A $(1+\epsilon)$-Approximation Algorithm for Partitioning Hypergraphs Using A New Algorithmic Version of the Lovász Local Lemma

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Lovász Local Lemma: $\mathcal{A} = \{A_1, \ldots, A_n\}$ a set of random events, A_i has probability at most p and is mutually independent of all but at most d other events. If $ep(d+1) \leq 1$, where $e = 2.7182\ldots$, then

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- Finding disjoint paths in expander graphs

The most typical example of applications of the LLL is:

Theorem 1: If $H(V, E)$ is a k-uniform hypergraph and each edge intersects less than $\frac{2^{k-1}}{e}$ other edges, then H is 2-colorable.

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There is a more general form of the LLL, by which we can show:

Theorem 2: If $H(V, E)$ is non-uniform and each edge E_i has size at least 3 and intersects at most $2^{O(k)}$ other edges of size k, then H is 2-colorable.

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- None of these algorithms work for the case that H is non-uniform.

The first algorithmic version for *non-uniform* hypergraphs:

Theorem 4 [Czumaj & Scheideler SODA'00]: We can find a 2-coloring of a non-uniform $H(V, E)$, as long as no edge $e \in E$ intersects more than $O(|e|2^{O(k)})$ edges of size at most k.

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That is, if $R(E_i)$ ($B(E_i)$) is the number of Red (Blue) vertices in E_i:

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- Theorem 4 does not extend to hypergraph partitioning

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- Algorithm is Randomized; Expected Running time is linear in size of H.
- Algorithm is simple; proof of correctness is too complicated to present here.

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- Recolor the vertices of these components by exhaustive search, s.t. no bad remains. Such a coloring exists by the LLL.
- This 2-coloring satisfies:

$$\forall E_i : R(E_i) = R(E_i^1) + R(E_i^2) + R(E_i^3) \approx (1 \pm 3\epsilon)\frac{|E_i|}{2}$$

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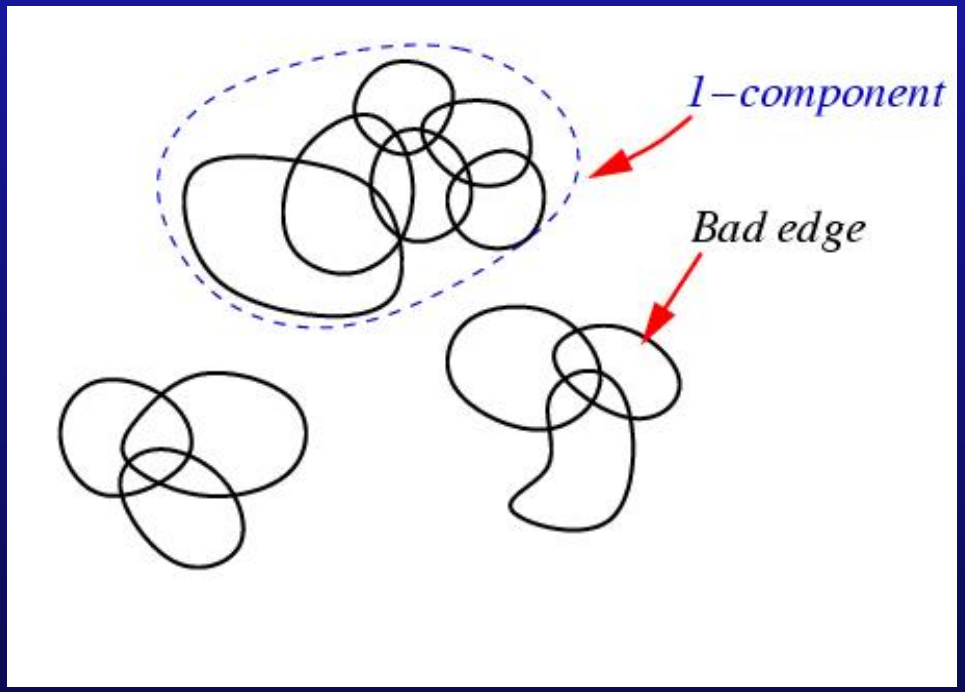

