Part 3: Map Representations & Geometric Path Planning

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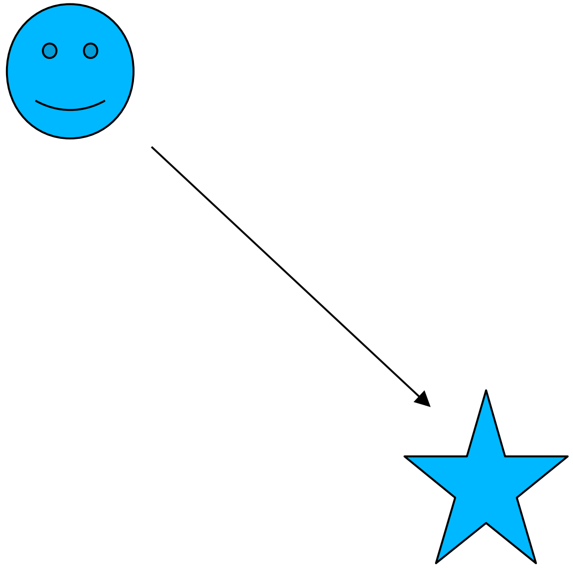
Game-playing, **A**nalytical methods,
Minimax search, and **E**mpirical **S**tudies

Outline

- Map Representations
  - Grids, polygon-based
  - Free space decompositions
  - Constrained Delaunay Triangulations
- Path Planning in Triangulations
  - A* applied to triangulations (TA*)
  - Triangulation Reductions and TRA*
- Outlook
  - Further improvements
  - Applications to high-level game AI

Pathfinding

- Want to get some object from one point to another, avoiding obstacles
- Robotics: non-point object, needs to avoid obstacles by some margin
- Games: needs to be very fast and use little memory



Map Representations

- Path planning algorithm is only half the picture
- Underlying **map representation** and **data structures** are just as important
- Important design questions:
  - Are optimal paths required?
  - Is the world static or dynamic?
  - Are worlds known ahead of time?
  - Are there real-time constraints?
  - How much memory is available?

Goal of pathfinding algorithms

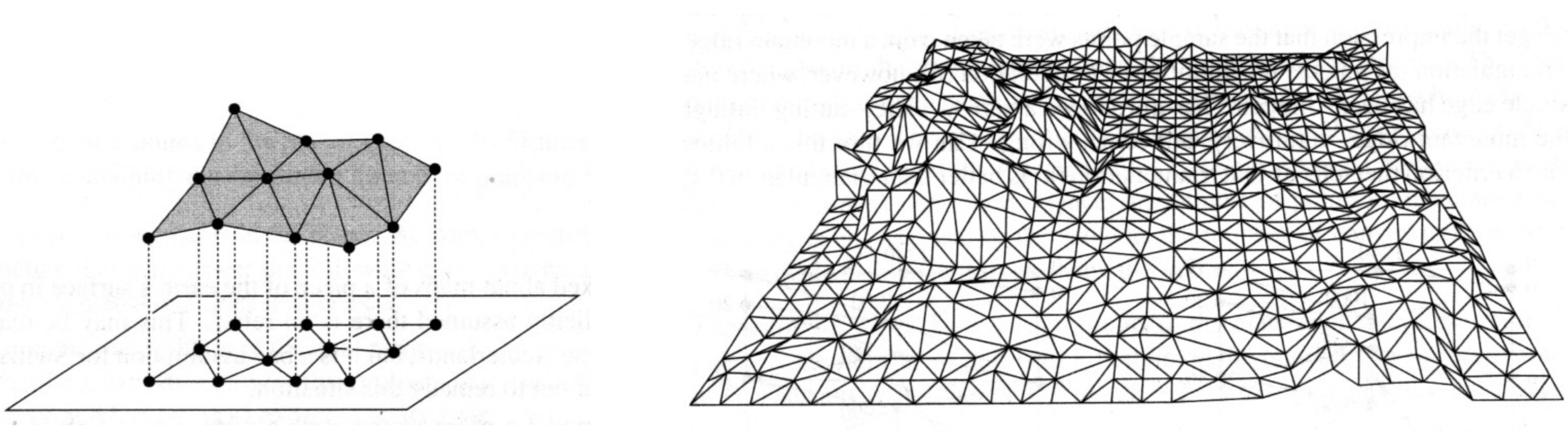
- Find (nearly) optimal path, where optimal usually means quickest
- Obey constraints (e.g. object size, fuel limit, exposure to enemy fire, real-time)
- Terrain features and some interactions with the environment can be expressed in terms of gaining or losing time
  - Moving on highways vs. swamps
  - Destructible obstacles along the way
- Tradeoff between search **space complexity** and **path quality**

State Space Generation

- Worlds can be huge
- Like to avoid cumbersome task of picking waypoints or room abstractions manually
- Should be **automatically generated** from world geometry

Finding Paths in Continuous Spaces

- Main approach: discretize continuous height field to create search graph
- Objects move on 2d surface, so mapping height field to plane is sufficient



Regular Grids

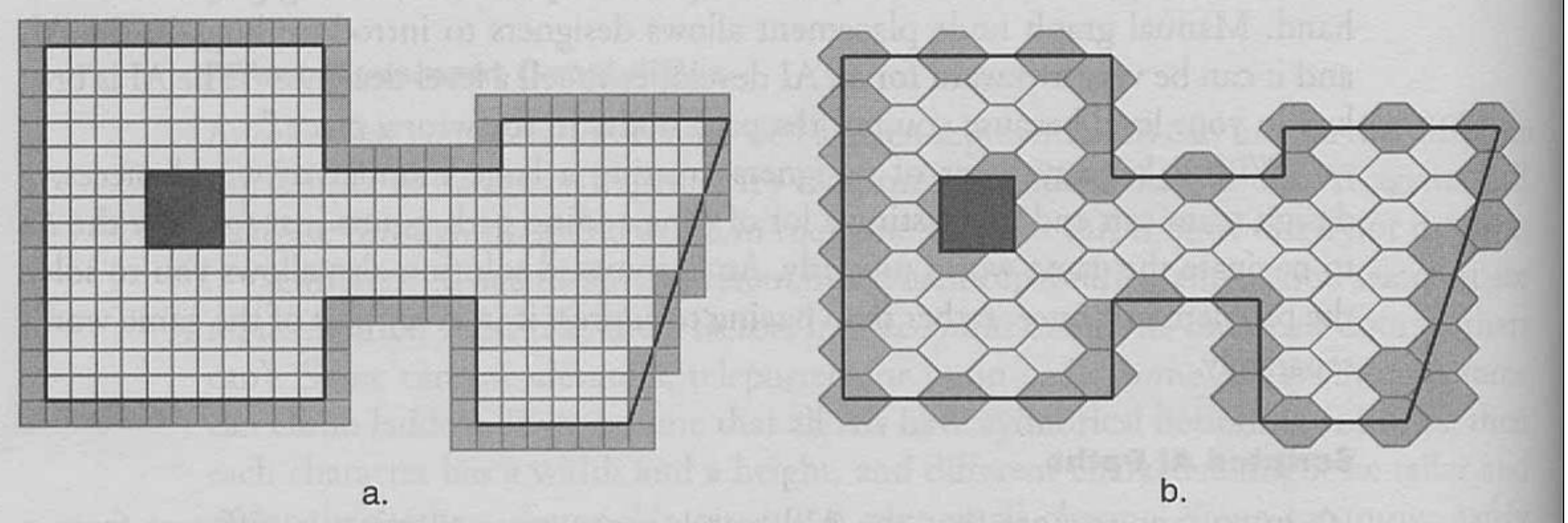
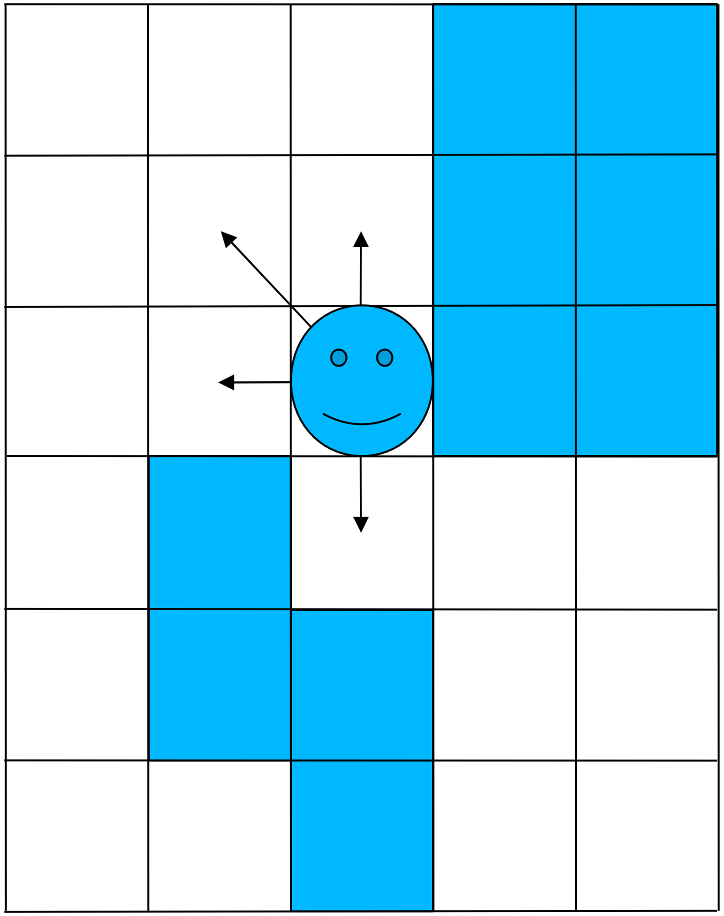


FIGURE 2.1.3 *Grid representations based on square and hexagonal cells.*

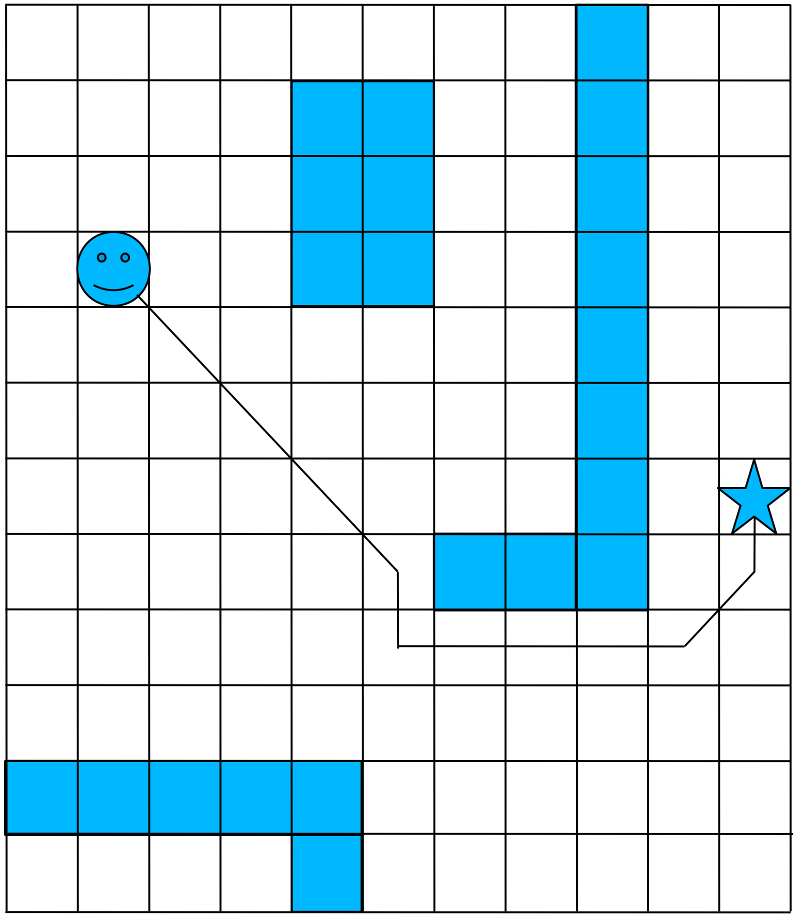
Grid-Based Methods

- Represent the environment by a grid of (usually square) cells
- Each cell is either traversable or obstructed
- Object (on a traversable cell) can move to any adjacent traversable cell



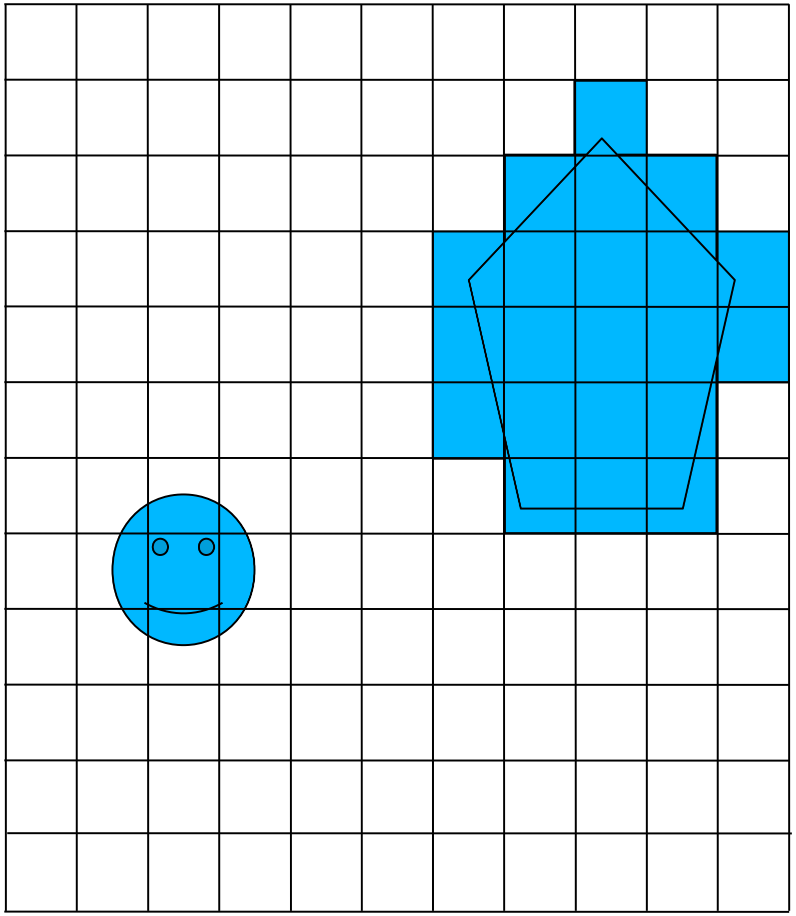
Grid-Based Methods: Advantages

- Conceptually simple representation
- Local changes have only local effects – well-suited for dynamic environments
- Perfectly represents tile-based environments
- Exact paths easy to determine for cell-sized objects



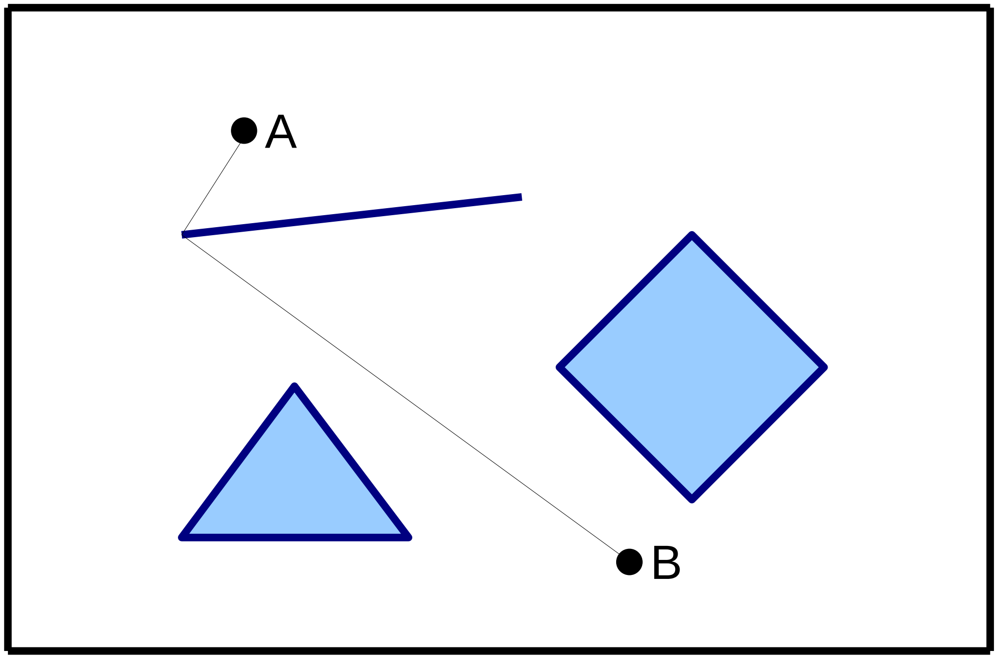
Grid-Based Methods: Disadvantages

- Imprecise representation of arbitrary barriers
- Increased precision in one area increases complexity everywhere – potentially large memory footprint
- Awkward for objects that are not tile-sized and shaped
- Need to post-process paths if environment allows arbitrary motion angles (or tweak A*)



Geometric Representations

- World is an initially empty simple shape
- Represent obstacles as polygons, i.e. sequences of line segments (also called constraints)
- Find path between two points that does not cross constraints



