Exploiting Glue Clauses to Design Effective CDCL Branching Heuristics

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Introduction

Boolean Satisfiability (SAT)

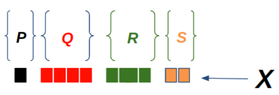
+ Determine assignments of the variables to satisfy a boolean formula, if one exists. Otherwise, unsatisfiable ....

Conflict Driven Clause Learning (CDCL) SAT Solvers

+ Conflict Generation at a fast rate is crucial for CDCL solvers.
  * conflict→learned clause→space pruning.
+ CDCL SAT solvers learns clauses at a fast rate.
  * May affect solver speed.
  * Learned clause DB management is necessary.

The LBD Criterion

+ One criterion for clause DB management is **Literal Block Distance (LBD)** score of the learned clasues.
  * Number of distinct decisions in a learned clause.
  * Learned clause ***X*** has 4 decisions : LBD(***X***) = 4



+ Literals which are assigned in a single dicision are like a connected block.
  * Eg., 3 literals forms a **block *R*** that are assigned in a decision .
  * Lower the LBD score → higher quality learned clause.

Glue Clauses

+ Learned clauses with LBD score 2.
+ Possess high pruning power.
+ Are permanently kept in the modern CDCL Solver.

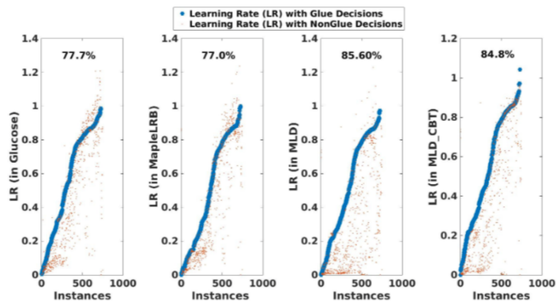
We relate glue clauses with branching variables.

+ **Glue Variables**: variables that appear in at least one glue clause.
+ **NonGlue Variable**: never appear in any of the glue clauses.

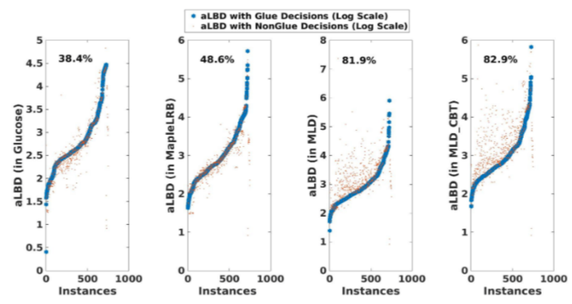
Empirical Properties of Glue Variables

Conflict efficiency of Glue Variables

+ Study LR and aLBD over Glue and NonGlue decisions.
  * Four solvers: **Glucose**, **MapleLRB**, **MapleLCMDist** (MLD) and **MapleLCMDistChronoBT** (MLD_CBT).
  * 750 SAT Comp-2017, 2018 maintrack instances.



* Glue decisions achieve higher LR for most instances.



* In general, Glue decisions achieve lower LR for most Instances.

Bias for Glue Variable Selection

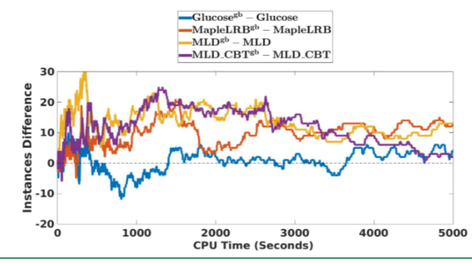
(A) Systems	(B) Average for Glue Variable	
	GP (B1)	Glue Decisions % (B2)
Glucose	25.32%	65.43%
MapleLRB	21.8%	63.14%
MLD	22.05%	47.60%
MLD_CBT	22.19%	48.76%

+ On average, given their smaller pool size, Glue Variables are selected disproportionately more often.

Empirical Evaluation

Systems	SAT Comp-17				SAT Comp-18				Combined			
	SAT	UNSAT	Total	PAR-2	SAT	UNSAT	Total	PAR-2	SAT	UNSAT	Total	PAR-2
Glucose	83	96	179	1893	97	95	192	2274	180	191	371	4167
Glucose^{gb}	**86 (+3)**	96 (+0)	**182 (+3)**	**1868**	96 (-1)	**97 (+2)**	193 (+0)	**2273**	**182 (+2)**	**193 (+2)**	**375 (+4)**	**4141**
MapleLRB	80	95	175	1897	114	95	209	2069	194	190	384	3966
MapleLRB^{gb}	**87 (+7)**	**97 (+2)**	**184 (+9)**	**1824**	**117 (+3)**	**96 (+1)**	**213 (+4)**	**2027**	**204 (+10)**	**193 (+3)**	**397 (+13)**	**3851**
MLD	99	106	205	1635	136	101	237	1807	235	207	442	3442
MLD^{gb}	**103 (+4)**	**107 (+1)**	**210 (+5)**	**1593**	**143 (+7)**	**102 (+1)**	**245 (+8)**	**1725**	**246 (+11)**	**209 (+2)**	**455 (+13)**	**3318**
MLD_CBT	**103**	113	216	1565	135	**102**	237	1800	238	215	453	3365
MLD_CBT^{gb}	102 (-1)	**114 (+1)**	216 (+0)	**1539**	**138 (+3)**	101 (-1)	**239 (+2)**	**1756**	**240 (+2)**	215 (+0)	**455 (+2)**	**3295**

+ Performance gain with all the 4 baselienes for 750 SAT-competiton-17, 18 instances.



