“A beginning is the time for taking the most delicate care that the balances are correct.”

Frank Herbert, *Dune*

CMPUT 628
Deep RL

Marlos C. Machado

Plan

Assignment 1

Overview / Refresher of (everything?) Reinforcement Learning

Warning! *This will be quick. It is meant to start establishing a common language between us, but it is too fast for you if you are seeing this for the first time.*

Reminder: You can still leave

- I know, I know, *Deep Reinforcement Learning* sounds fun, modern, and hyp-ey

But…

- But this course won't be so well-structured as you (or I) would hope
- I won't teach you how to code fancy deep RL algorithms
- I'm not as much fun as you might think
- I don't care about grades – I might have a reputation :-)
  - *There won't be a practice midterm*
- I don't care if this course ends up being difficult

Please, interrupt me at any time!

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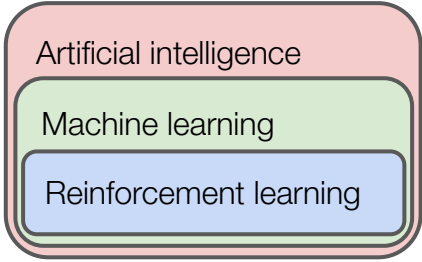
Assignment 1

Reinforcement learning problem formulation

Marlos C. Machado



Reinforcement learning

Reinforcement learning is a computational approach to learning from interaction to maximize a numerical reward signal (Sutton & Barto; 2018)

- The idea of learning by interacting with our environment is very natural
- It is based on the idea of a learning system that wants something, and that adapts its behavior to get that

Some features are unique to reinforcement learning:

- Trial-and-error
- The trade-off between exploration and exploitation
- The delayed credit assignment / delayed reward problem

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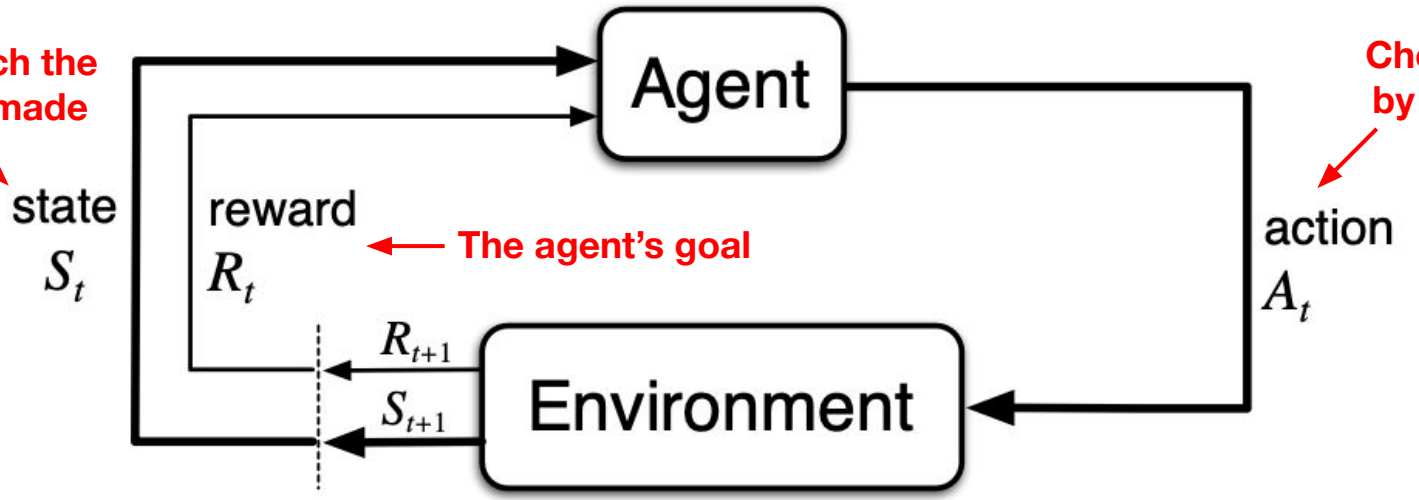
Reinforcement learning (RL)

- RL is about learning from *evaluative* feedback (an evaluation of the taken actions) rather than *instructive* feedback (being given the correct actions).
  - Exploration is essential in reinforcement learning.

