13.5	4



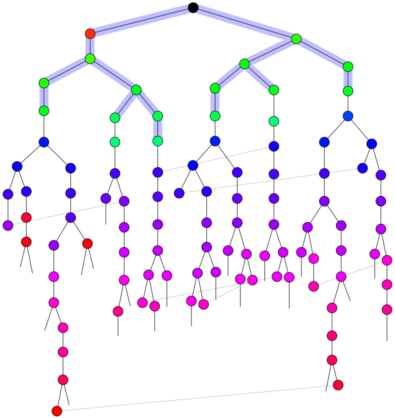
IBEX (main idea): With an oracle

budget	f-cost	expansions
2	11.2	2
4	13.5	4
8	13.9	8



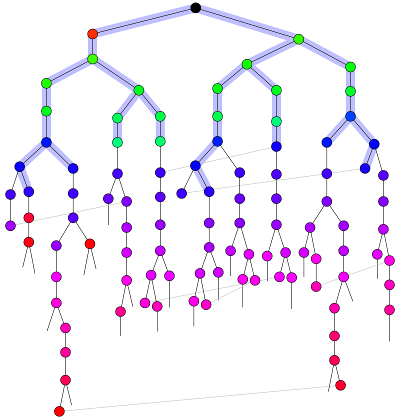
IBEX (main idea): With an oracle

budget	f-cost	expansions
2	11.2	2
4	13.5	4
8	13.9	8
16	14.4	16



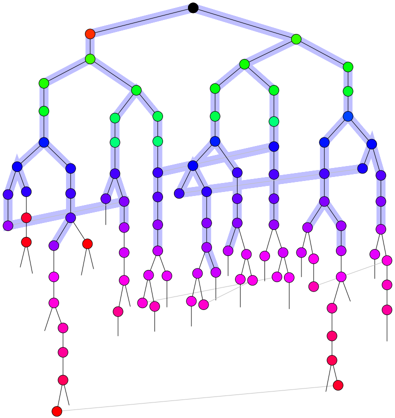
IBEX (main idea): With an oracle

budget	f-cost	expansions
2	11.2	2
4	13.5	4
8	13.9	8
16	14.4	16
32	16.8	30



IBEX (main idea): With an oracle

budget	f-cost	expansions
2	11.2	2
4	13.5	4
8	13.9	8
16	14.4	16
32	16.8	30
64	17.7	62