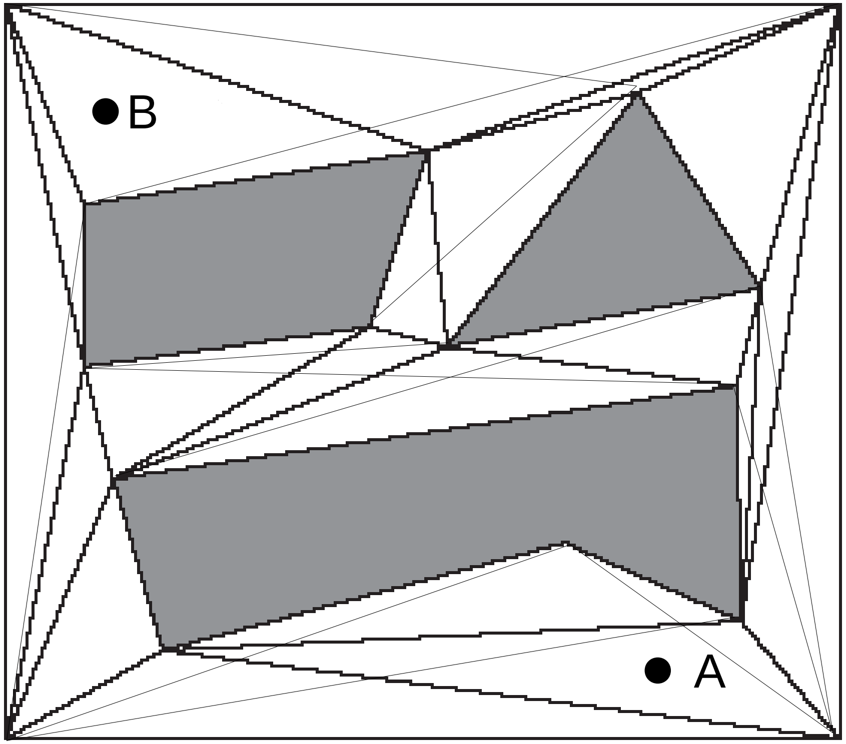
Geometric Methods

- **Advantages**
  - Arbitrary polygon obstacles
  - Arbitrary motion angles
  - Memory efficient
  - Finding optimal paths for circular objects isn't hard
  - Topological abstractions

- **Disadvantages**
  - Complex code
  - Robustness issues
  - Point localization takes more than constant time

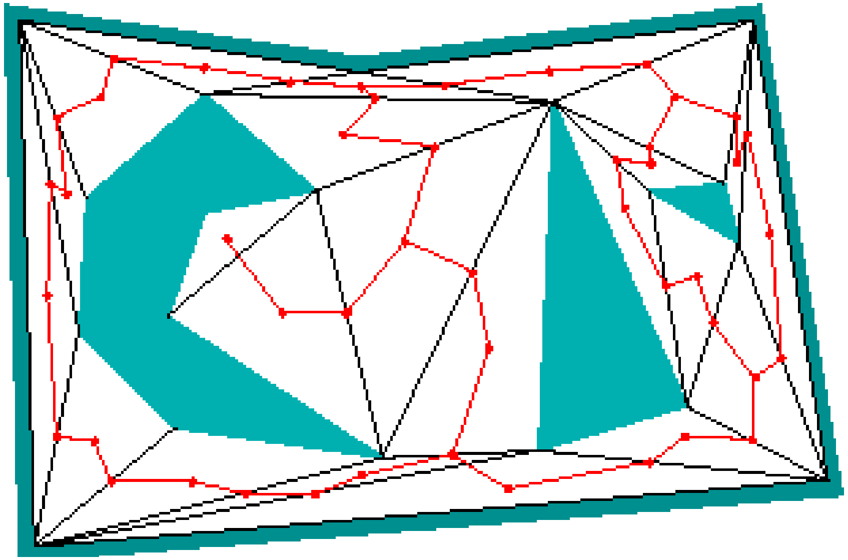
Visibility Graphs

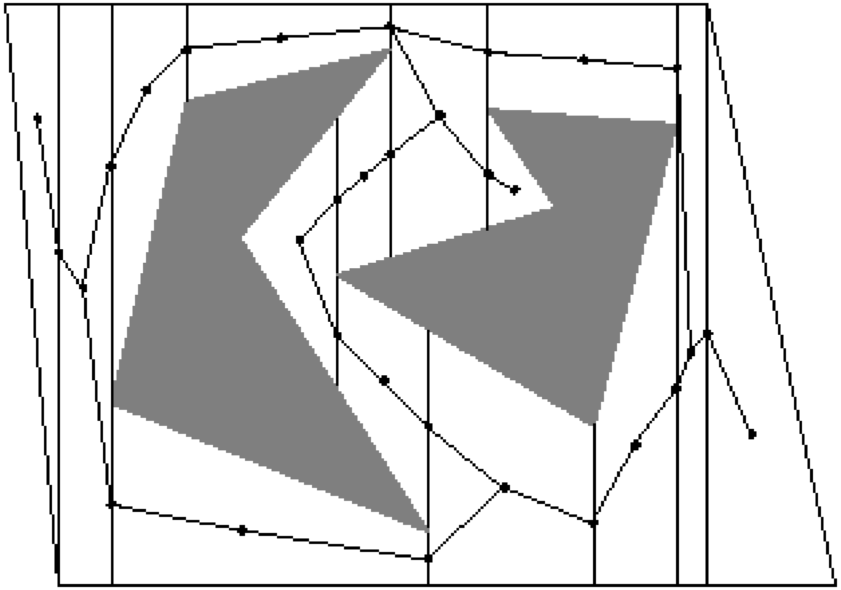
- Place nodes at corners of obstacles
- Place edges between nodes that can “see” each other
- Find path from A to B:
  - add these nodes to graph, connect to visible nodes
  - A* on resulting graph
- Path **provably optimal**
- But adding and changing world can be expensive as graph can be **dense**



Free-Space Decompositions

- Decompose empty areas into simple **convex shapes** (e.g. triangles, trapezoids)
- Create waypoint graph by placing nodes on unconstrained edges and in the face interior, if needed
- Connect nodes according to direct reachability
- Find path from A to B:
  - Locate faces in which A, B reside
  - Connect A, B to all face nodes
  - Run A*, smooth path





Local path finding

- Path planning algorithms must be able to deal with dynamic obstacles
- Adding / removing objects can be expensive in abstractions or geometry-based systems
- Can use simple object avoidance methods that try to follow high-level paths and resolve local conflicts

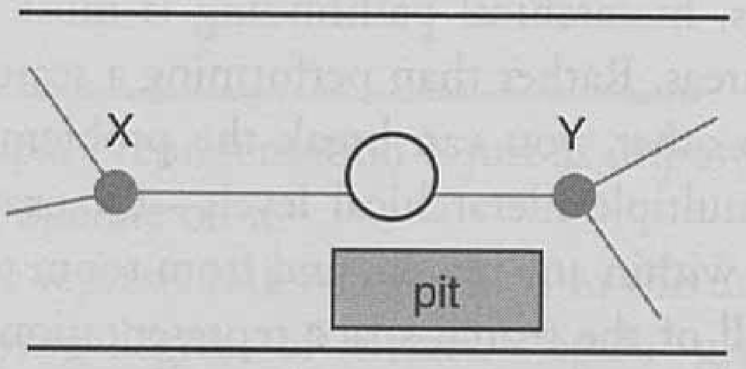
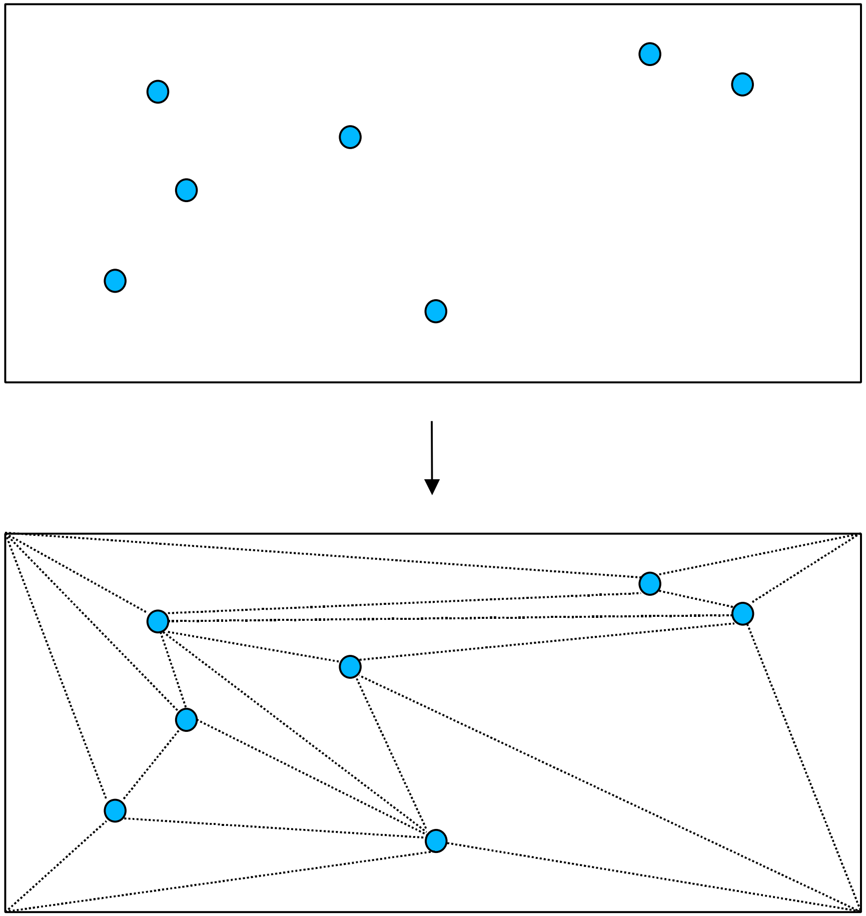


FIGURE 2.1.18 *A dynamic obstacle (the outlined circle) along an edge in a waypoint graph.*

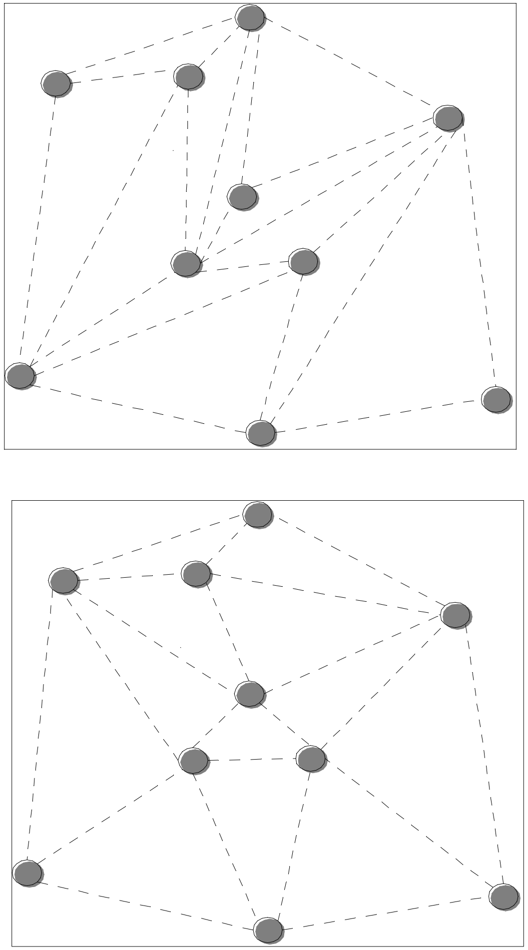
Triangulations

- Starting with an area (like a rectangle) and a collection of points
- Add edges between the points without such edges crossing
- Continue until no more such edges can be added

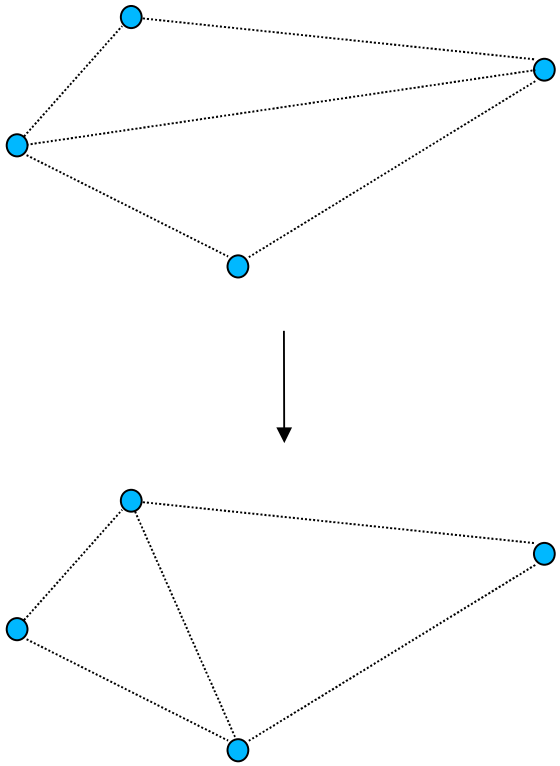


Triangulation Quality

- For a given point set many triangulations exist
- We would like to avoid sliver-like triangles which decrease locality and the quality of distance heuristics



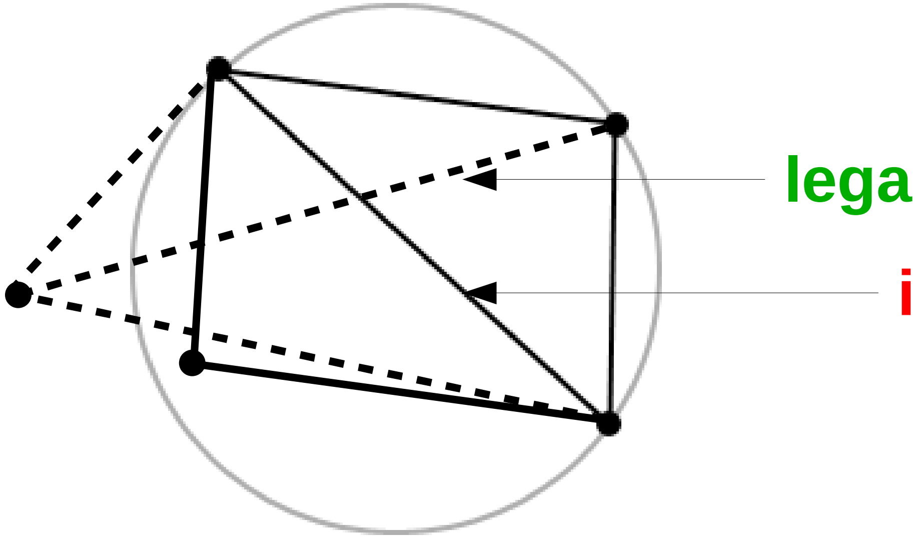
Delaunay Triangulations



- Triangulations in which the minimum interior angle of all triangles is maximized
- Makes “nice” triangulation: tends to avoid thin, sliver-like triangles
- Can be done locally by “edge flipping” diagonals across quadrilaterals

Delaunay Triangulation Characterization

A triangulation maximizes the minimal angle iff the circumcircle of any triangle does not contain another point in its interior



legal

illegal – needs to be flipped

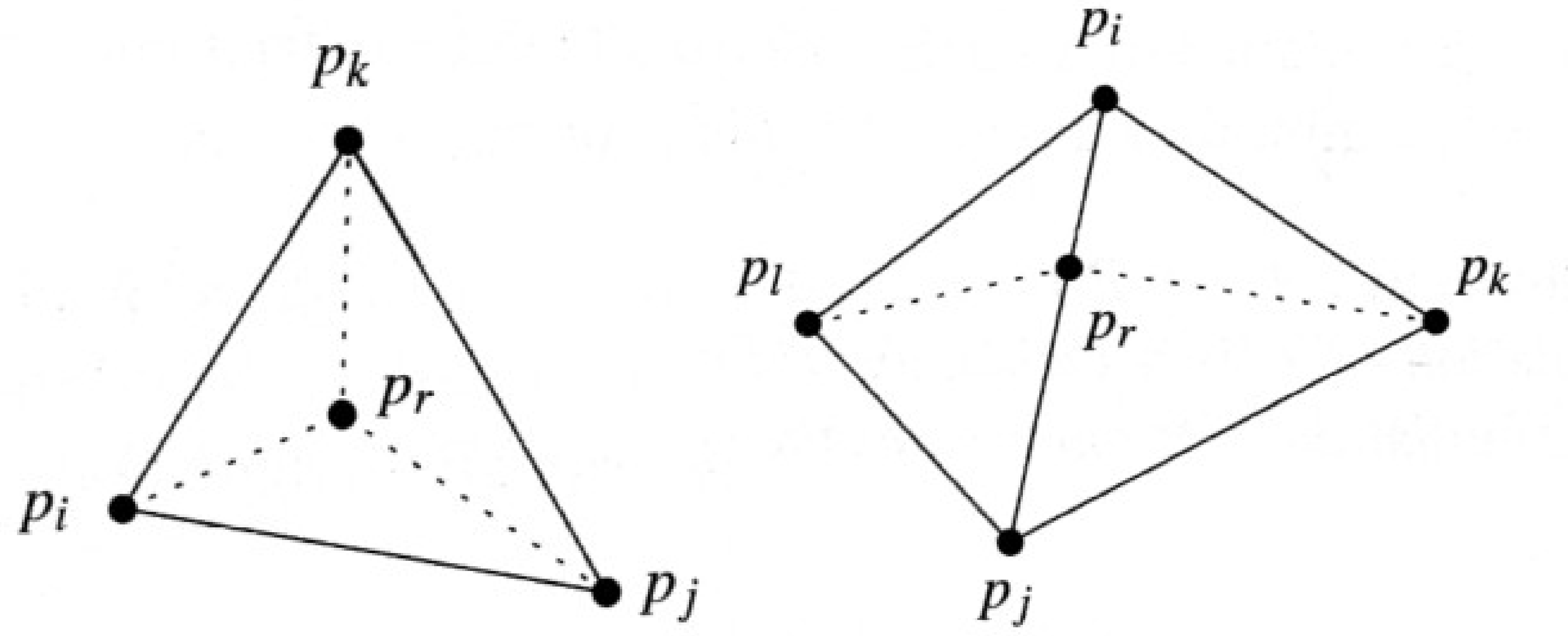
Computing Delaunay Triangulations

1. Initialize triangulation *T* with a "big enough" helper bounding triangle that contains all points of P
2. Randomly choose a point p_r from P
3. Find the triangle Δ that p_r lies in
4. Subdivide Δ into smaller triangles that have p_r as a vertex
5. Flip edges until all edges are legal
6. Repeat steps 2-5 until all points have been added to *T*

Randomized algorithm. Expected runtime O(n log n)