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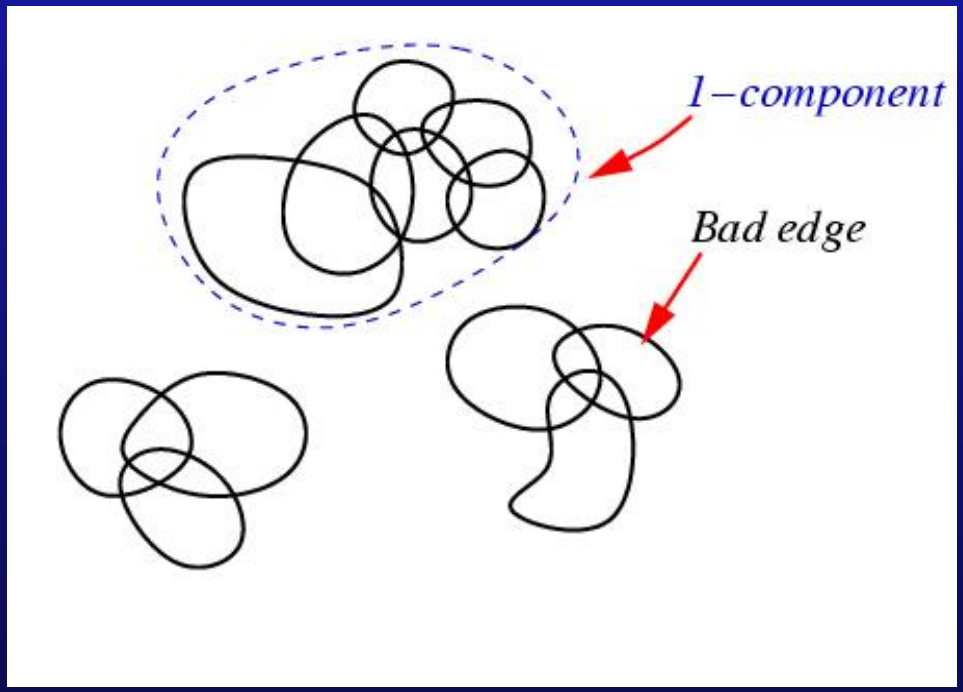

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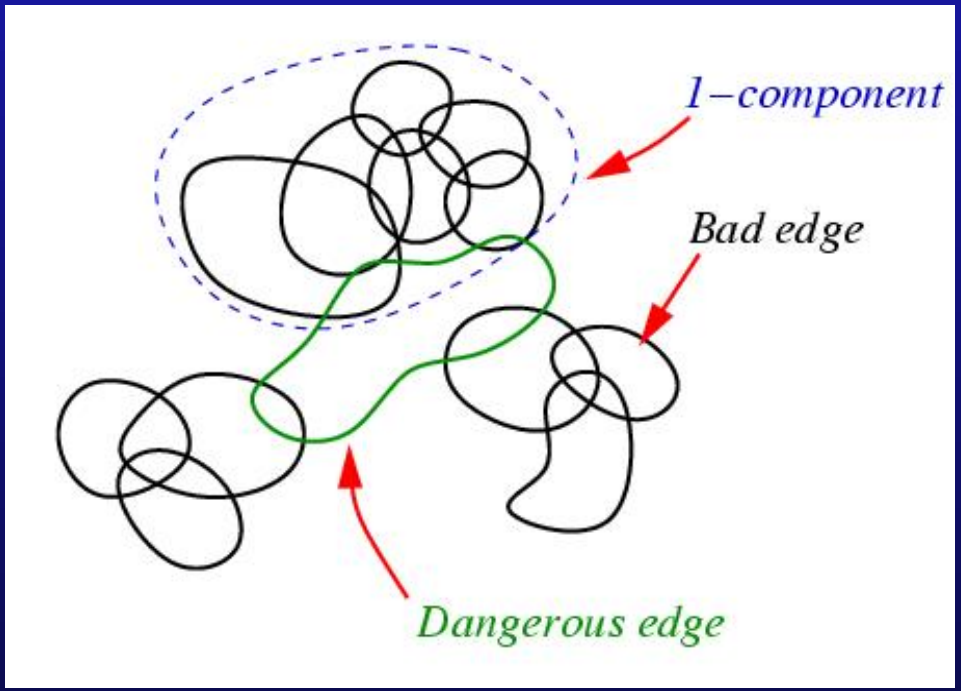