- It is not necessarily about online learning, as it is sometimes said, but more generally about sequential decision-making.

- Reinforcement learning potentially allows for continual learning but in practice, quite often we deploy our systems.
  - Continual learning is important, but this course is not about this.

The Agent-Environment Interface



Basis on which the choices are made

Choice made by the agent

The agent's goal

Figure 3.1: The agent–enviro

$$S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, \ldots$$

The ultimate goal: Maximize Returns

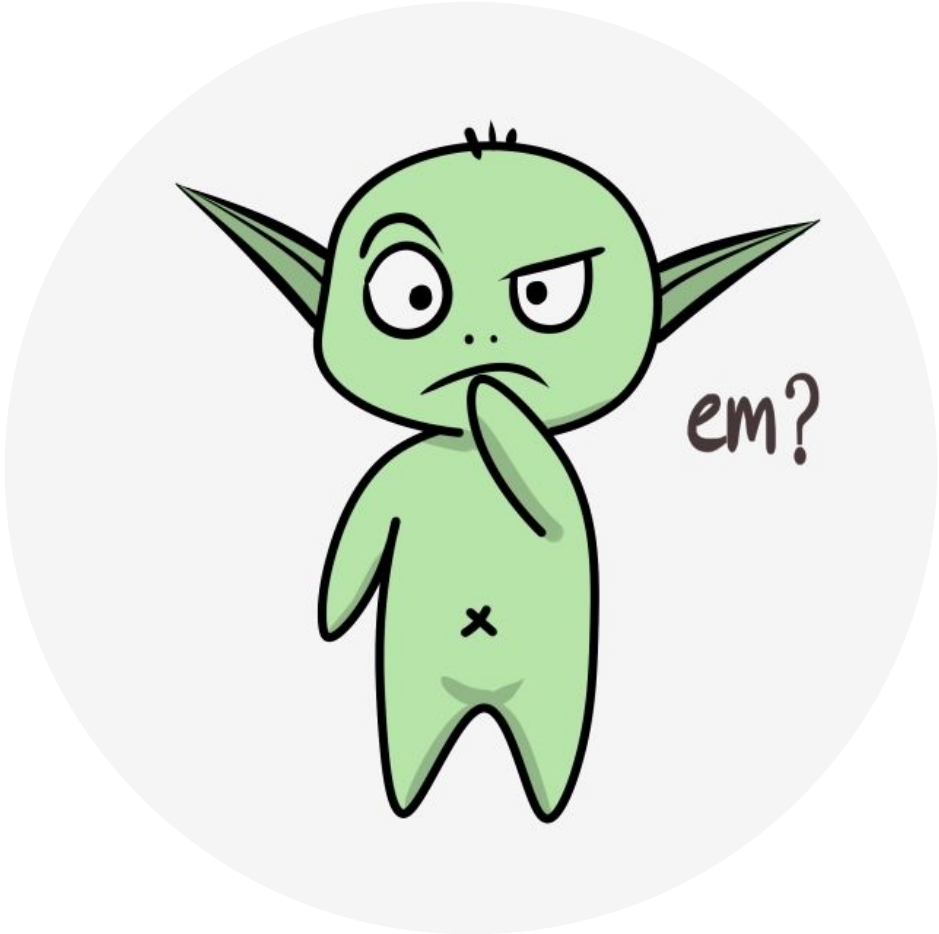
End of an episode

$$G_t \doteq R_{t+1} + R_{t+2} + R_{t+3} + \cdots + R_T$$

Continuing task

$$G_t \doteq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

$$\begin{aligned} G_t &\doteq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \gamma^3 R_{t+4} + \cdots \\ &= R_{t+1} + \gamma \left(R_{t+2} + \gamma R_{t+3} + \gamma^2 R_{t+4} + \cdots \right) \\ &= R_{t+1} + \gamma G_{t+1} \end{aligned}$$