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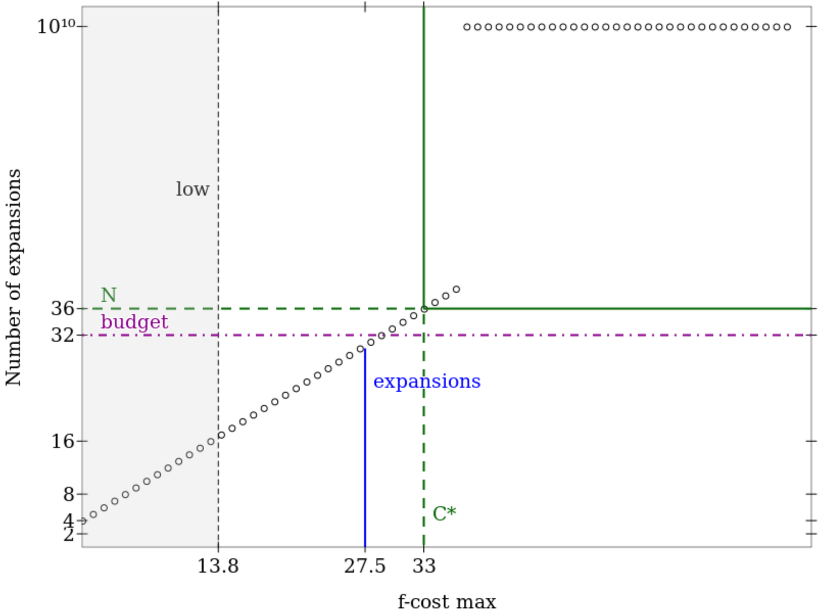
```

Oracle(budget) returns largest cost for which budget is sufficient.

Oracle(budget) $=$ (budgeted) exponential search

IBEX (main idea)



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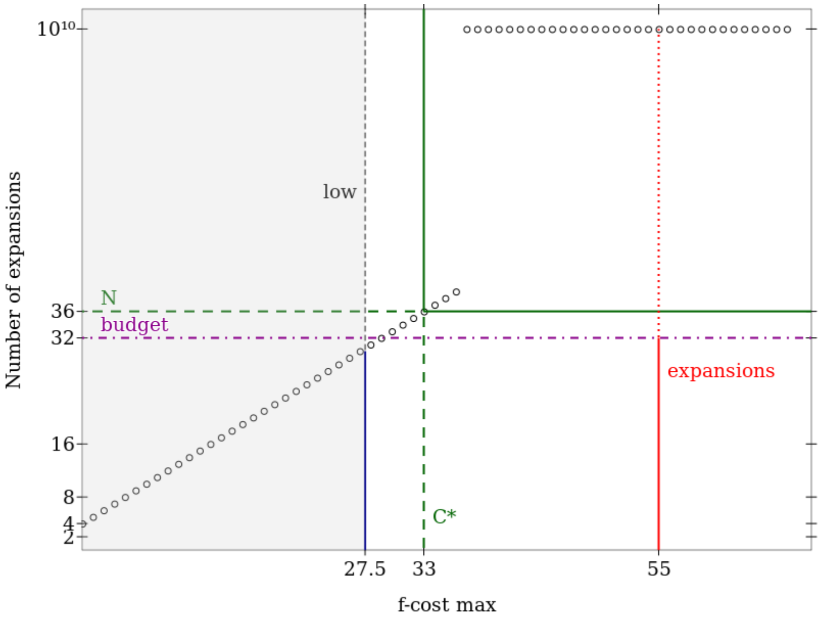

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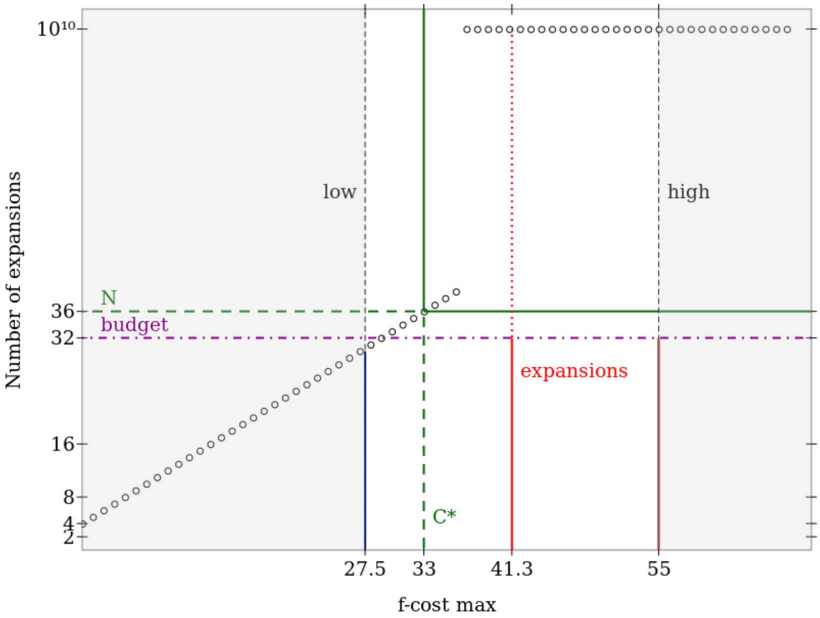

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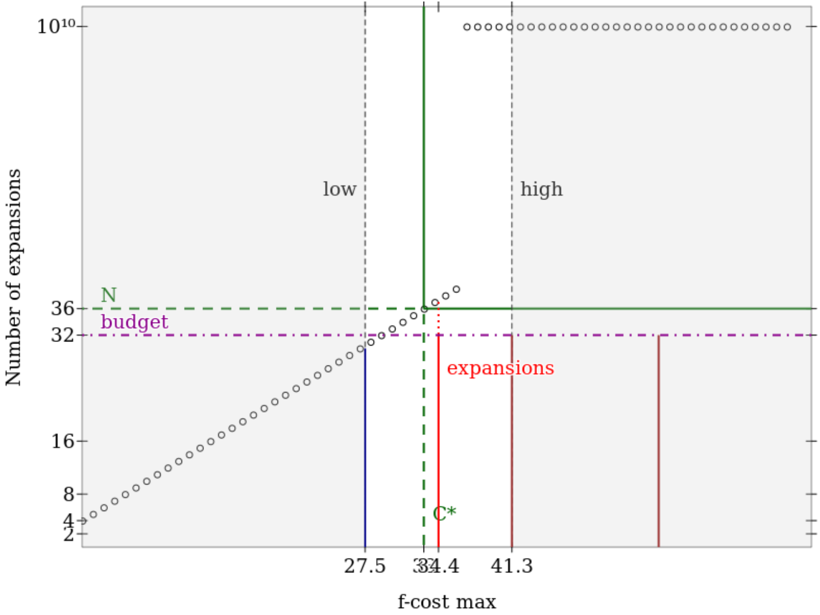

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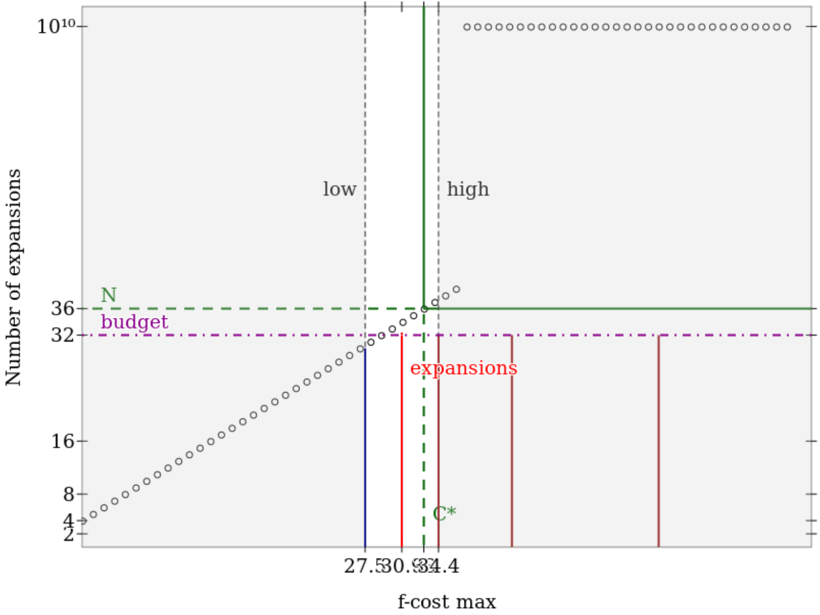

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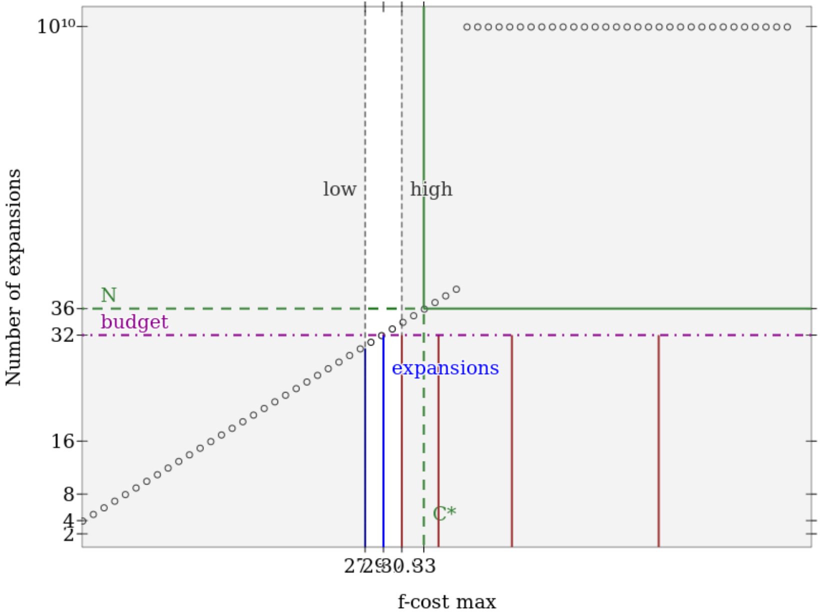

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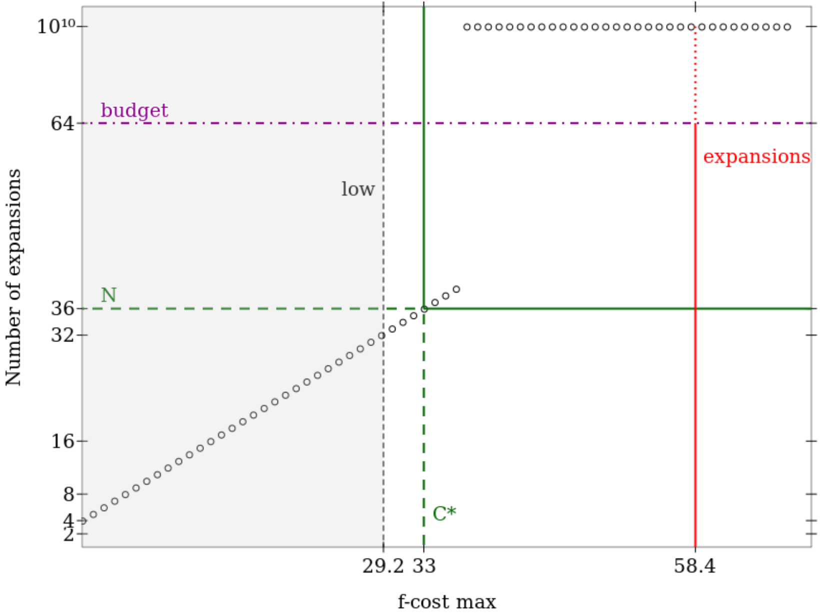

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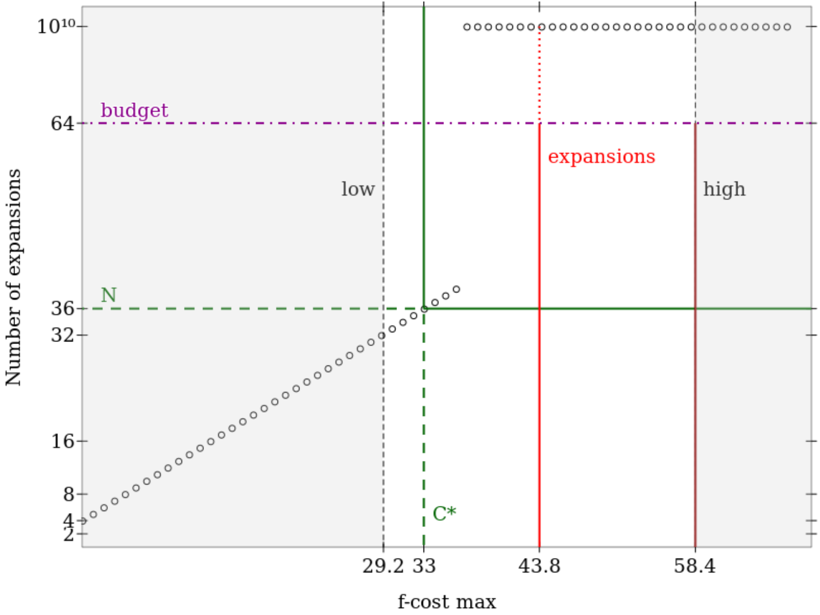

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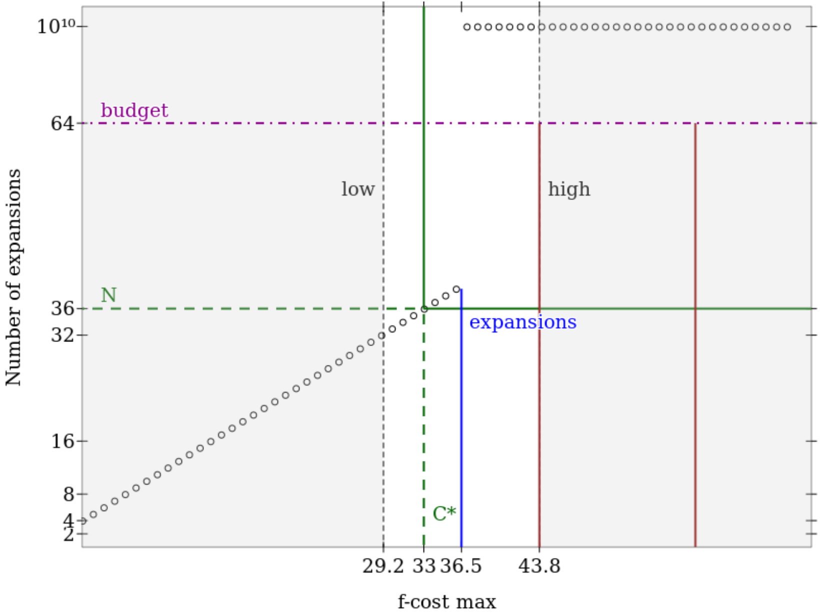