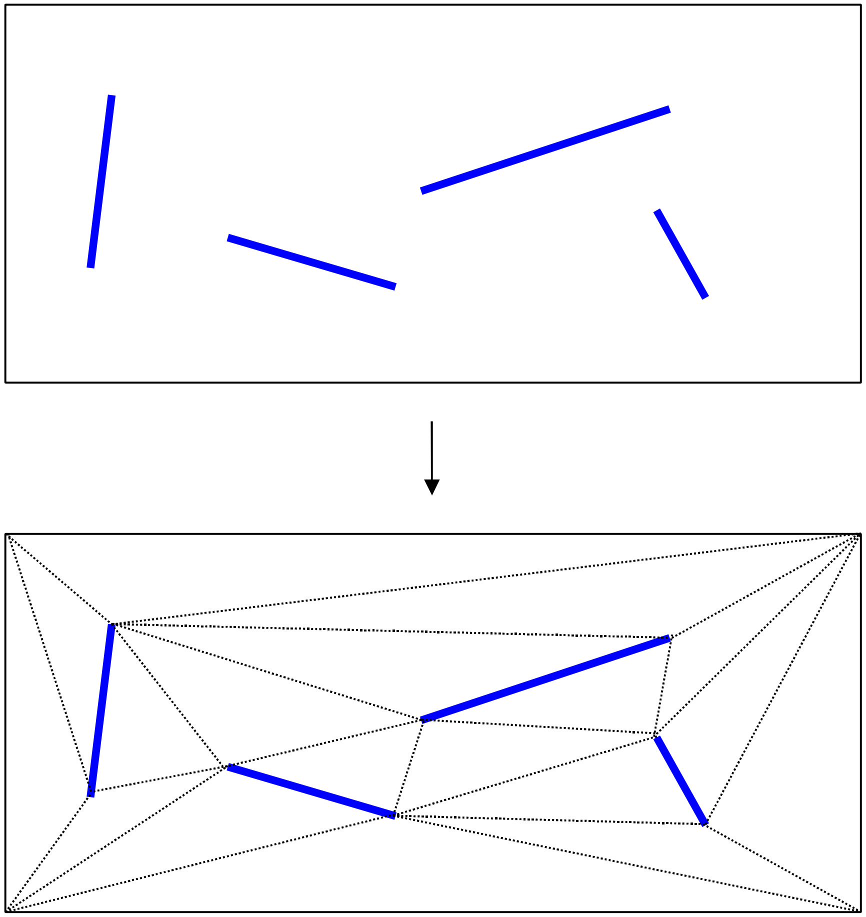
Can also be computed using Divide & Conquer

Inductive Step



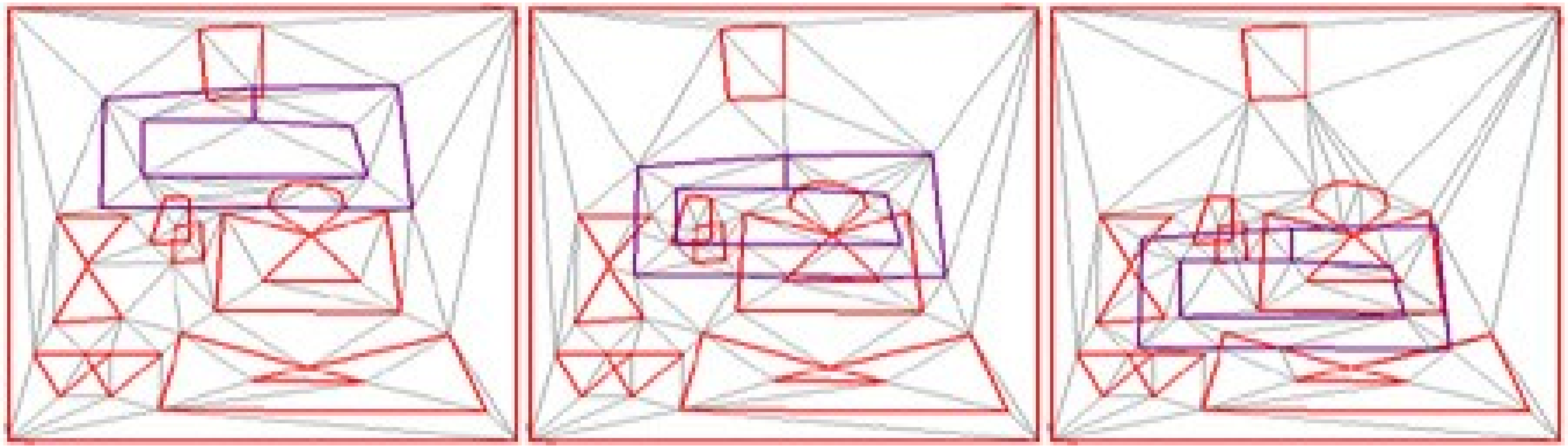
Constrained Triangulations

- Triangulations where certain (constrained) edges are required to be in the triangulation
- Then other (unconstrained) edges are added as before
- Constrained Delaunay Triangulations maximize the minimum angle while keeping constrained edges
- Above algorithm can be used with modifications



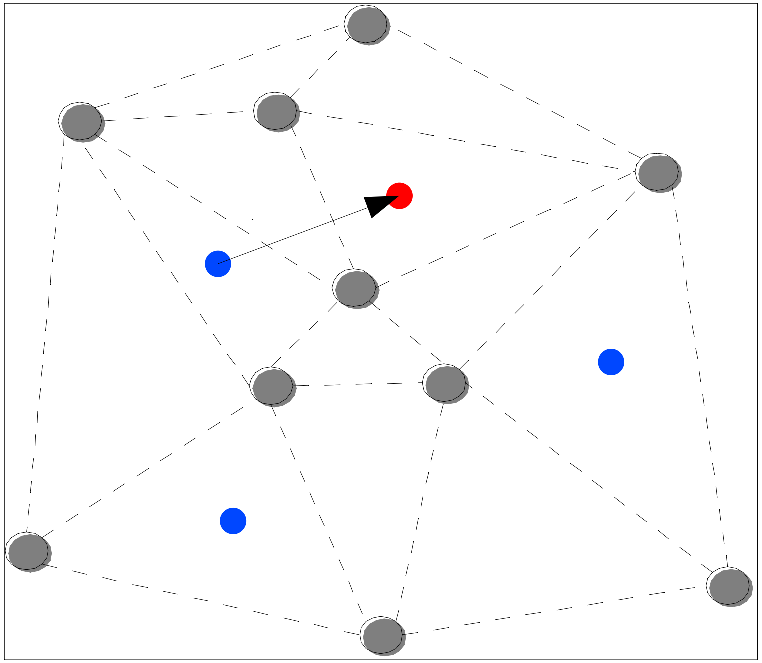
Dynamic Constrained Delaunay Triangulations (DCDT)

- Marcelo Kallmann's DCDT software can repair a triangulation dynamically when constraints change
- Repairs can be made using local information allowing it to work in a real-time setting



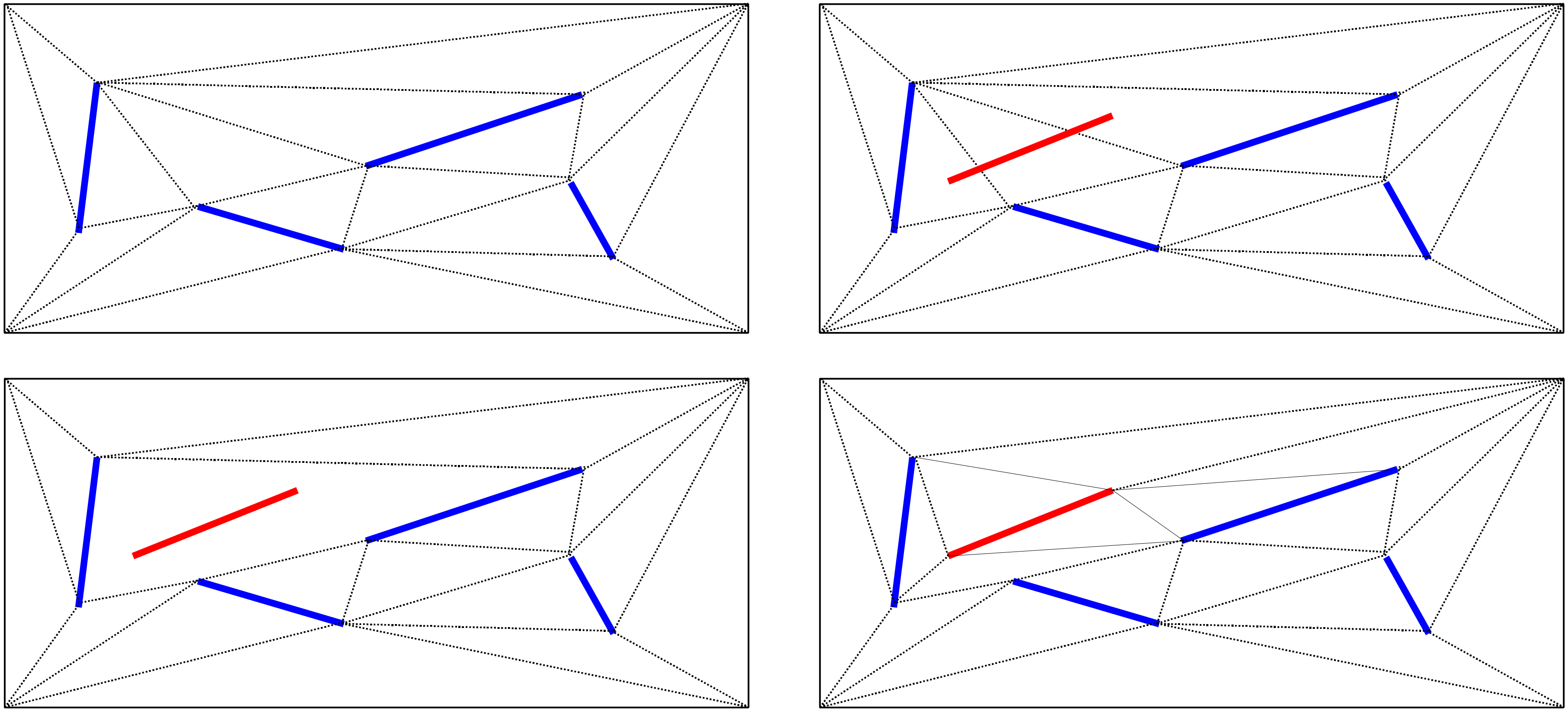
How DCDT Works

- **Point localization.** Algorithms usually construct a DAG for localizing points in time O(log n)
  - Maintaining this DAG is complicated
  - "Jump and Walk" algorithm much simpler and quite efficient ($O(n^{1/3})$ in DTs)
- **Repairing the triangulation** after changing constraints is not trivial either but takes amortized constant time (mostly local operations)



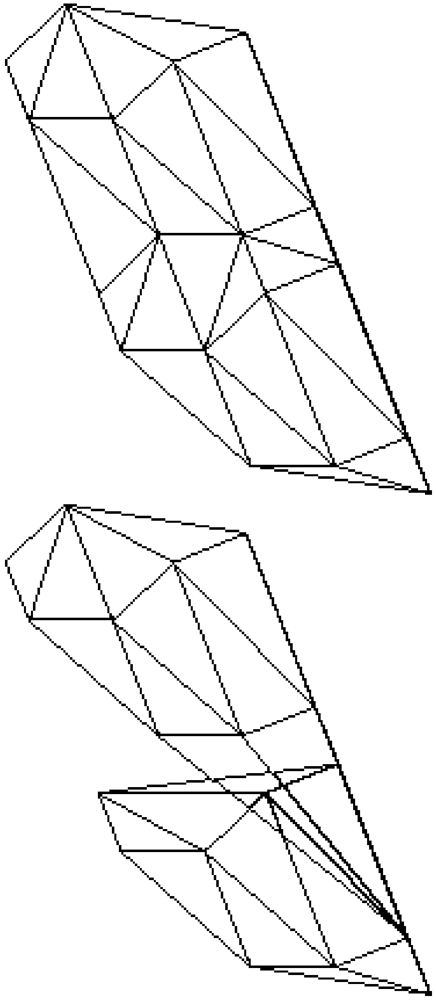
Sample triangles and walk towards the location starting with the closest triangle

Example: Add Constraint



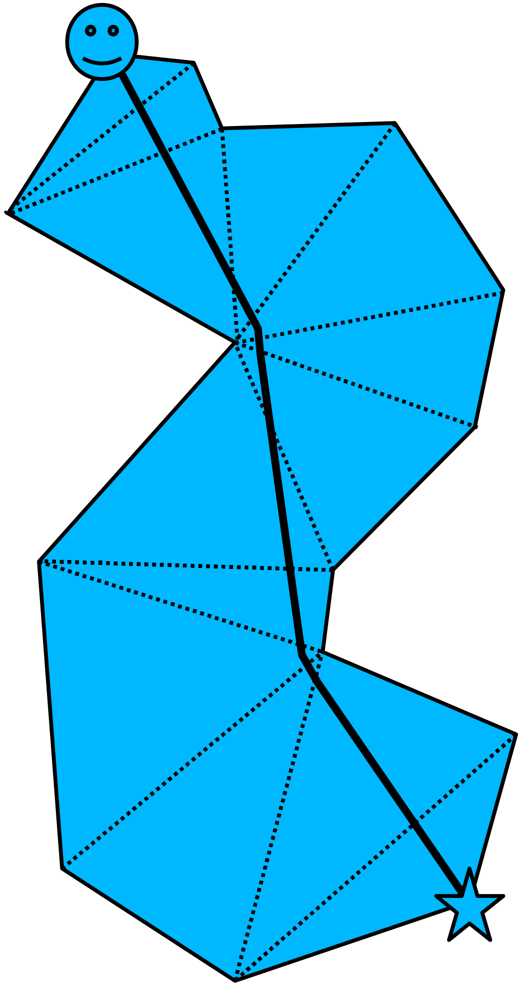
Robustness of Geometric Computations

- Using fixed-length floating point arithmetic can cause geometric algorithms
  - to **crash**
  - to **hang**
  - to produce **incorrect** output
- Kallmann's DCDT software suffers from this in rare cases
- We are working on a GPL'ed DCDT implementation that overcomes this problem by using rational and interval arithmetic



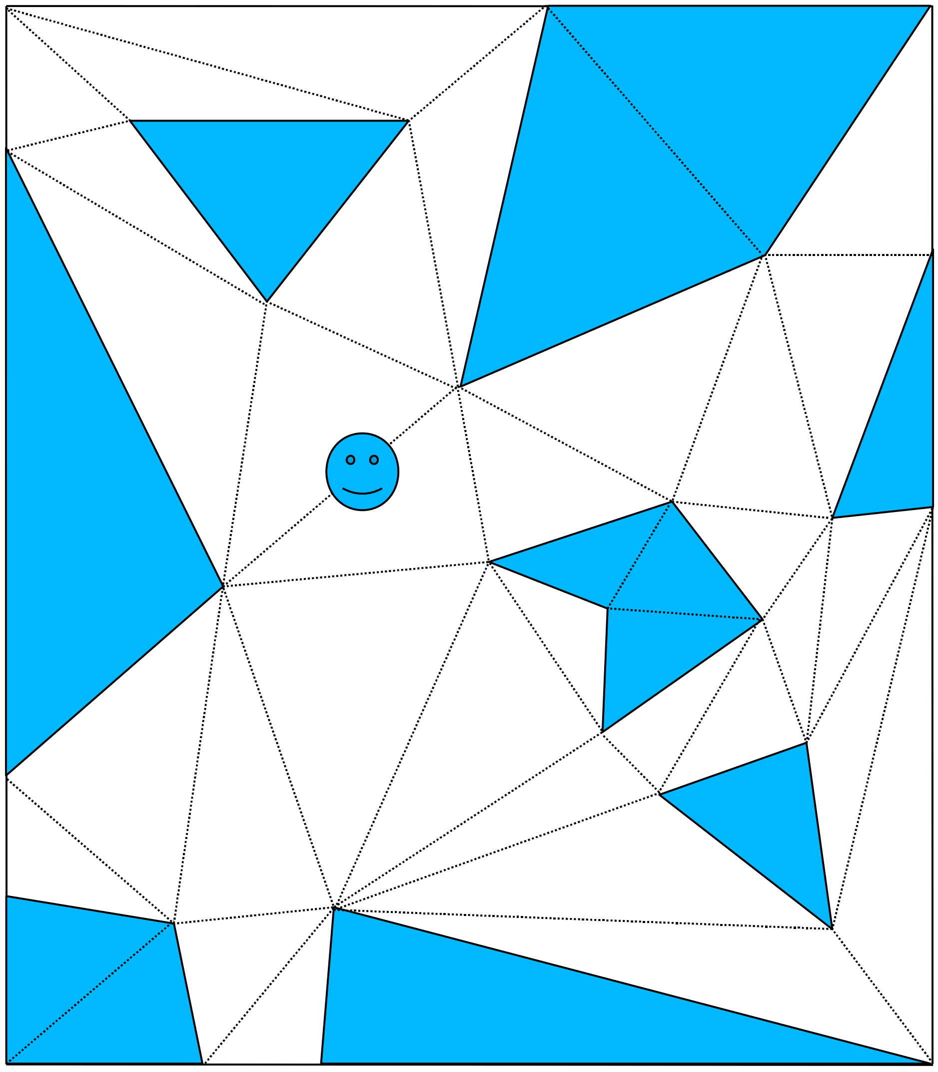
Triangulation-Based Pathfinding

- Using a constrained triangulation with barriers represented as constraints
- Find which triangle the start (and goal) point is in
- Search adjacent triangles across unconstrained edges
- Finds a ***channel*** of triangles inside which we can easily determine the shortest path



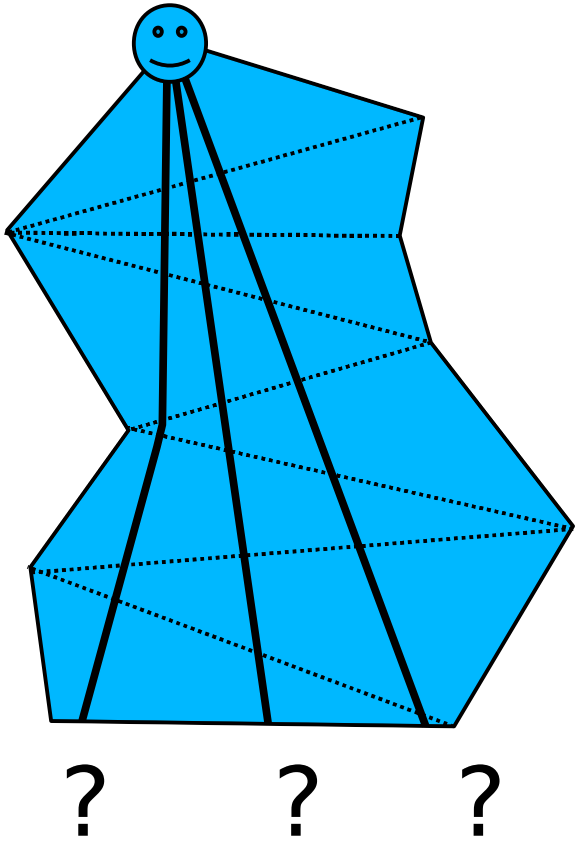
Triangulation-Based Pathfinding: Advantages