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Tabular value-based model-free reinforcement learning

Marlos C. Machado

Value Functions and Policies

- *Value functions are "functions of states (or state-action pairs) that estimate how good it is for the agent to be in a given state".*
- "How good" means expected return.
- Expected returns depend on how the agent behaves, that is, its *policy*.

Policy

- A policy is a mapping from states to probabilities of selecting each possible action:

$$\pi : \mathcal{S} \to \Delta(\mathcal{A})$$

in other words, π(a|s) is the probability that $A_t = a$ if $S_t = s$.

Value Function

- The value function of a state s under a policy π, denoted $v_\pi(s)$ or $q_\pi(s, a)$, is the expected return when starting in s, taking a (for q_π), and following π thereafter.

state-value function for policy π

$$v_\pi(s) \doteq \mathbb{E}_\pi[G_t \mid S_t = s] = \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s\right]$$

$$q_\pi(s, a) \doteq \mathbb{E}_\pi[G_t \mid S_t = s, A_t = a] = \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s, A_t = a\right]$$

action-value function for policy π

Optimal Policies and Optimal Value Functions

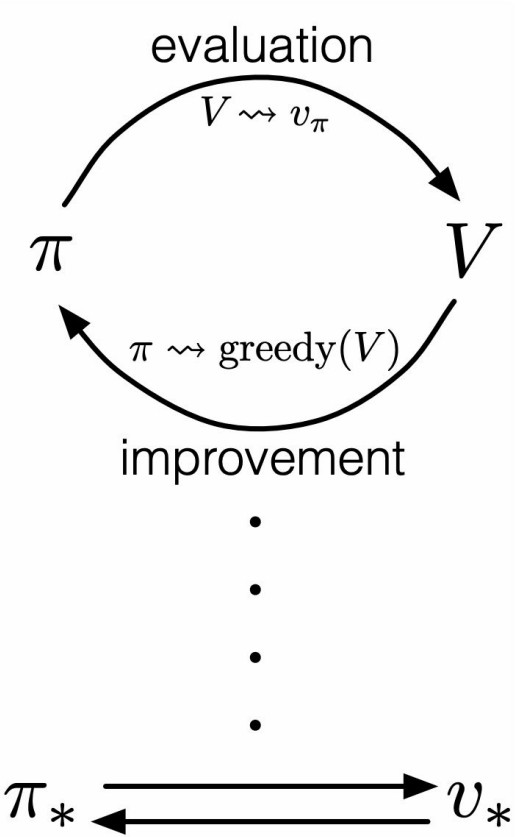
- Value functions define a partial ordering over policies.
  - $\pi \geq \pi'$ iff $v_\pi(s) \geq v_{\pi'}(s)$ for all $s \in \mathcal{S}$.
  - There is always at least one policy that is better than or equal to all other policies. The *optimal policy*.