IBEX (main idea)

N

$\downarrow$

For budget = 2, 4, 8, 16, 32, 64 : $\leq 4N$

With **exponential search**:

Refine lower and upper bound on cost

Proves no solution when lower = upper

$\Big\}\ O(\log C^*)$

Application: Tree Search

IBEX + Depth-First Search = *Budgeted Tree Search (BTS)*

Alg.	**Worst case**
IDA*$_{(1985)}$	$\Theta(N^2)$
EDA*$_{(2014)}$	unbounded
BTS	$O(N \log C^*)$

C^*: cost of the optimal solution
N: number of nodes of cost at most C^*

All algorithms return an optimal solution (under usual assumptions).

Application: Graph Search

IBEX + Uniform-Cost Search = *Budgeted Graph Search (BGS)*

Alg.	Admissible Heuristic	
	Consistent	Inconsistent
A*$_{(1968)}$	$O(N)$	$\Omega(2^N)$
B$_{(1977)}$	$O(N)$	$\Theta(N^2)$
BGS	$O(N)^\dagger$	$O(N \log C^*)$

C^*: cost of the optimal solution
N: number of nodes of cost at most C^*

All algorithms return an optimal solution (under usual assumptions).

Enhancements: Tree search

- Recover IDA* when exponential unit-cost domains
  1) Infinite iteration at next f-cost
  2) If expansions grow exponentially, go to 1) else exponential search

Enhancements: Tree search

- Recover IDA* when exponential unit-cost domains
  1) Infinite iteration at next f-cost
  2) If expansions grow exponentially, go to 1) else exponential search
- Budget window $[2, 8]$ to be more aggressive

Experiments

Algorithm	15-Puzzle (unit)		15-Puzzle (real)		Coconut	
	Solved	Exp. $\times 10^6$	Solved	Exp. $\times 10^6$	Solved	Exp. $\times 10^4$
BTS	100	**242.5**	100	**673.1**	100	84.7
EDA*	99	5 586.0	100	2 882.3	3	$\geq$ 554 249.0
IDA*_CR	100	868.4	100	700.6	3	$\geq$ 516 937.7
IDA*	100	**242.5**	57	62 044.3	100	5 484.2
N	100	100.8	100	258.1	100	2.7

BTS *(Budgeted Tree Search)* = IBEX + Depth-First Search

Experiments

Algorithm	15-Puzzle (unit)		15-Puzzle (real)		Coconut	