- Remedies grid-based methods' deficiency with off-axis barriers
- Representing detailed areas better doesn't complicate “open” areas
- Triangulations have much fewer cells and are more accurate than grids
- Can deal with non-point objects quite easily (below)



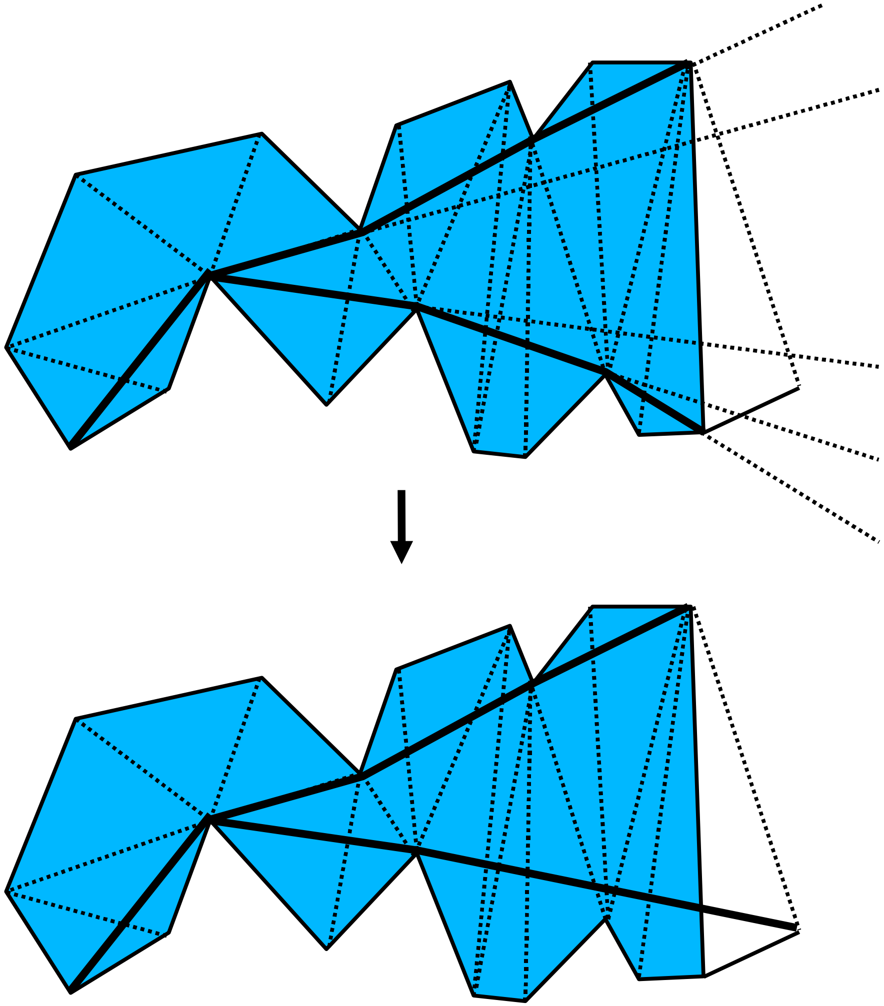
Triangulation-Based Pathfinding: Disadvantages

- Curved obstacle barriers must be approximated by straight segments
- We do not know what path we will take through the triangles until after we have found the goal
- Can lead to either suboptimal paths or multiple paths to triangles



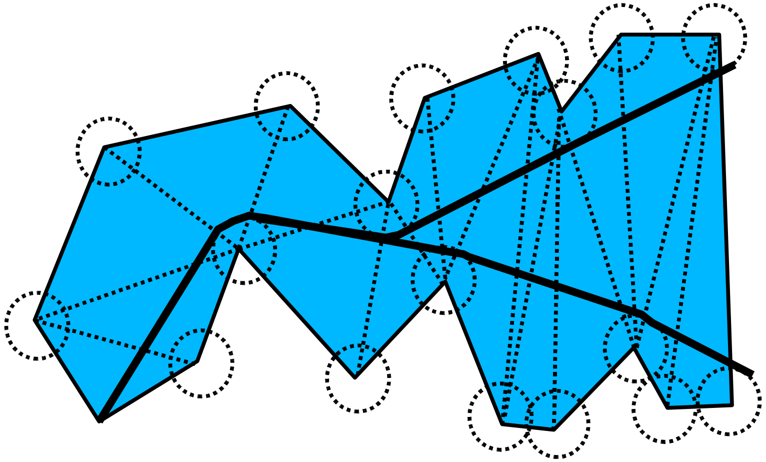
Funnel Algorithm

- To find the exact path through a channel of triangles, we use the *funnel algorithm*
- Finds the shortest path in this simple polygon in time linear in the number of triangles in it
- Maintains a ***funnel*** which contains the shortest path to the end of the channel so far
- Funnel is updated for each new vertex in the channel



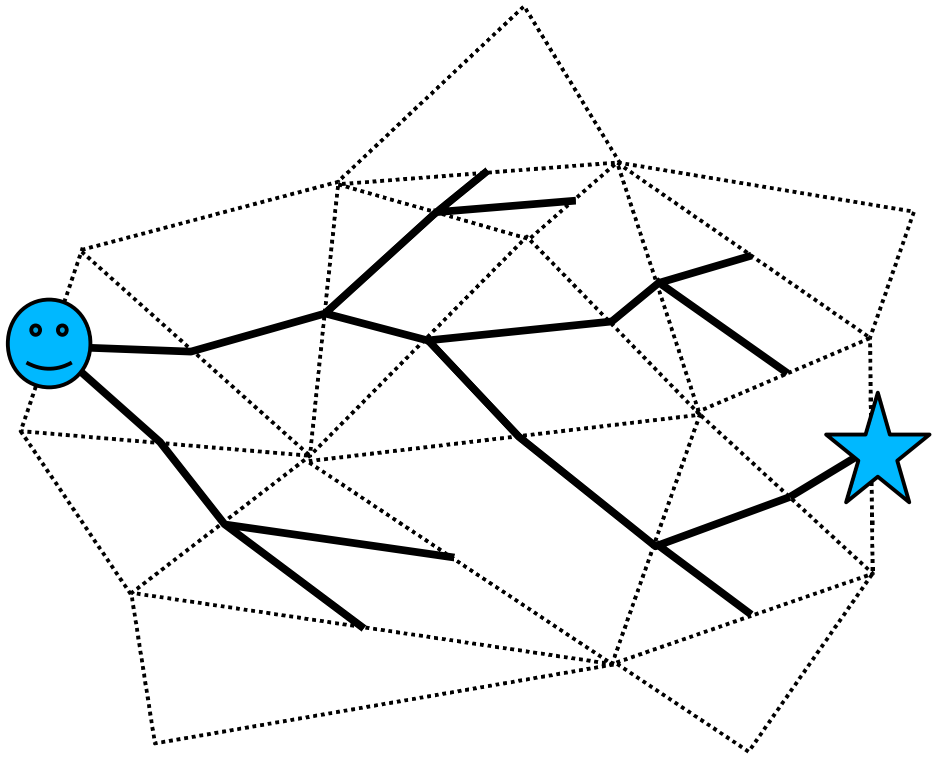
Modified Funnel Algorithm

- For circular units with non-zero radius
- Conceptually attach circles of equal radius around each vertex of the channel
- Consider segments tangent to these circles and arcs along them



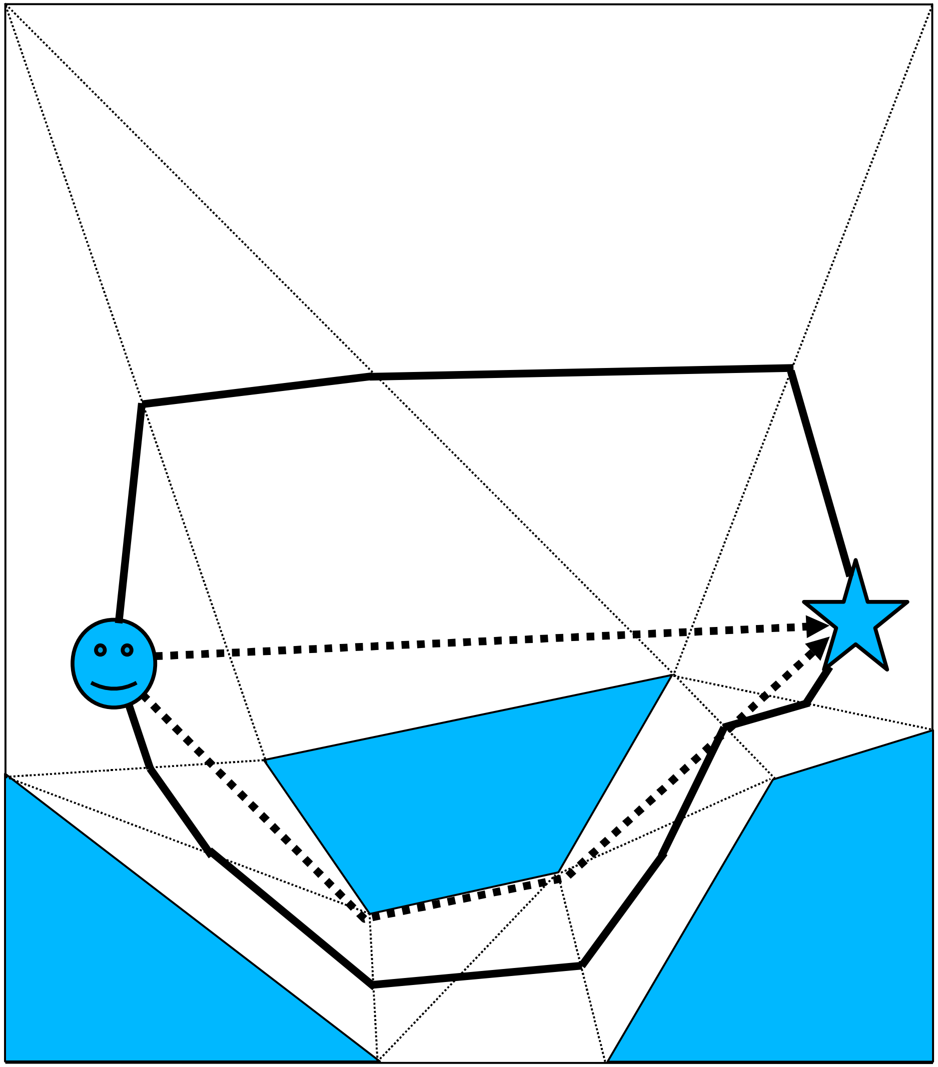
“Naive” Search

- Assume, while searching, that we know the exact path through the triangles
- Use this to prune search states
- For example, assume straight-segment paths between edge midpoints



“Naive” Search: Advantages and Disadvantages

- Considers each triangle once and has fairly good distance measures
- So finds paths quickly
- However, in cases like the example on the right, thinks a path through the bottom channel is shorter than one through the top
- So it may result in suboptimal paths



How To Find Optimal Paths?

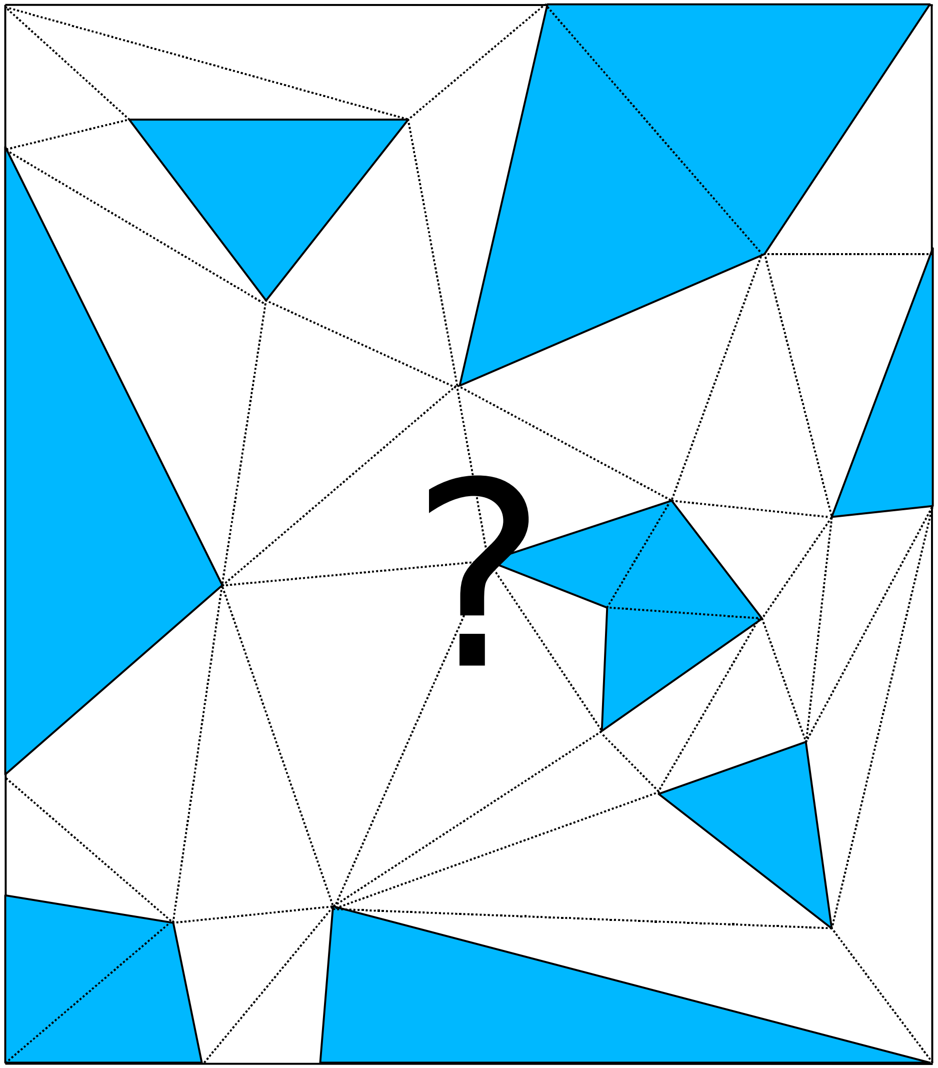
- (Under)estimate the distance travelled so far
- Allow multiple paths to any triangle
- When a channel is found to the goal, calculate the length of the shortest path in this channel
- If it is the shortest path found so far, keep it, otherwise, reject it (anytime algorithm)
- When the distance travelled so far for the paths yet to be searched exceeds the length of the shortest path, the algorithm ends and we have an optimal path

Triangulation A* (TA*)

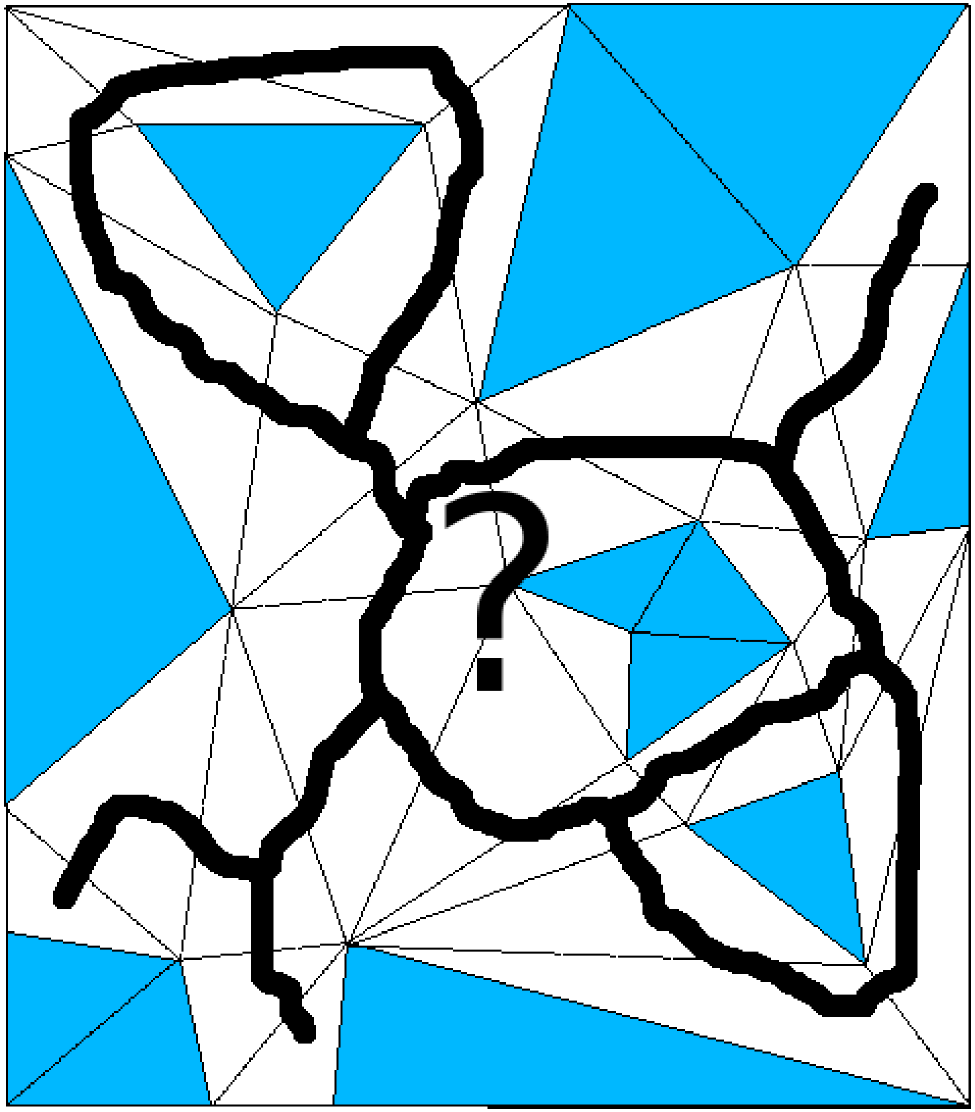
- Search running on the base triangulation
- Uses a triangle for a search state and the adjacent triangles across unconstrained edges as neighbors
- Using anytime algorithm and considering multiple paths to a triangle as described earlier
- For a heuristic (h-value), take the Euclidean distance between the goal and any point on the triangle's entry edge
- Calculate an underestimate for the distance-travelled-so-far (g-value)
- Only considers triangles once until the first path is found