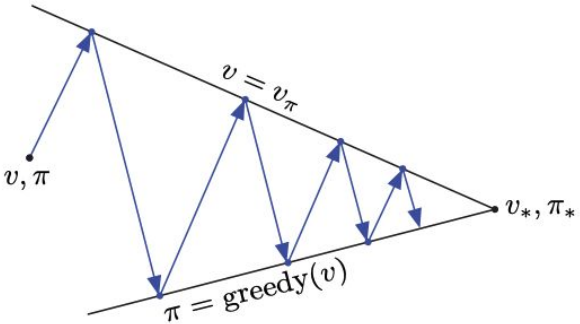
$$v_*(s) \doteq \max_\pi v_\pi(s)$$

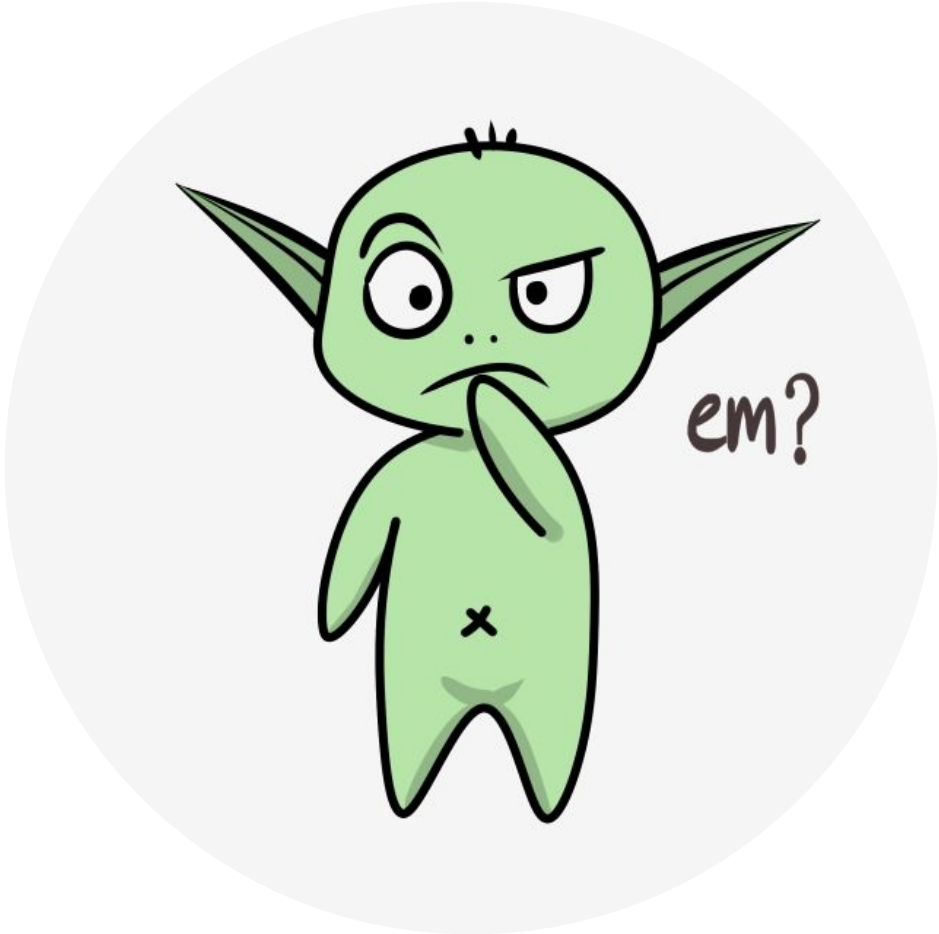
$$q_*(s,a) \doteq \max_\pi q_\pi(s,a)$$

$$q_*(s,a) = \mathbb{E}[R_{t+1} + \gamma v_*(S_{t+1}) \mid S_t = s, A_t = a]$$

Generalized Policy Iteration







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TD Prediction

A simple every-visit Monte Carlo method is:

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[G_t - V(S_t)\Big]$$

What if we don't want to wait until we have a full return (end of episode)!

$$NewEstimate \leftarrow OldEstimate \; + \; StepSize \; \Big[Target - OldEstimate\Big]$$