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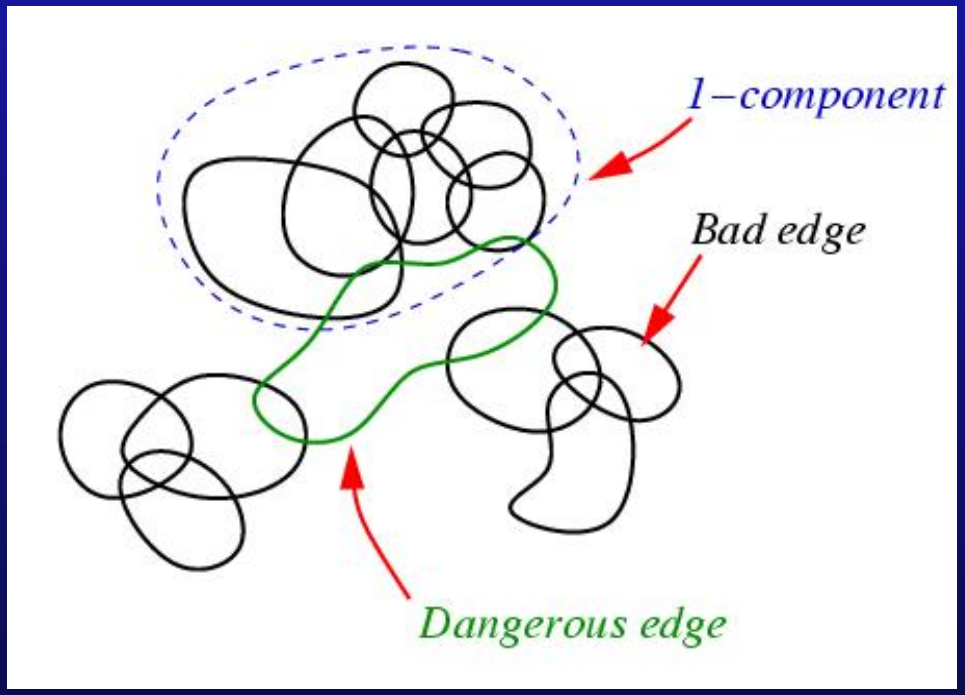

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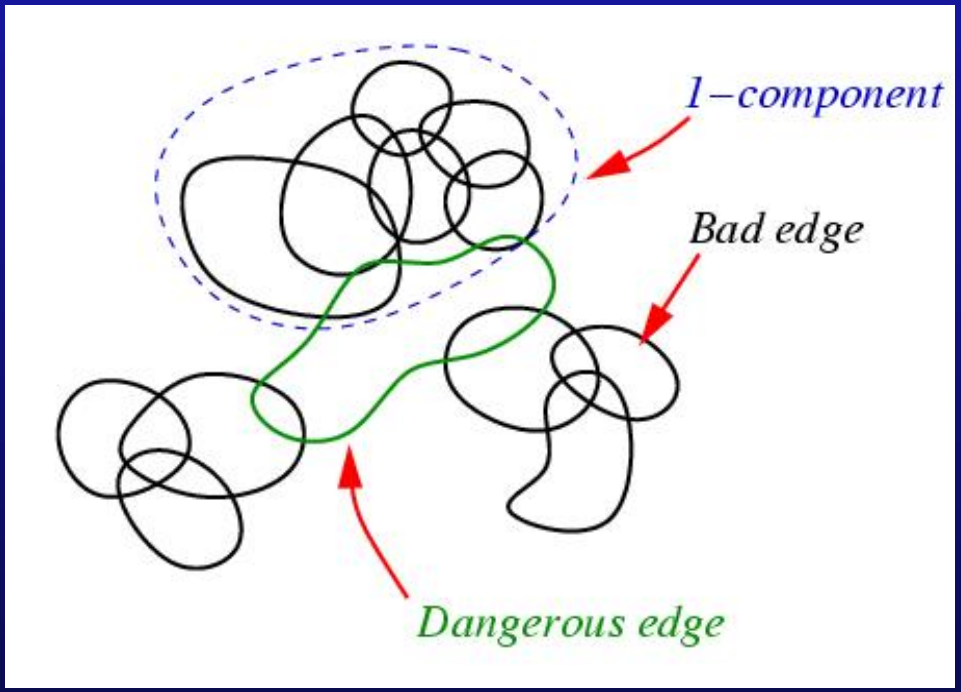