Triangulation Reduction

- Want to reduce the triangulation without losing its topological structure
- Determine triangles as being decision points, on corridors, or in dead ends
- Map a triangle to a degree-*n* node when it has exactly 3-*n* triangles adjacent across unconstrained edges that are not mapped to degree-1

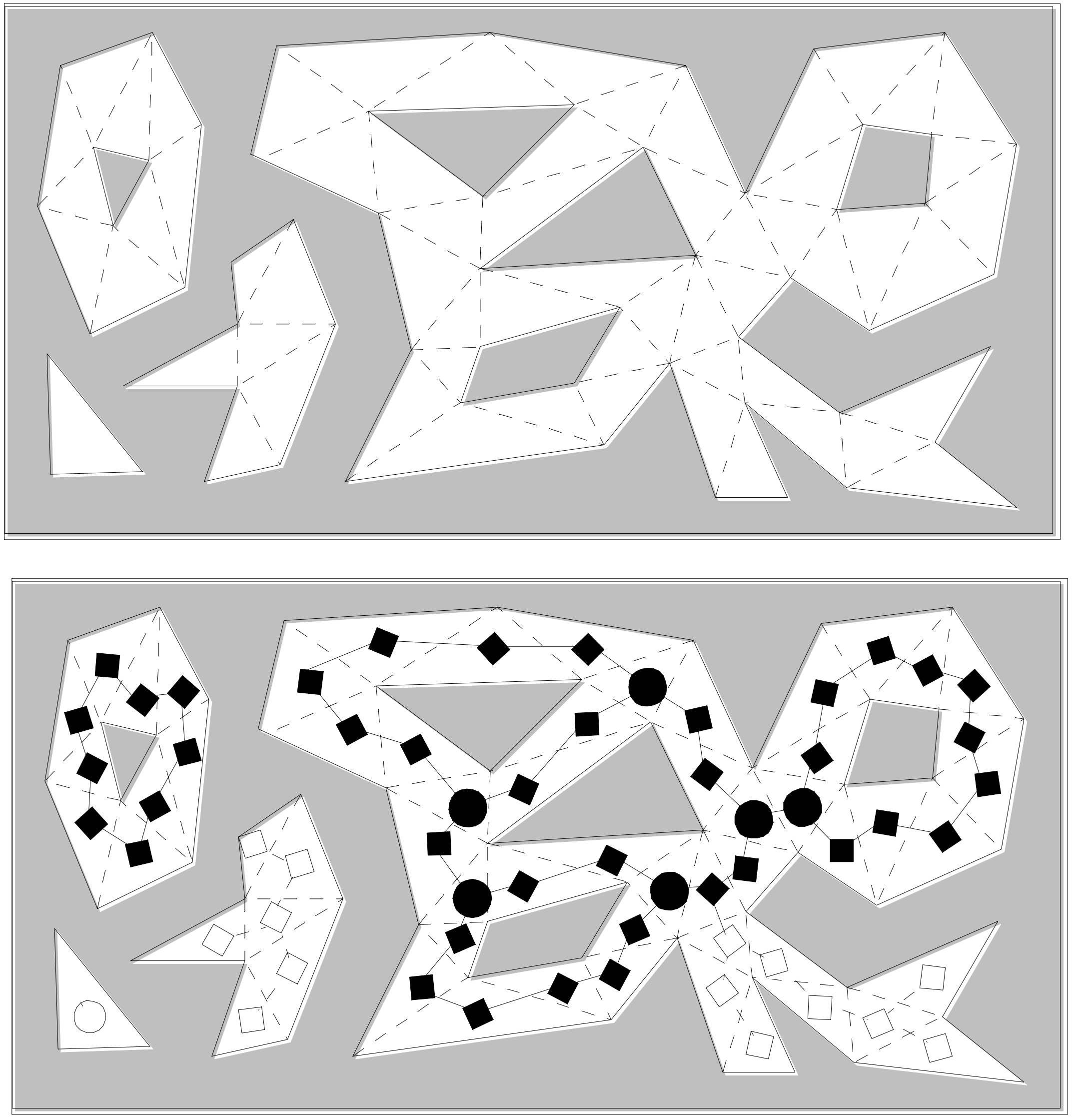


Topological View

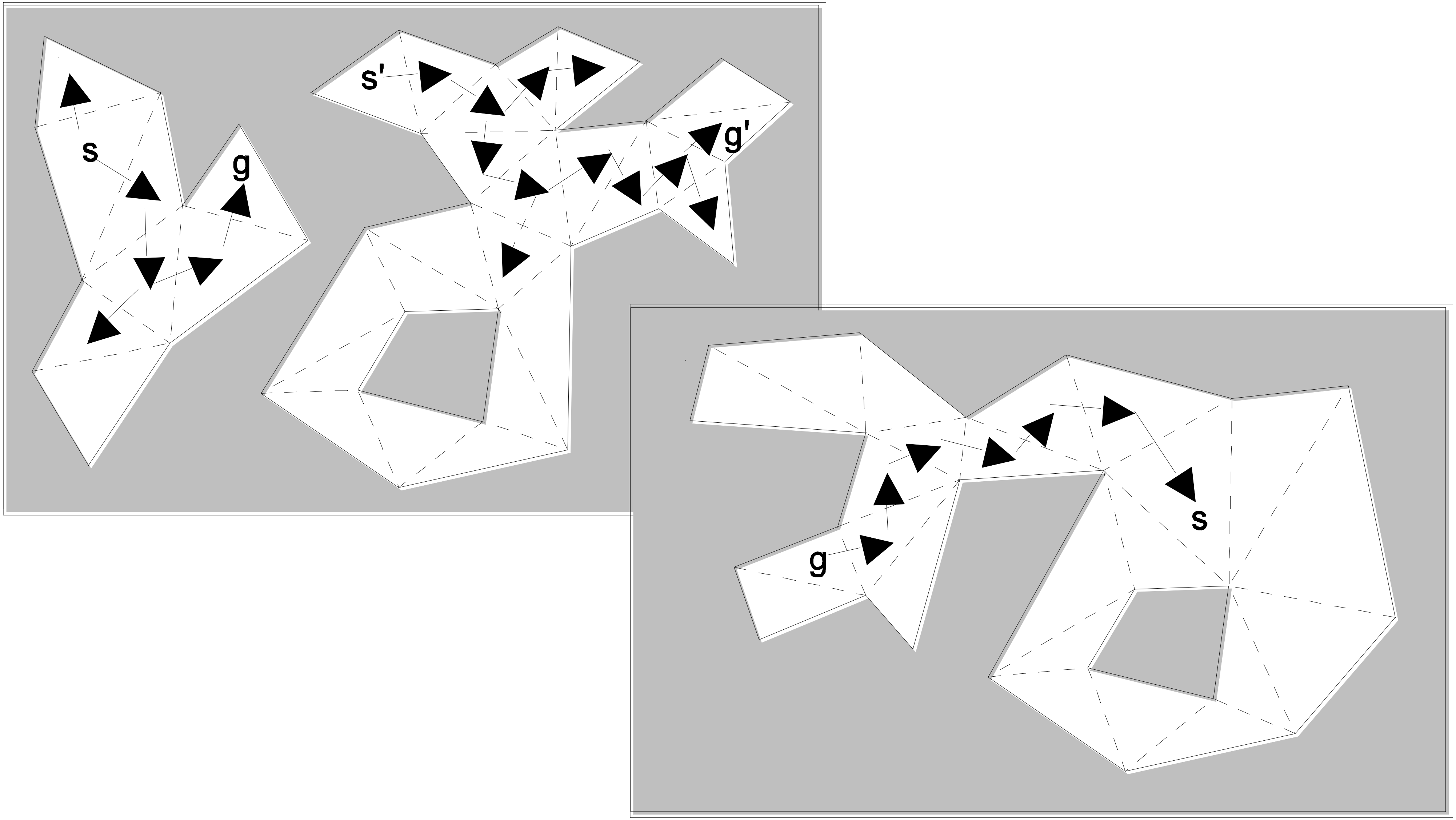


Reduction Example

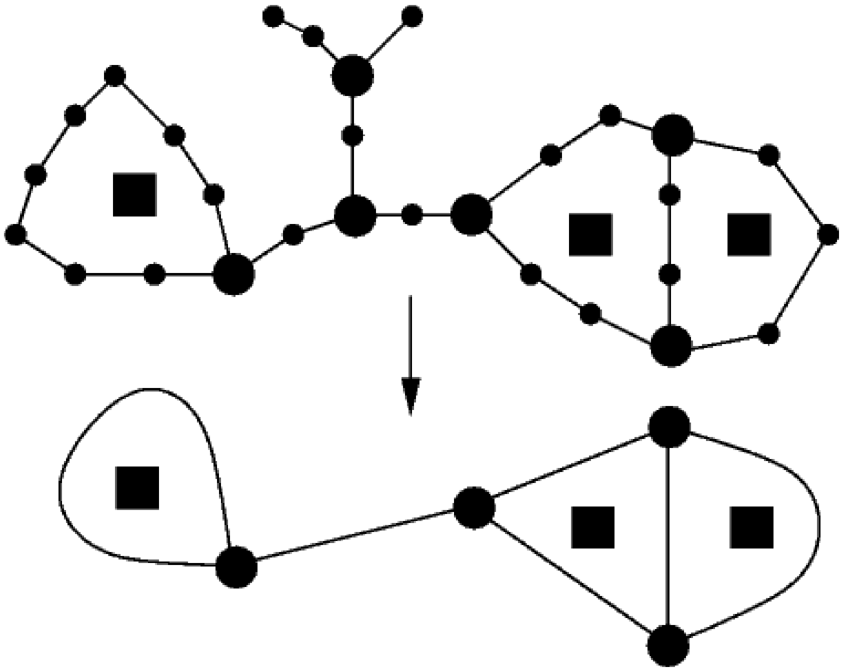
- Pathfinding in **tree components** (degree-1, empty squares) and **corridors** (degree-2, solid squares) is trivial
- The only real choice points are **degree-3 triangles** (solid circles)
- The resulting search graph has size linear in the number of islands!



Simple Special Cases: No Search Required

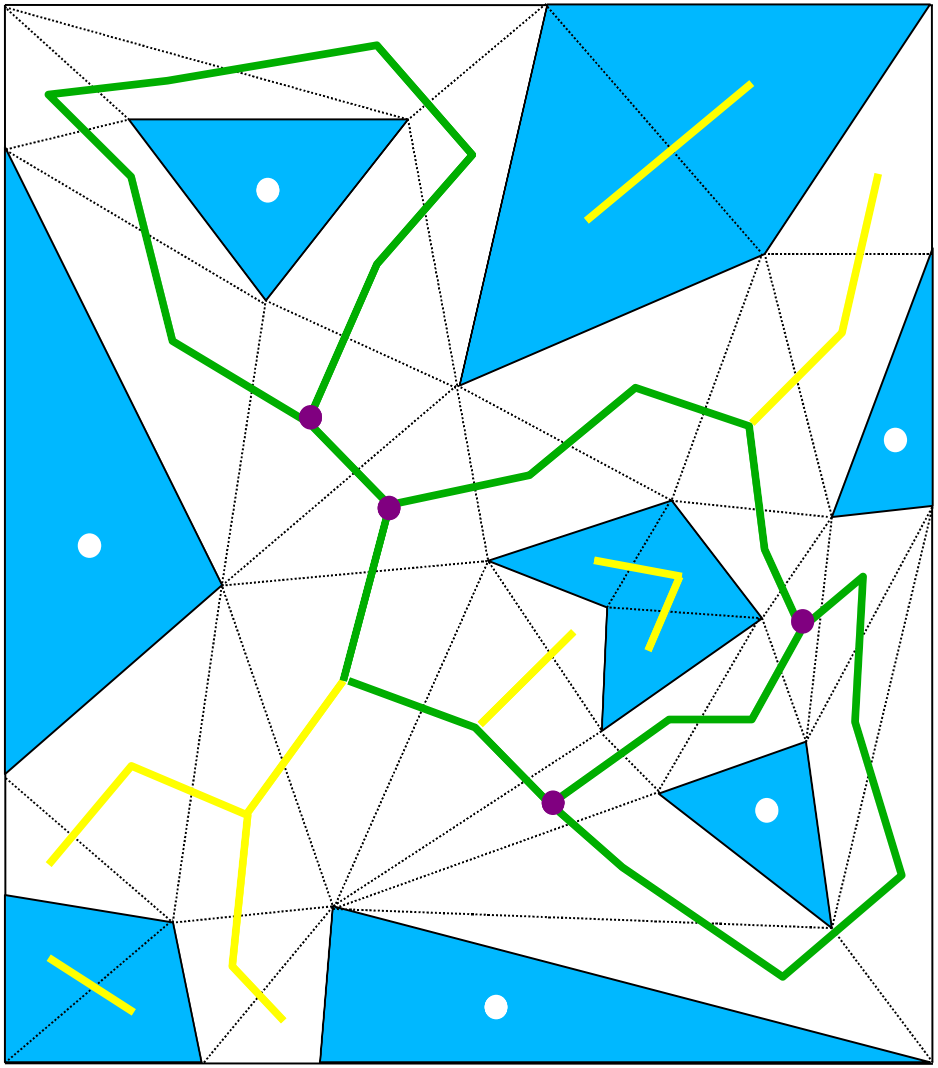


Typical Triangulation Graph and its Reduced Form



Abstraction Information

- Adjacent structures
- Choke points (the narrowest point between this triangle and the adjacent structure)
- A lower bound on the distance to each adjacent structure
- The triangle “widths”
- Using this graph can find paths for differently sized objects



Triangulation Reduction A* (TRA*)

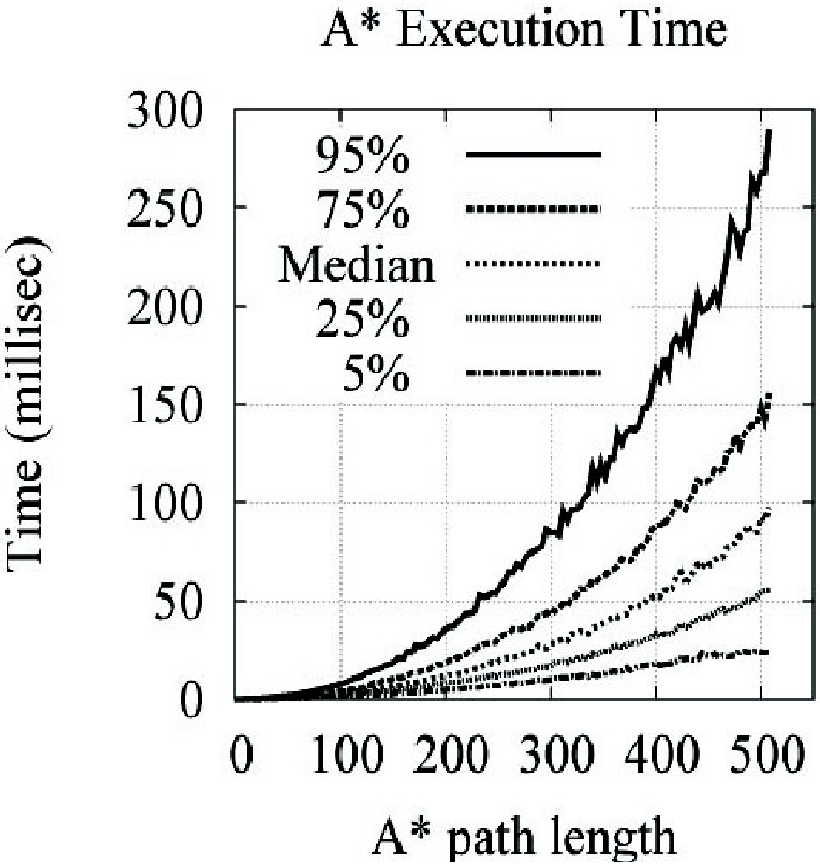
- TA* running on the abstraction just described
- First check for a number of “special cases” where no actual search needs to be done
- Move from the start and goal to their adjacent degree-3 nodes
- Use degree-3 nodes as search states and generate their children as the degree-3 nodes adjacent across corridors
- As with TA*, use an anytime algorithm, allowing multiple paths to a node, and use the same g- and h-values

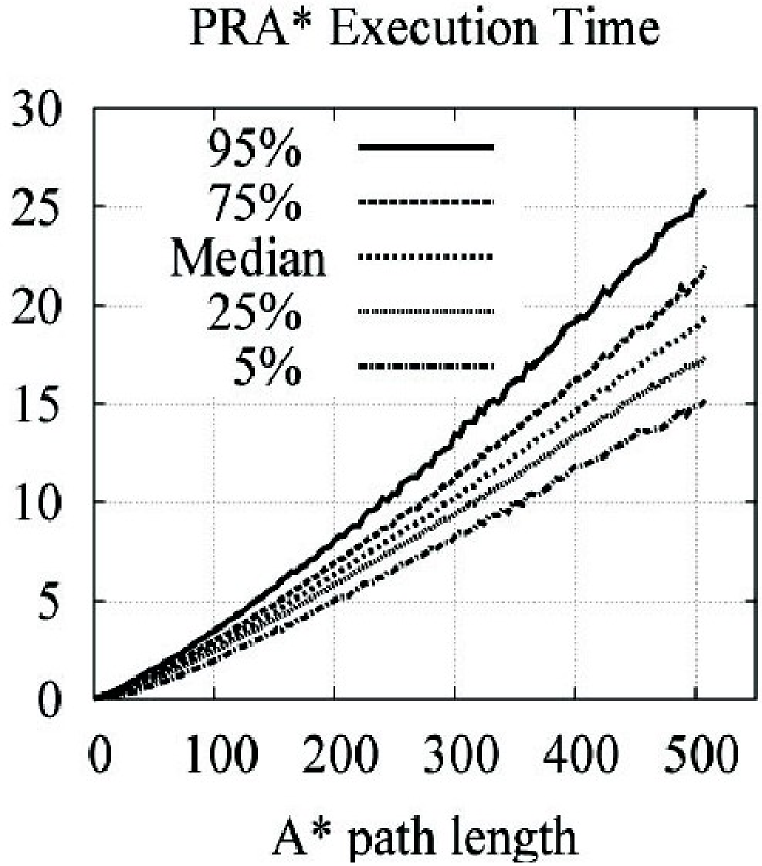
Experimental Setup

- 116 maps scaled to 512 x 512 tiles:
  - 75 Baldur's Gate maps (grid of tiles marked traversible or untraversible)
  - 41 WarCraft III maps (grid of types of terrain and heights where paths cannot cross height differences without ramps or boundaries between different types of terrain)
- 1280 paths in each, with A* length between 0 and 511 and categorized into one of 128 buckets based on length
- Compared TA* and TRA* to A* and PRA* using these same maps and paths