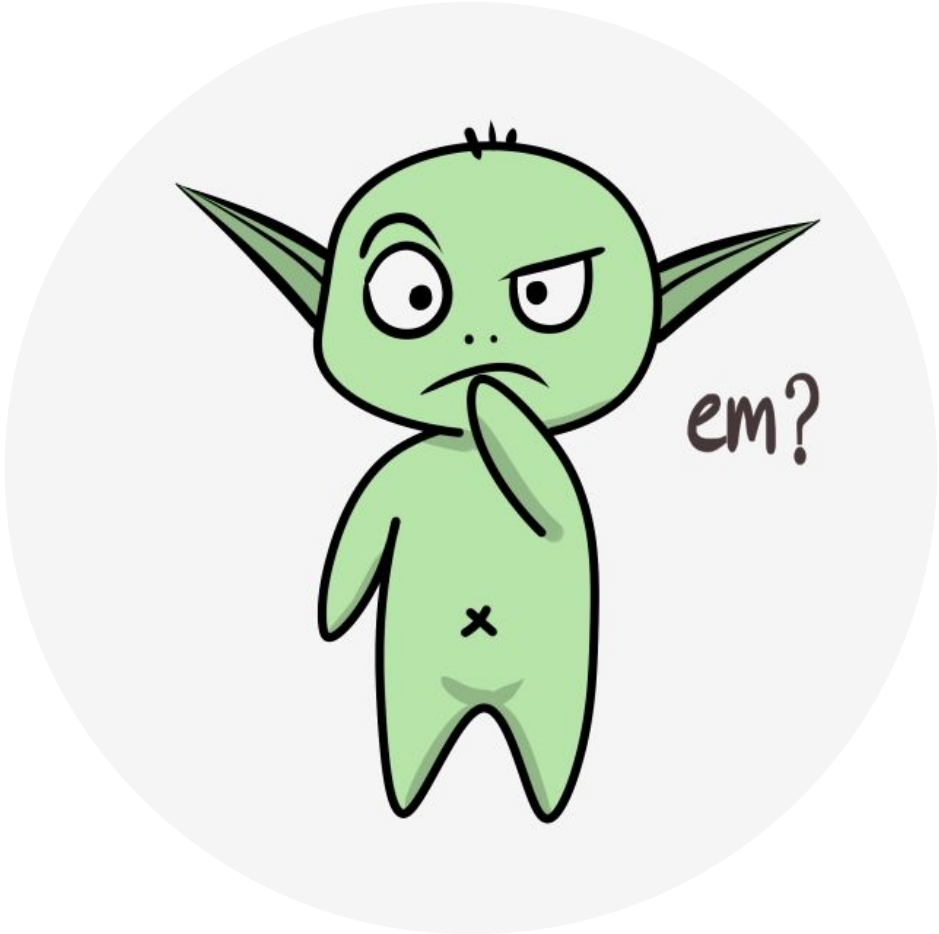
TD Prediction

A simple every-visit Monte Carlo method is:

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[\underbrace{G_t}_{\textbf{Target}} - V(S_t)\Big]$$

Temporal-Difference Learning:

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[\underbrace{R_{t+1} + \gamma V(S_{t+1})}_{\textbf{Target}} - V(S_t)\Big]$$



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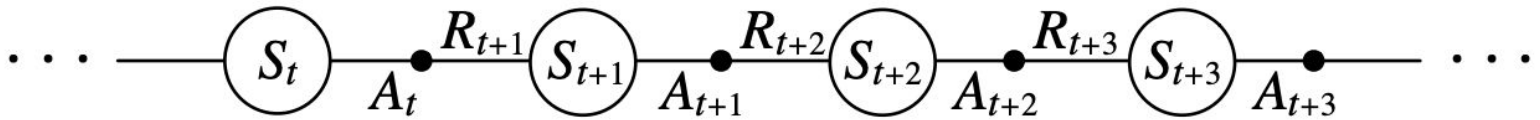
Sarsa: On-policy Control

- We again use generalized policy iteration (GPI), but now using TD for evaluation.
- We need to learn an action-value function instead of a state-value function.