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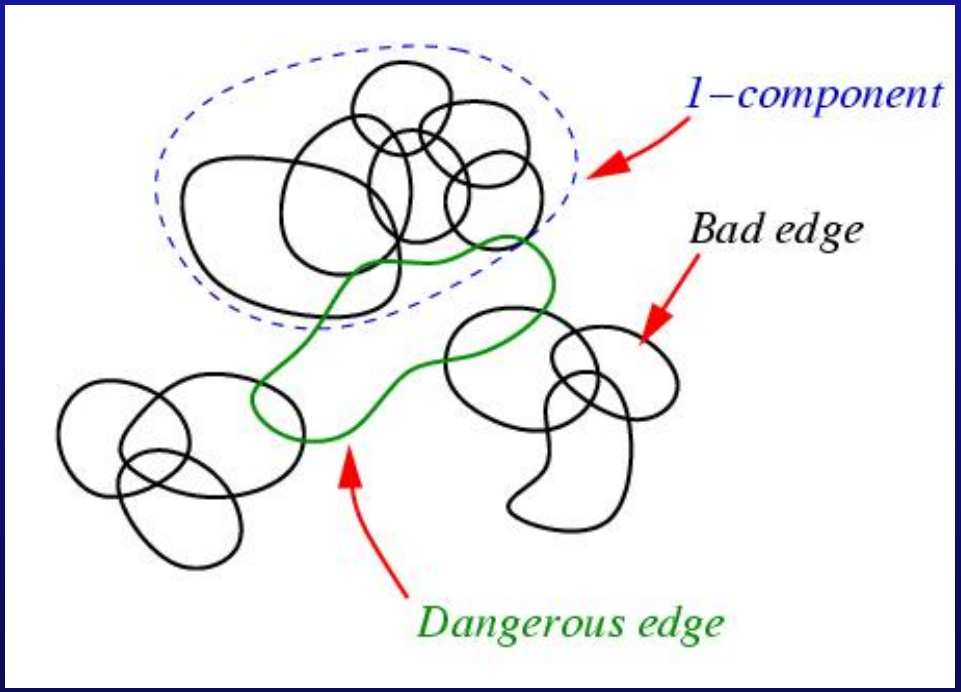

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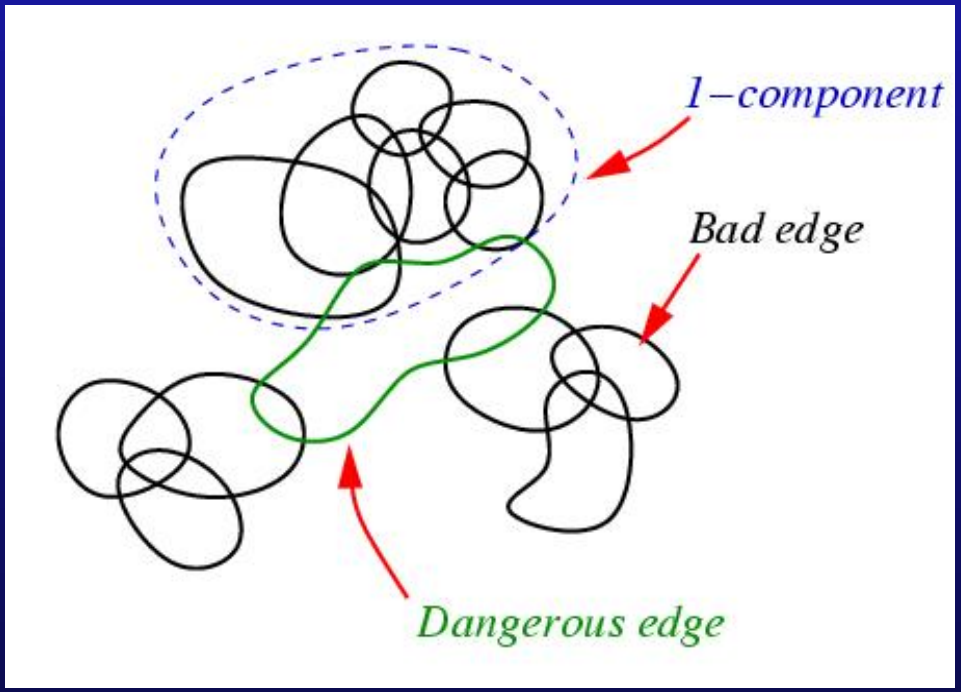