Experimental Results

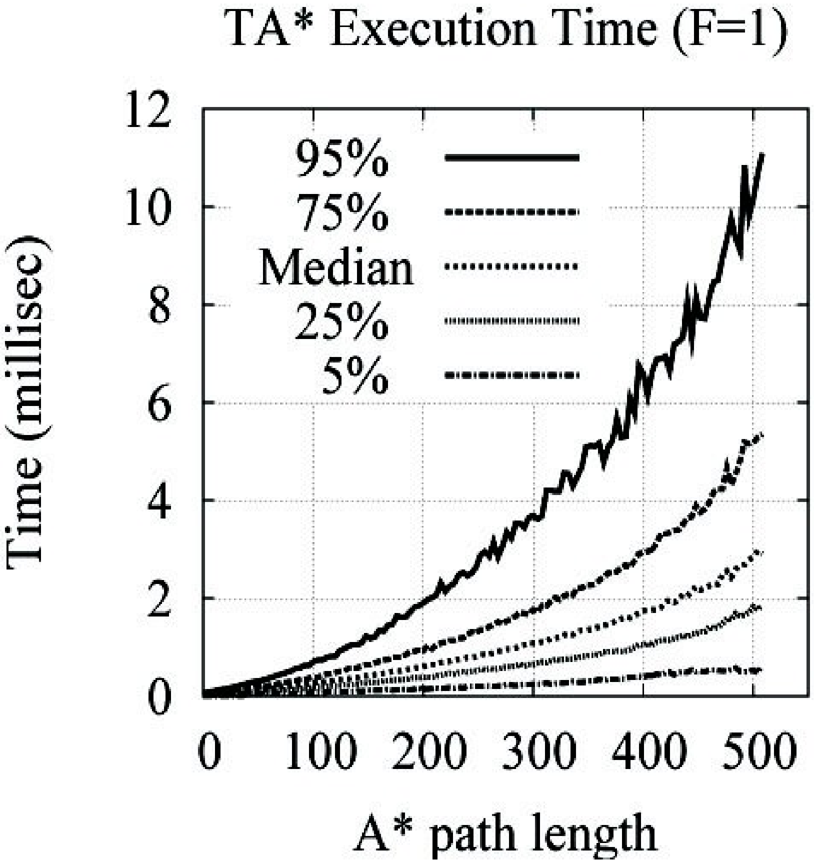
- Execution times of standard A* and PRA*

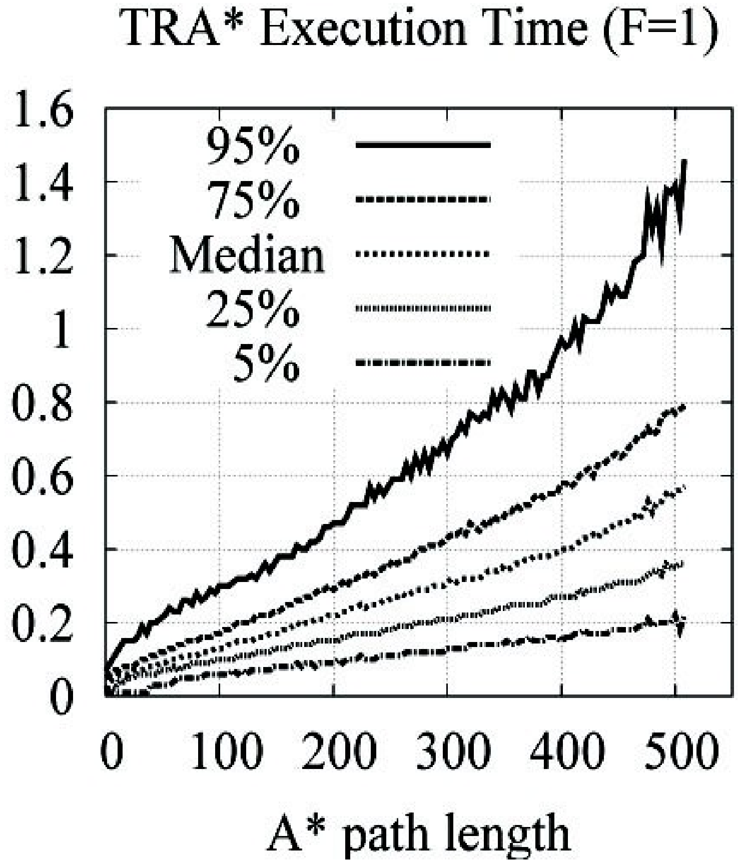

A* Execution Time


PRA* Execution Time

Experimental Results, Cont'd

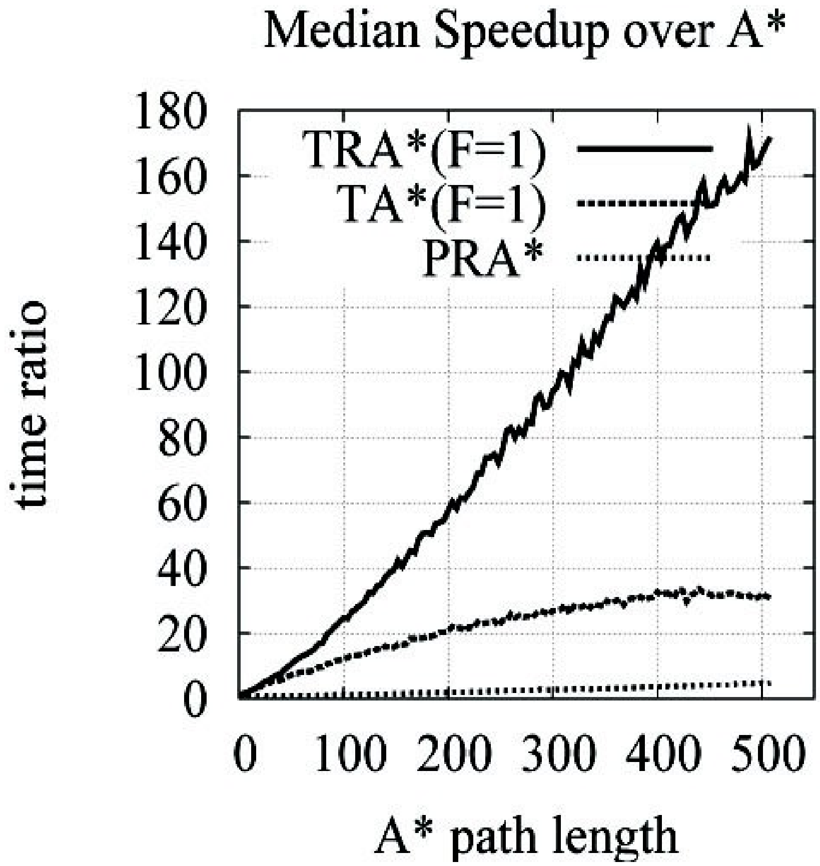
- First paths found by TA* & TRA* (not searching duplicates)

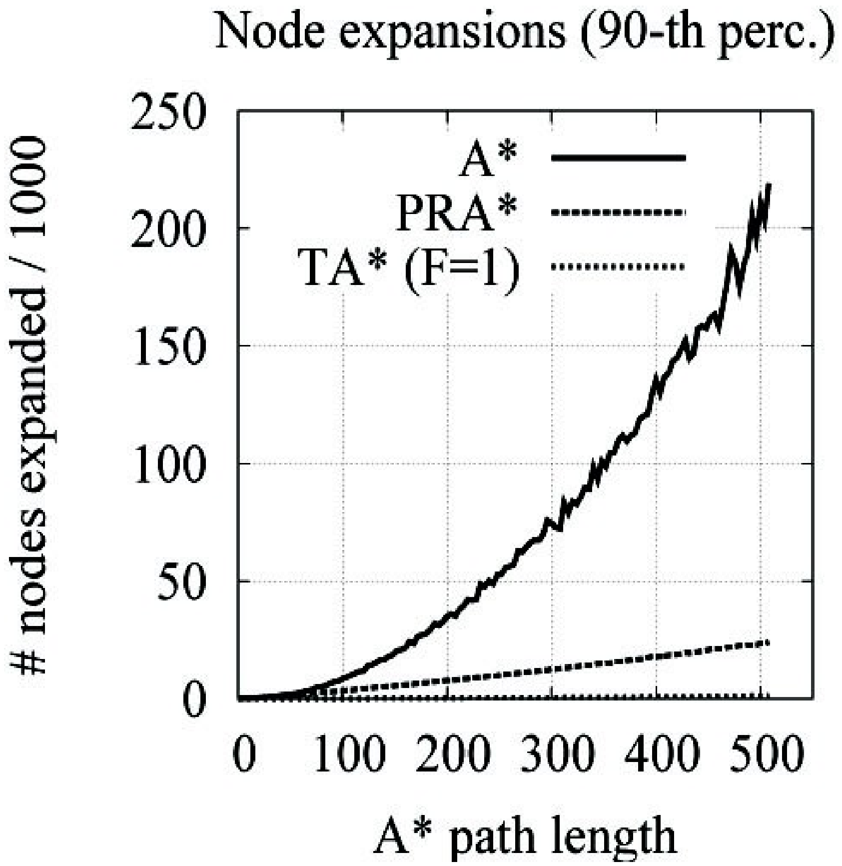

TA* Execution Time (F=1)


TRA* Execution Time (F=1)

Experimental Results, Cont'd

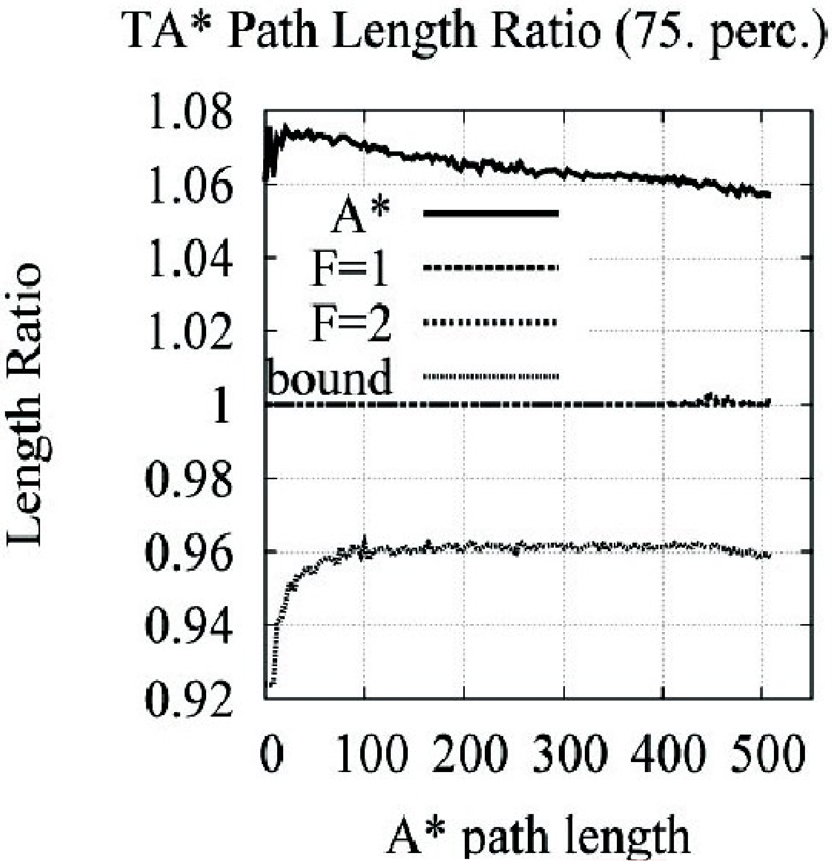
- Speedup comparison and nodes expanded

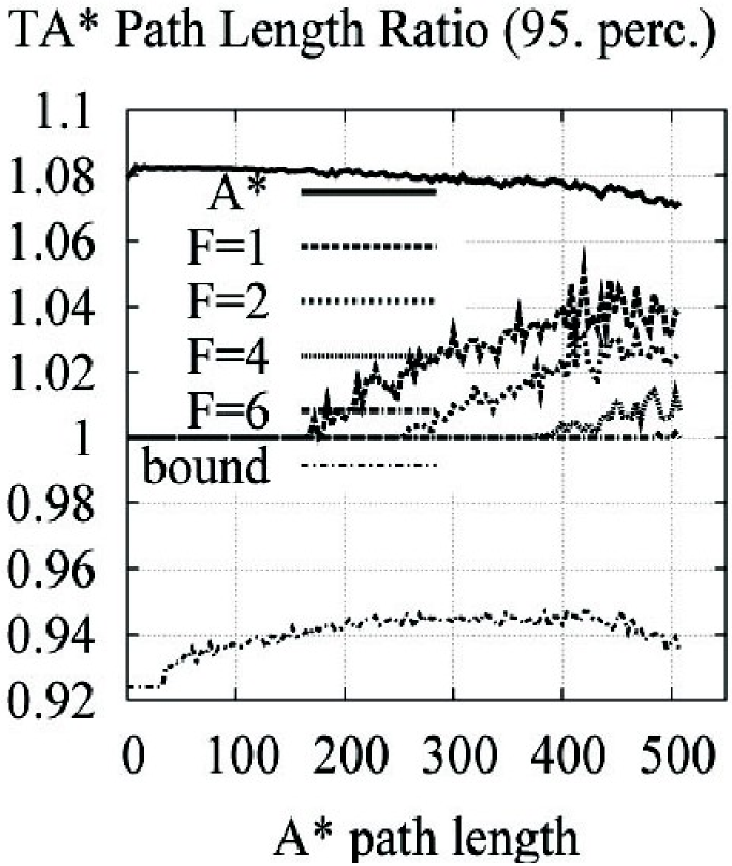

Median Speedup over A*


Node expansions (90-th perc.)

Experimental Results, Cont'd

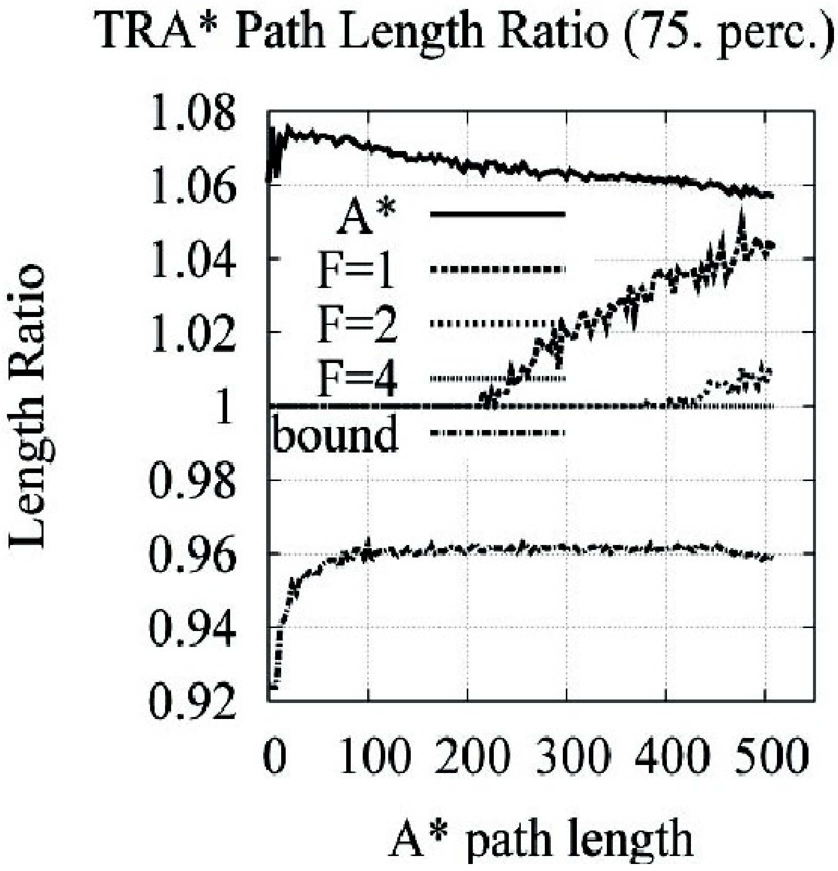
- TA* path length ratios compared to A* and lower bound

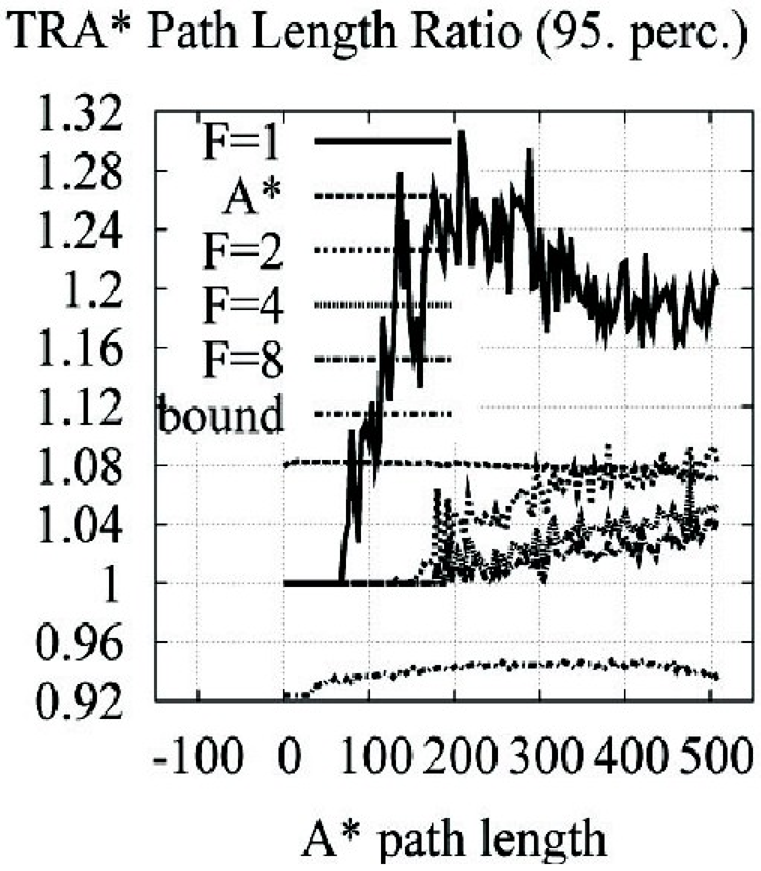

TA* Path Length Ratio (75. perc.)