$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \Big[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t) \Big]$$

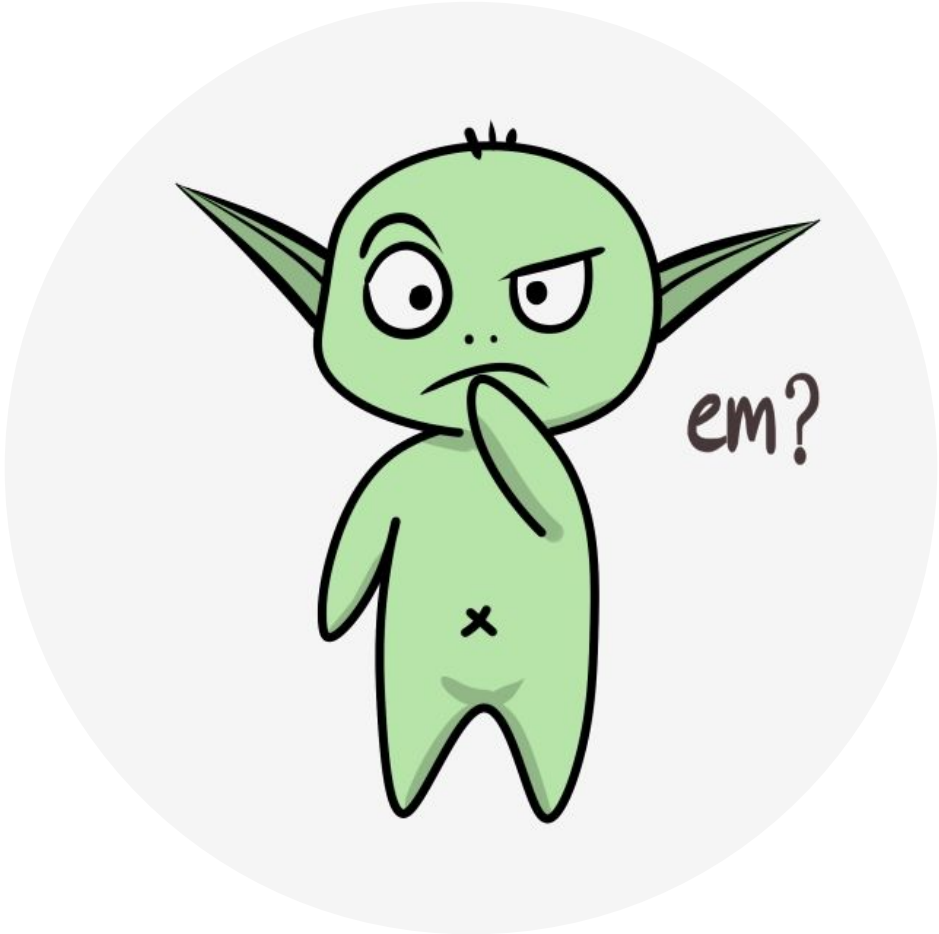
Sarsa: On-policy Control

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
  Initialize S
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Loop for each step of episode:
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal

We need to explore!



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Q-Learning: Off-Policy Control

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \Big[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t) \Big]$$