Surprising observation for GLR and aLBD

+ Study of extreme cases for obtaining insights.

GB$_{exclusive}$: Instances are solved by the GB extension, not by its baseline.
Baseline$_{exclusive}$: Instances are solved by the baseline, not by its GB extension.

+ Average GLR and average aLBD are **largely inconsistent** wrt. Liang 2017 et. al.

(A) Systems	(B) Employed Heuristics	(C) GB$_{exclusive}$				(D) Baseline$_{exclusive}$			
		#inst	avg. GLR	avg. aLBD	avg. G2L	#inst	avg. GLR	avg. aLBD	avg. G2L
Glucose	{VSIDS}	33	**0.56**	28.60	0.0005	29	0.59	**18.52**	**0.0015**
Glucose^{gb}	{VSIDS}^{gb}		0.53	**24.69**	**0.0016**		**0.62**	20.14	0.00078
MapleLRB	{LRB}	27	**0.50**	26.06	0.00073	14	0.47	**30.75**	**0.00046**
MapleLRB^{gb}	{LRB}^{gb}		0.46	**20.38**	**0.00126**		**0.48**	32.02	0.00037
MLD	{Dist/VSIDS/LRB}	28	**0.55**	**23.60**	0.00029	15	0.53	26.70	**0.0011**
MLD^{gb}	{Dist/VSIDS/LRB}^{gb}		0.51	26.04	**0.00032**		**0.58**	**23.21**	0.0009
MLD_CBT	{Dist,VSIDS,LRB}	26	**0.49**	**26.08**	0.0006	24	0.51	29.64	**0.00065**
MLD_CBT^{gb}	{Dist/VSIDS/LRB}^{gb}		0.43	36.24	**0.0011**		**0.55**	**25.42**	0.00037

+ **Better heuristic** for an instance set **consistently achieves** higher G2L, on average.

Peculiarity of Glucose

+ Lowest gains with Glucose^{gb}→why?

+ Glucose **already increases** the score of some of the (glue) variables during conflict analysis.

+ Hypothesis: GB in Glucose^{gb} creates an imbalance.
  * We **lower the bumping factor** by dividing the glue level with high normalizing factor.
    →**improved performance** with Glucose^{gb}
  * 11 additional inatacnes(4 additioanl with previous version)

Related Work

+ Glucose bumps scores of those variables that are propagated from glue clauses. (Audemard 2009 et. al.)

+ Propagated and branched variables have high Eigen Centrality (Katsirelos 2012 et. al.).

+ VSIDS more often branches on variables which are bridges between communities (Liang 2015 et. al.).

Contributions

Contribution I

We empirically show that
(a) Decisions with Glue variables are more **conflict efficient**.
(b) CDCL branching heuristics **show bias** toward Glue variables.

Contribution II

(a) Developed a structure aware **variable bumping** scheme
  + Glue Bumping(GB)
  + **Prioritizes** selection of Glue variables
(b) Empirically evaluated the GB method on four state-of-the-art CDCL SAT solvers.

Contribution III

Have introduced the G2L metric
(a) Glue to Learned: fraction of the learned clauses that are glue.
(b) consistently explains the performance of the tested solvers.

Activity Score Bumping: Glue Bump

Glue Level (gl)

+ Let ***G*** be the set of learned glue clauses so far.
+ ***gl(v)*** of a variable ***v*** is the appearance count of **v** in the glue clauses in ***G***.

Glue Bump

+ Bumps activity scores of a **glue variable *v***
  * Based on its **activity score** and **glue level**.
  * Prioritize selection of **active glue variables with high *gl***

Alg. 1: Increase Glue Level
Input: *A newly learned glue clause* θ
1 **For** $i \leftarrow 1$ *to* $|\theta|$
2 $v \leftarrow varAt(\theta, i)$
3 $gl(v) \leftarrow gl(v) + 1$
4 **End**

glue clause Θ is learned

Alg. 2: Bump Glue Variable
Input: *A glue variable* v

1 $bf_v \leftarrow activity(v) * \left(\frac{gl(v)}{|G|}\right)$
2 $activity(v) \leftarrow activity(v) + bf_v$

glue variable *v* is unassigned

Notations

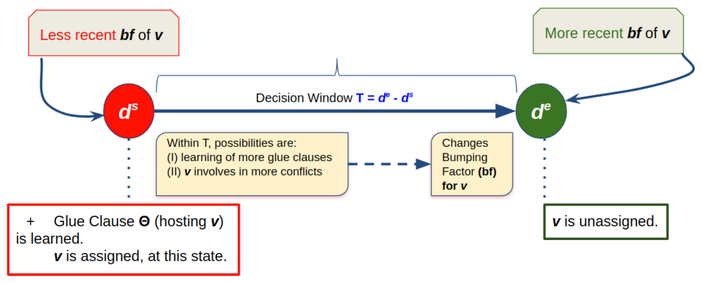
+ **Glue and NonGlue Decision:** a decision that
  * selects a Glue variavle for branching is called a **Glue decsion**.
  * selects a NonGlue variavle for branching is called a **NonGlue decsion**.

+ **Learning Rate (LR)**:
  * number of conflicts per branching decision.

+ **Average LBD (aLBD):**
  * Average LBD score of the learned clauses derived from the generated conflicts in a given run of a solver.

Delayed Bymping

+ Let ***v*** is a glue variable.



+ Hence, GB bumps at **d^e** (delayed).

Future Work

+ Investigate relationships between normalized glue level and other centrality measures.

+ Design clause deletion heuristics based on the notion of glue level?

+ New branching heuristics based on G2L?