TA* Path Length Ratio (95. perc.)

Experimental Results, Cont'd

- TRA* path length ratios compared to A* and lower bound



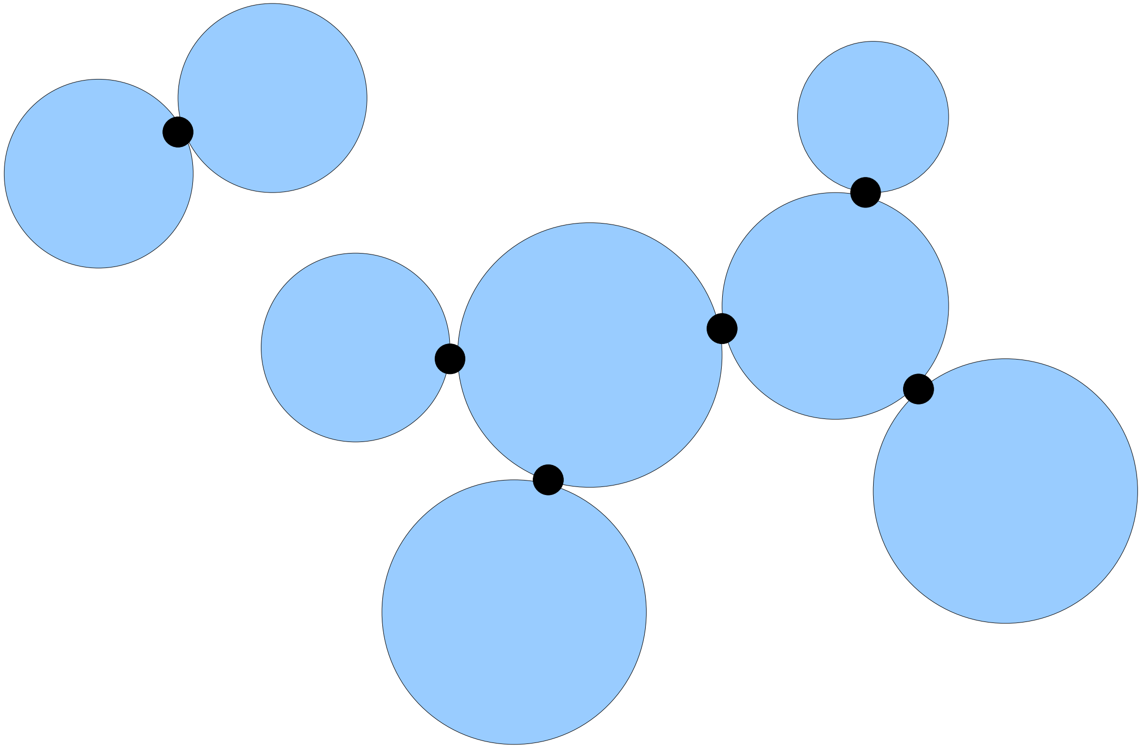


Conclusions

- Triangulations can accurately and efficiently represent polygonal environments
- Triangulations offer unique possibilities for pathfinding for a non-point (especially circular) object
- Triangulation-based pathfinding finds paths very quickly and can also find optimal paths given a bit more time
- Our abstraction technique identifies useful structures in the environment: dead-ends, corridors, and decision points
- This abstraction can be used to find paths even more quickly, only depending on the number of obstacles

Future Work (1)

- **Further abstraction** is possible by collapsing strongly-connected components of the abstract graph into single nodes of an even more abstract graph (a forest)
  - Identify "rooms" in the environment (similar to HPA*)
  - Pathfinding across tree nodes is trivial, and paths between entry points of the components could even be cached



Future Work (2)

- Channels resulting from TA* or TRA* are useful in pathfinding involving **multiple objects** because channel widths are known
- **Terrain analysis** is possible with the abstraction information (e.g. identifying choke points)
- More **edge annotations** can reduce the need for triangulation updates (e.g. enemy presense in corridors)
- It may be useful to construct **waypoint graphs** from triangulations that produce close to optimal paths in one shot

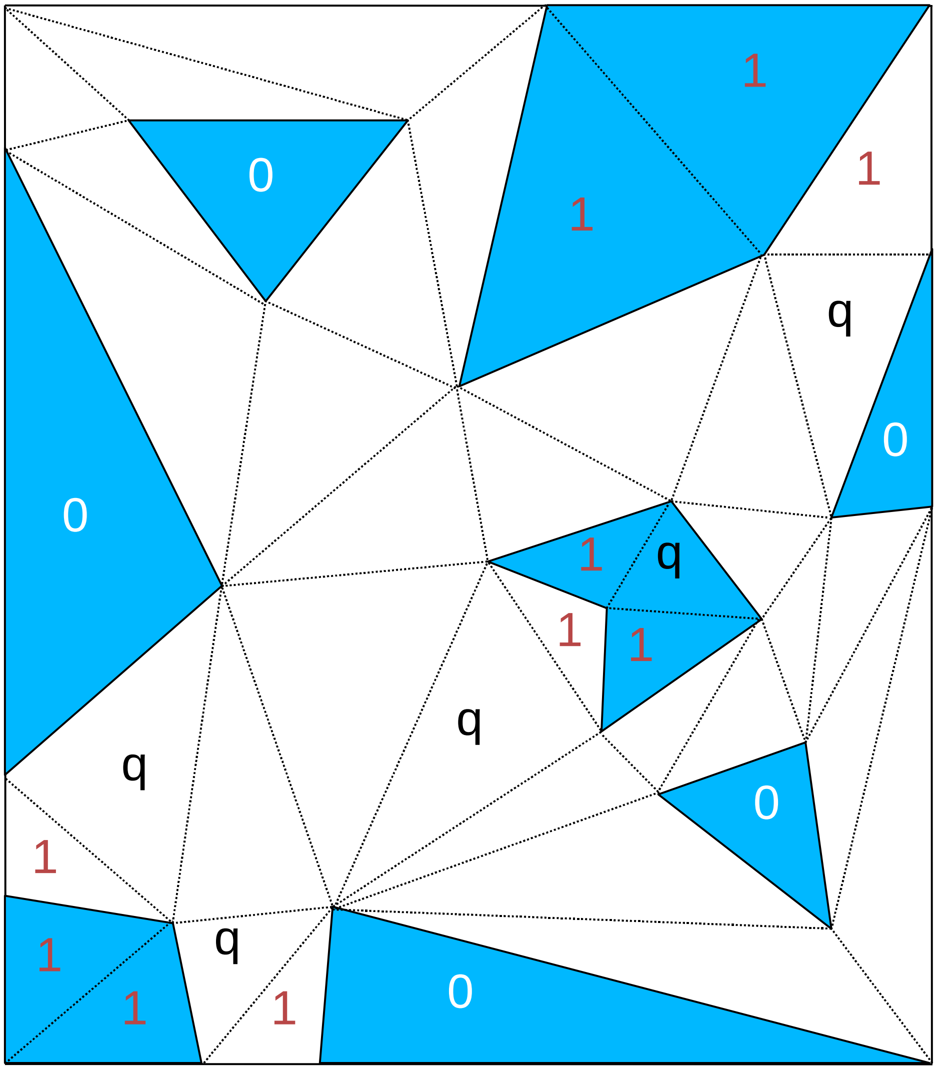
References

- **AI Game Programming Wisdom** Book Series
- M. de Berg et al., **Computational Geometry**, 3rd edition, Springer Verlag 2008
- M. Kallmann, H. Bieri, D. Thalmann, **Fully Dynamic Constraint Delaunay Triangulations**, in Geometric Modeling for Scientific Visualization, Springer Verlag 2003
- M. Kallmann, **Pathplanning in Triangulations,** IJCAI 2005
- D. Demyen, **Triangulation-Based Pathfinding,** MSc. Thesis, 2006, which is summarized in:
- D. Demyen and M. Buro, **Efficient Triangulation-Based Pathfinding**, AAAI 2006

Extra Material

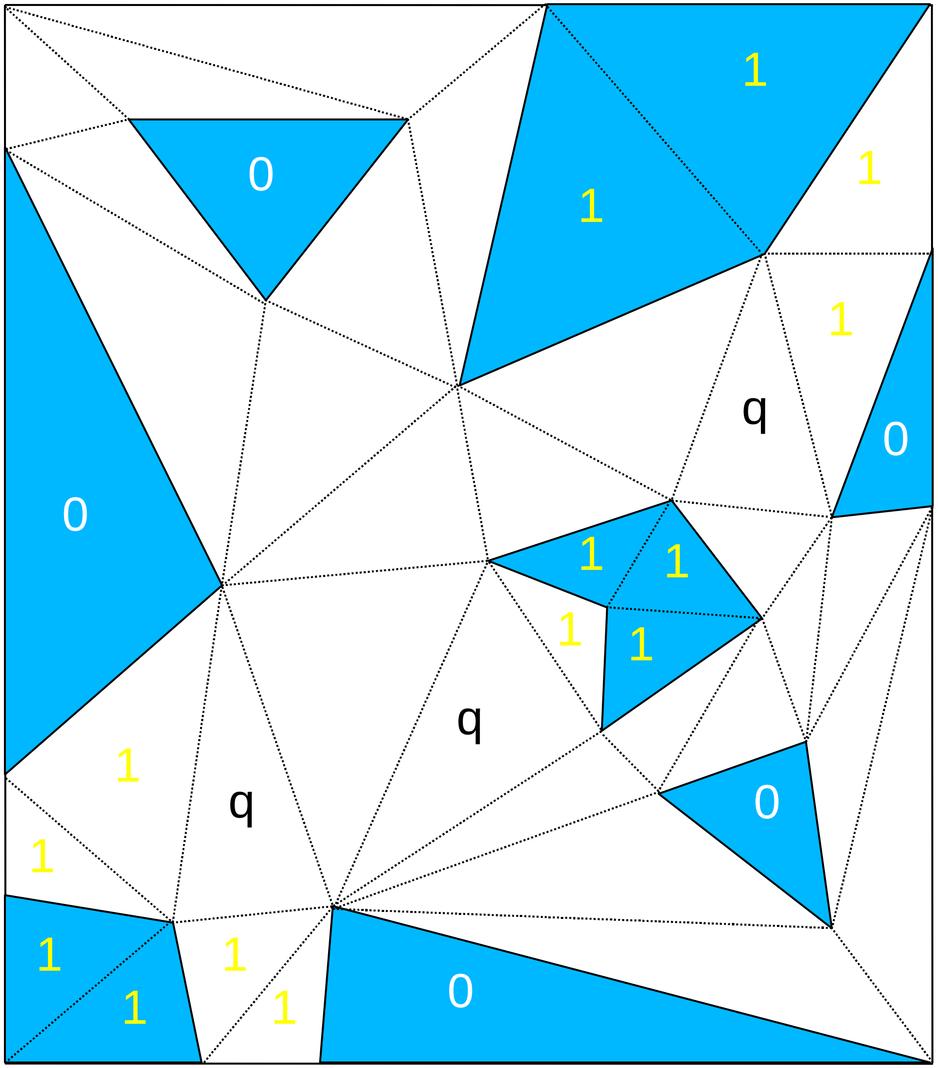
Reduction Algorithm

- Abstract triangles with 3 constrained edges as degree-0
- Abstract triangles with 2 constrained edges as degree-1
- Put the triangle adjacent the unconstrained edge on a queue



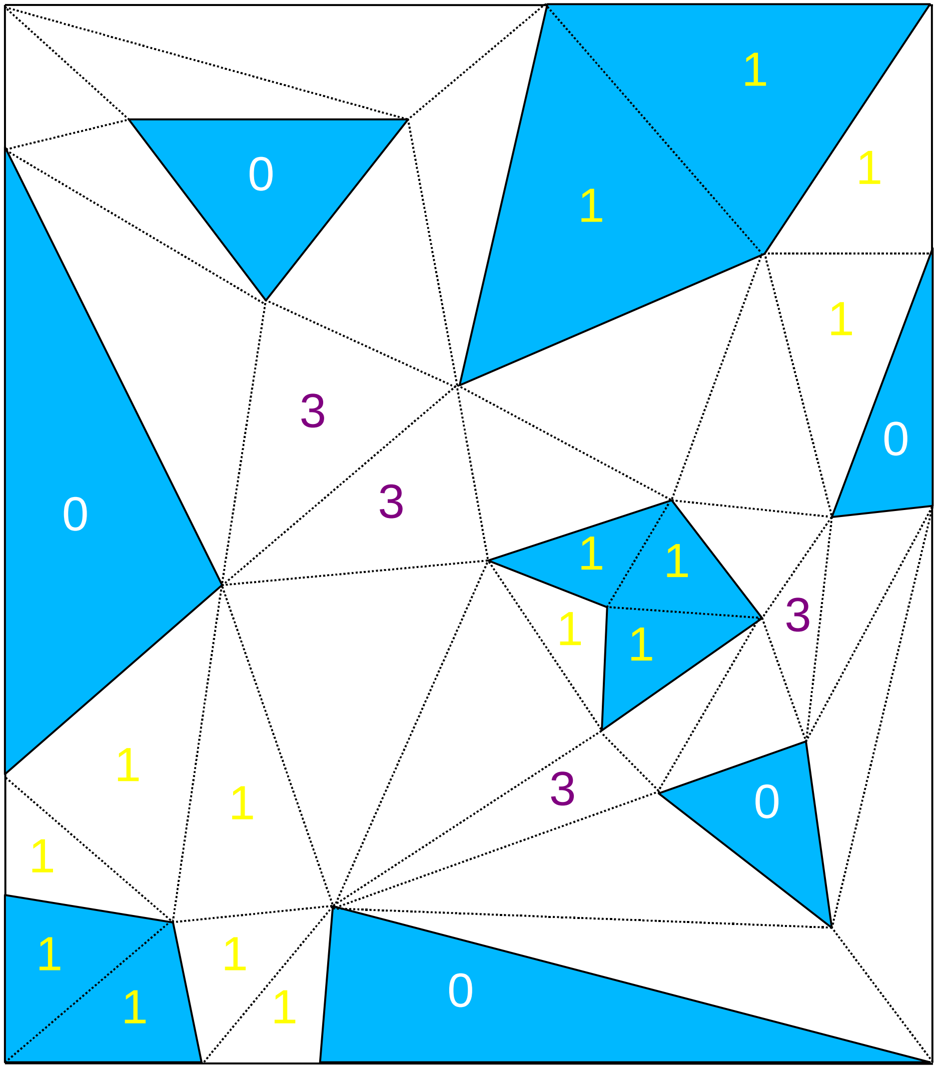
Reduction Algorithm, Cont'd

- Go through the queue
  - If the triangle is now degree-1, abstract it as one
  - And put the unabstracted face across the unconstrained edge onto the end of the queue
  - Otherwise, just remove it
- Sometimes a connected component is “collapsed” into all degree-1 triangles



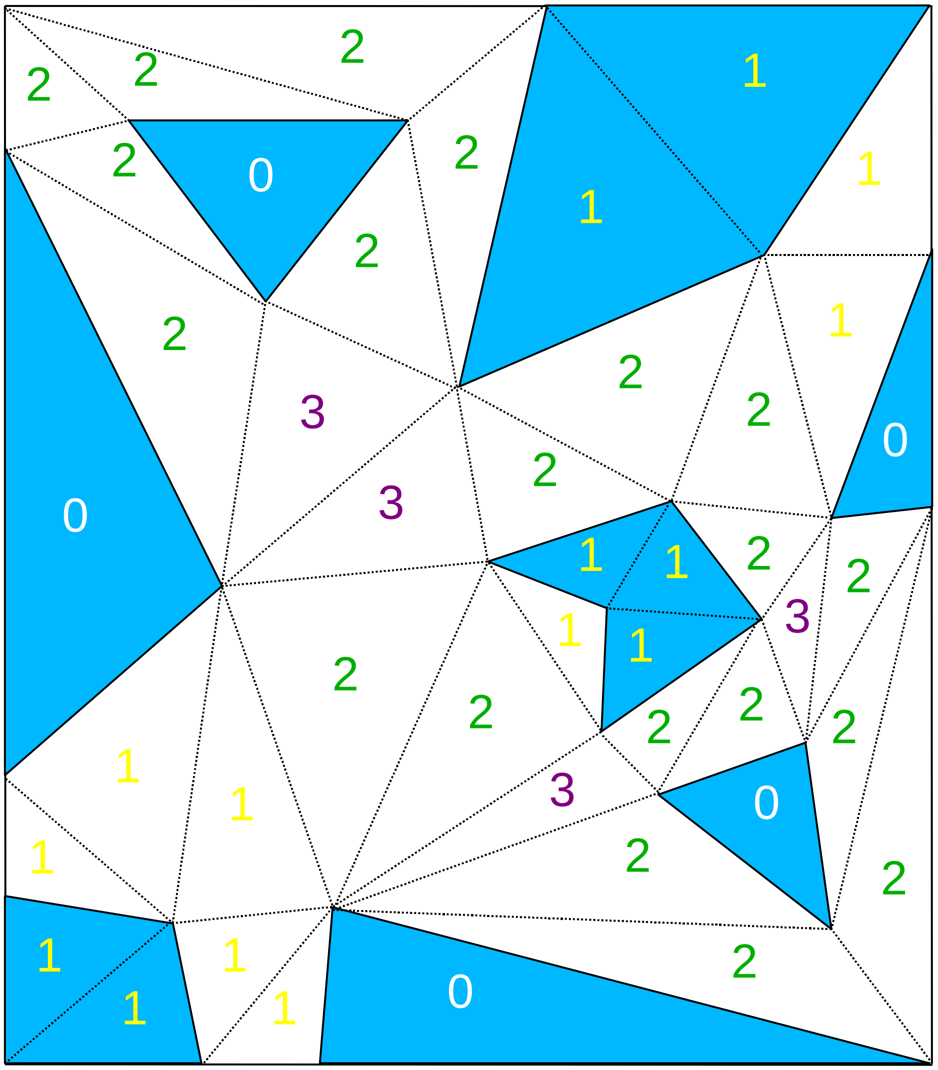
Reduction Algorithm, Cont'd

- Go through the other triangles
- Determine which ones have neither constrained edges nor adjacent degree-1 triangles
- Abstract these as degree-3
- There are 2*n* – 2 for a component with *n* obstacles