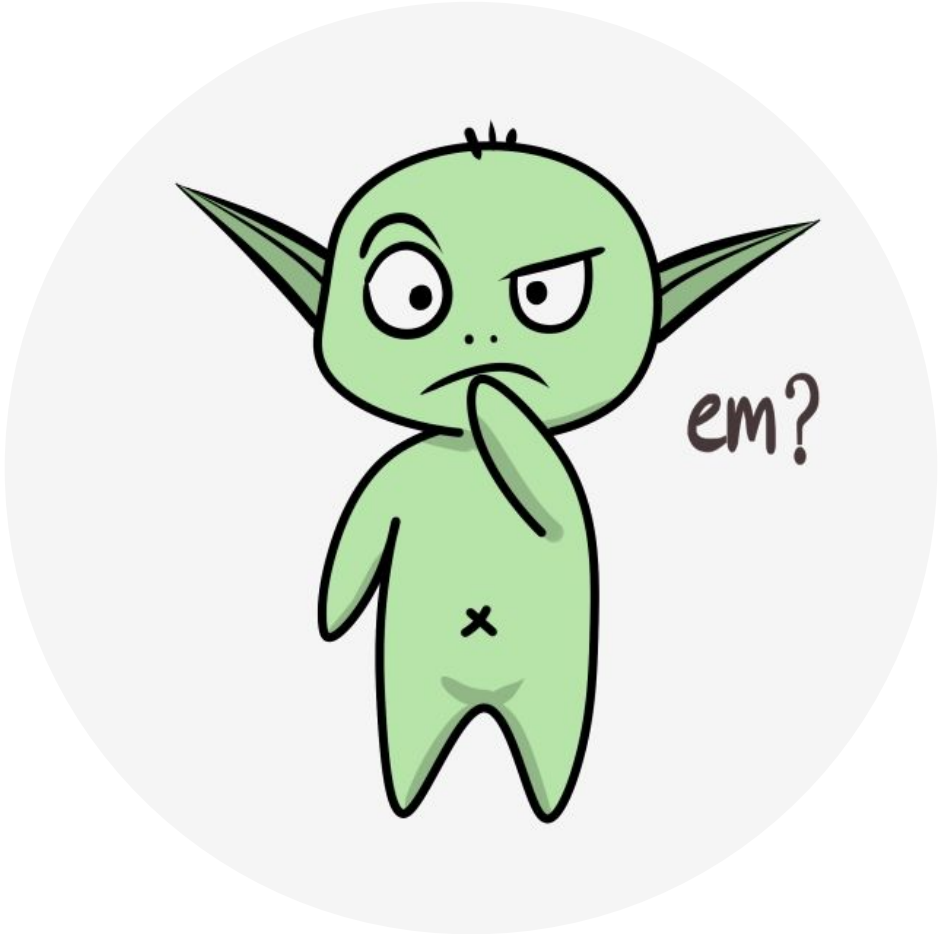
- Q directly approximates q_*, regardless of the policy being followed.
- Notice we do not need importance sampling. We are updating a state–action pair. We do not have to care how likely we were to select the action; now that we have selected it we want to learn fully from what happens.

Q-Learning: Off-Policy Control

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
    $S \leftarrow S'$
  until S is terminal