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IDA*_CR	100	868.4	100	700.6	3	$\geq$ 516 937.7
IDA*	100	**242.5**	57	62 044.3	100	5 484.2
N	100	100.8	100	258.1	100	2.7

BTS *(Budgeted Tree Search)* = IBEX + Depth-First Search

Experiments

Algorithm	15-Puzzle (unit)		15-Puzzle (real)		Coconut	
	Solved	Exp. $\times 10^6$	Solved	Exp. $\times 10^6$	Solved	Exp. $\times 10^4$
BTS	100	**242.5**	100	**673.1**	100	84.7
EDA*	99	5 586.0	100	2 882.3	3	$\geq$ 554 249.0
IDA*_CR	100	868.4	100	700.6	3	$\geq$ 516 937.7
IDA*	100	**242.5**	57	62 044.3	100	5 484.2
N	100	100.8	100	258.1	100	2.7

BTS *(Budgeted Tree Search)* = IBEX + Depth-First Search

Conclusion

- IBEX: Budgeting $\rightarrow O(N \log C)$
- Tree search: drop-in replacement for IDA*
- DovIBEX: For applications where oracle returns only if a solution is found
  - Interleaves budget iterations

Do you have a problem where the current worst case is $O(N^2)$, due to being cautious?

Maybe (Dov)IBEX can help!

Come and see our poster!