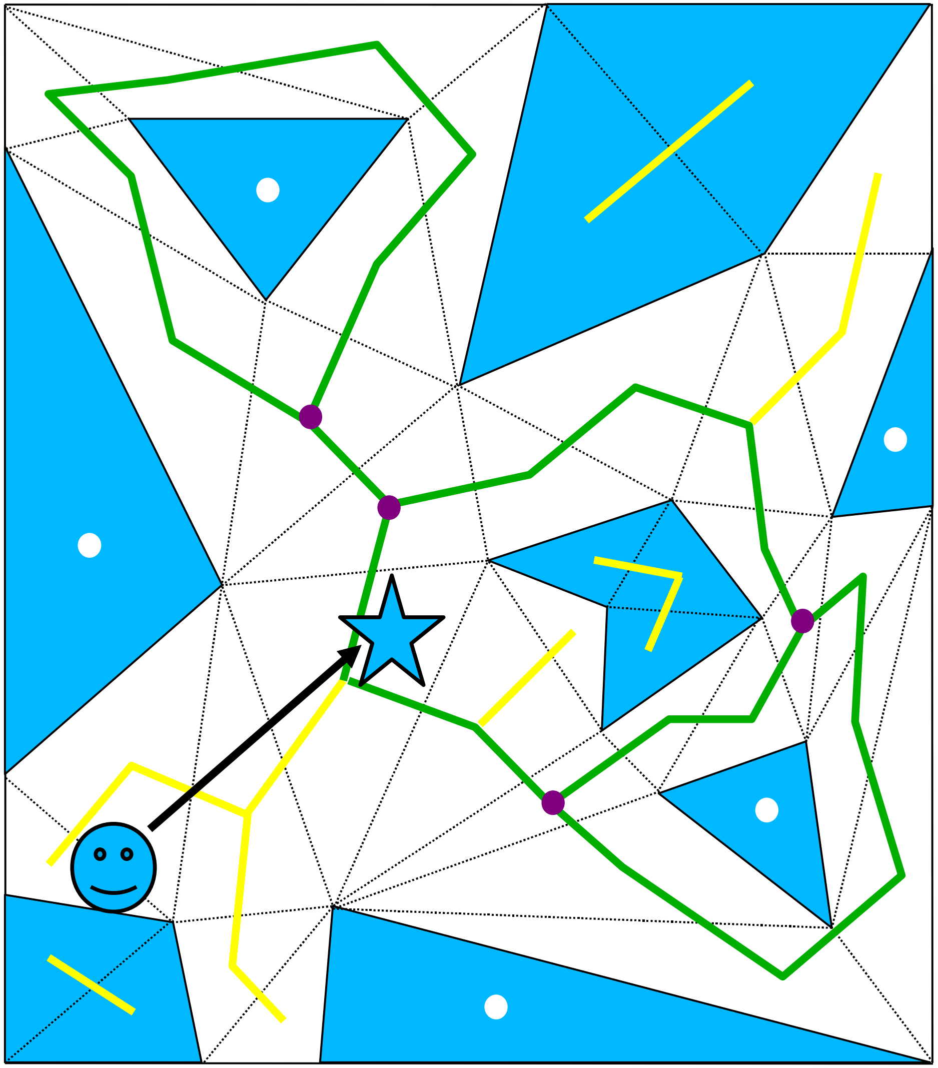
Reduction Algorithm, Cont'd

- From degree-3 triangles, move through the corridors of unabstracted triangles to the next degree-3 triangles
- Abstract these triangles as degree-2
- If there are still any unabstracted nodes, abstract them into one or more “rings” of degree-2 triangles



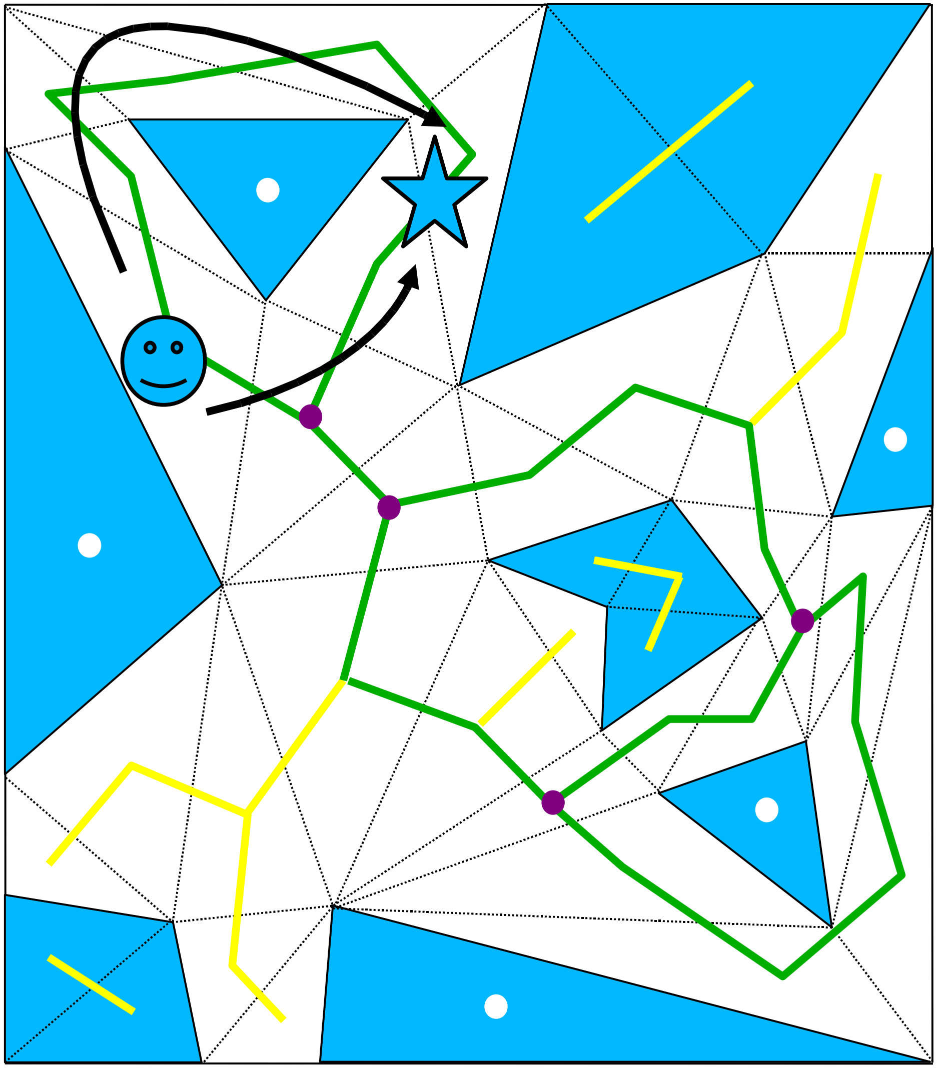
TRA* Special Cases

- For TRA* there are a number of special cases
- One must check for these first, a degree-3 search may not be required
- For example: If the start or goal is the root of a tree containing the other, we can "walk" to the root for the only path



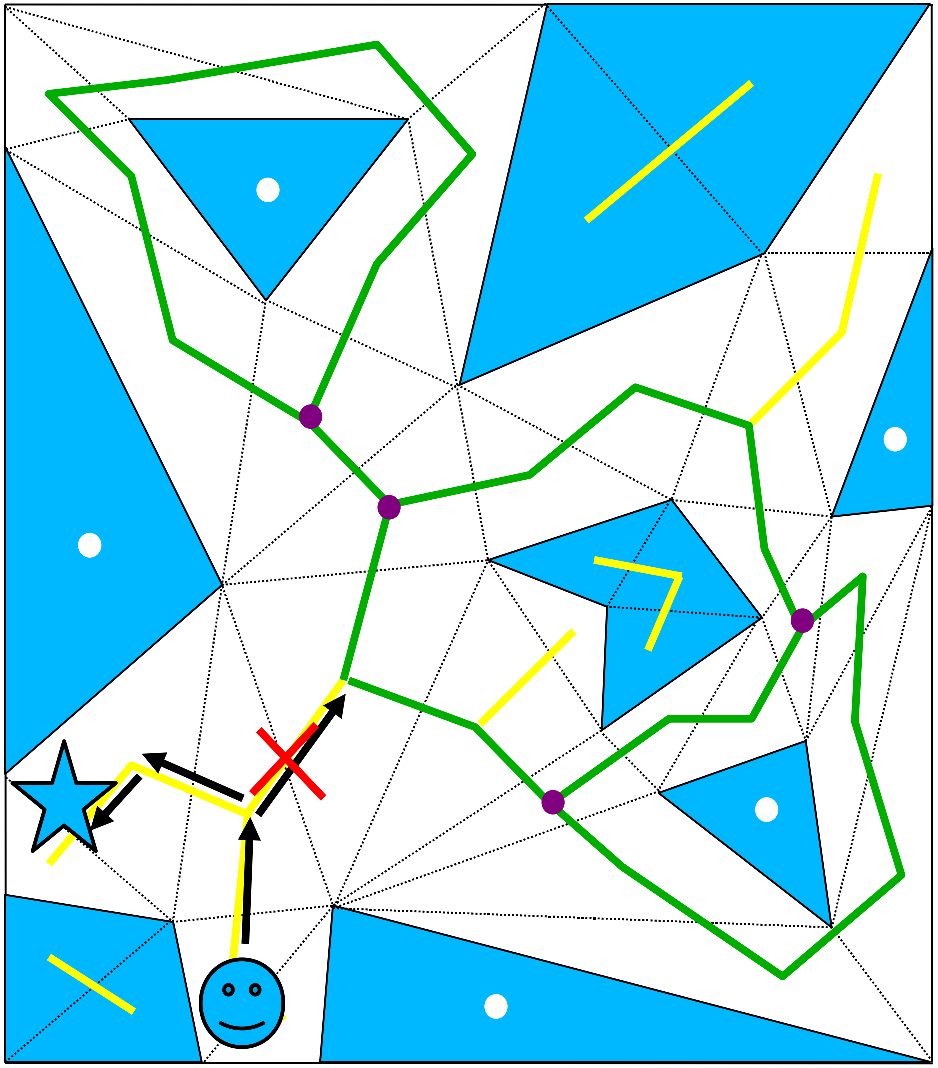
TRA* Special Cases, Cont'd

- Another one occurs when the start and goal are on the same “loop”
- We walk both ways around and pick the shorter path
- Works the same for degree-2 rings



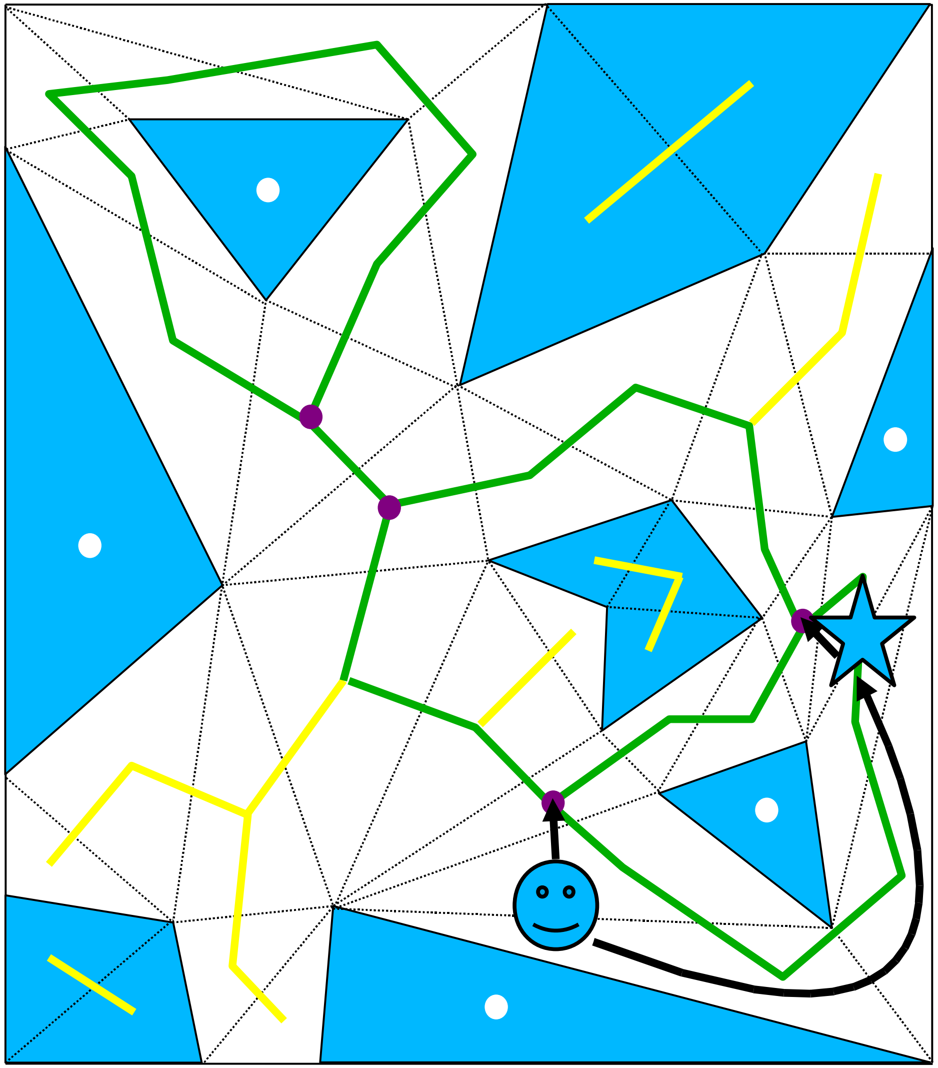
TRA* Special Cases, Cont'd

- If they are in the same degree-1 tree, we can do a simple search to find the path
- We stay within the tree
- Since there is only one path in a tree, we don't need to worry about duplicates



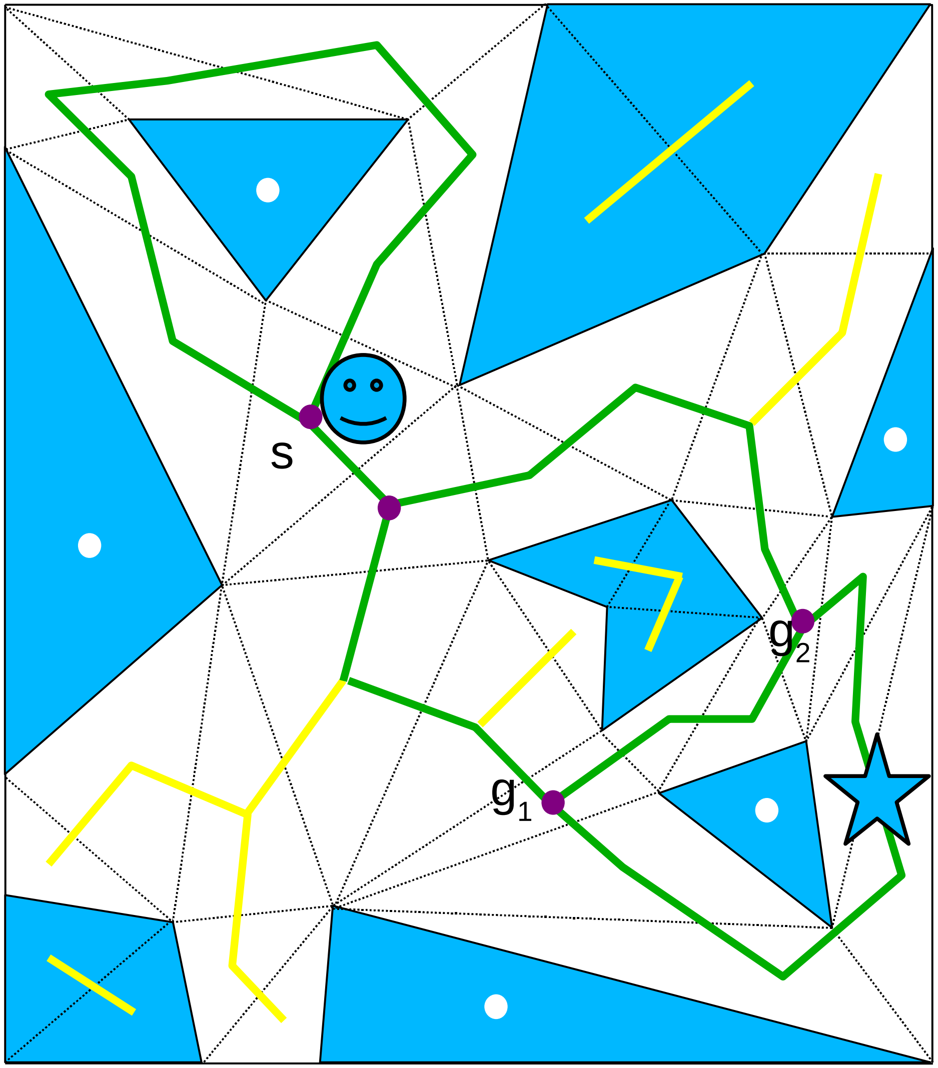
TRA* Special Cases, Cont'd

- If they are on the same degree-2 corridor, we take one path by walking through the corridor
- The degree-3 search then starts from the endpoints to attempt to find a shorter one
- The regular search starts if none if these cases applies



TRA* Degree-3 Node Search

- Start on a degree-3 node: search queue initialized with a state using that node
- Goal on a degree-2 corridor: degree-3 nodes on both ends of that corridor are possible goals for the search



TRA* Degree-3 Node Search, Cont'd

- Start in degree-1 tree: search queue initialized with states using degree-3 nodes at ends of corridor at the root of the tree
- Goal is one degree-3 node
- Now search moves only between degree-3 nodes