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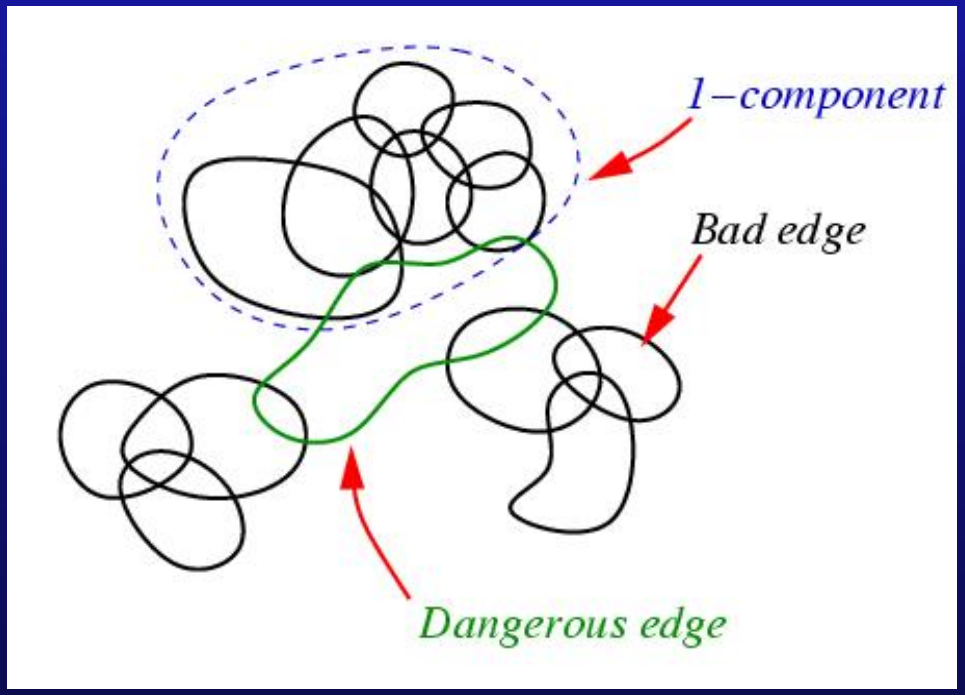

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- We can consider each 2-component independently.
- Using the LLL there exists a partitioning of the edges of 2-components.
- With prob at least $1 - \frac{1}{m^{\epsilon}}$, no 1-component has size larger than $O(\log m)$.

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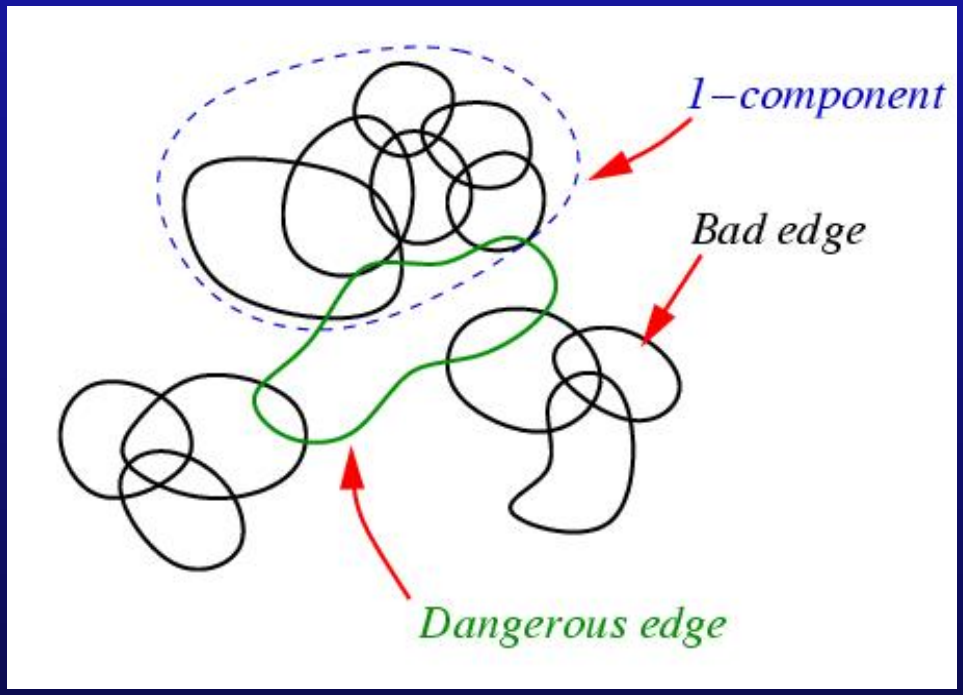
- Re-coloring 1-components that are intersecting a common dangerous edge may create a new bad edge.



- Therefore, we find maximal connected components of 1-components and dangerous edges; These are 2-components.
- We can consider each 2-component independently.
- Using the LLL there exists a partitioning of the edges of 2-components.
- With prob at least $1 - \frac{1}{m^{\epsilon}}$, no 1-component has size larger than $O(\log m)$.
- We repeat the same procedure on the new 2-components; with high probability all 2-components will have size $O(\log \log m)$.

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- How about a completely different approach?