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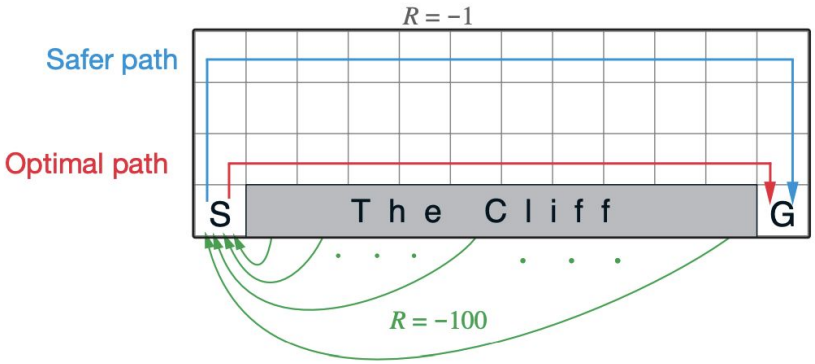
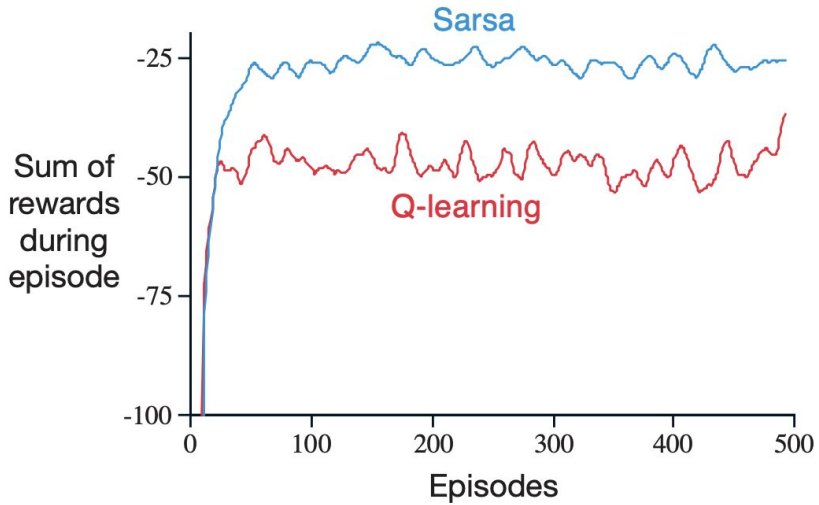
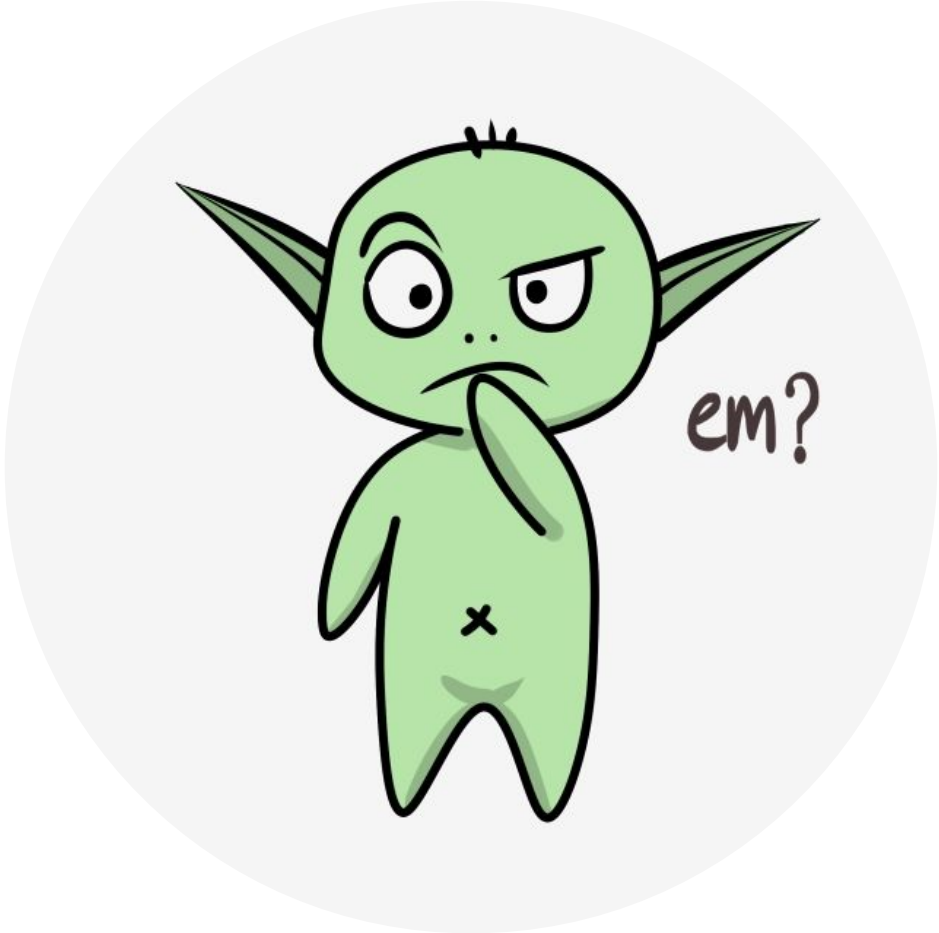
Example – Q-Learning vs Sarsa







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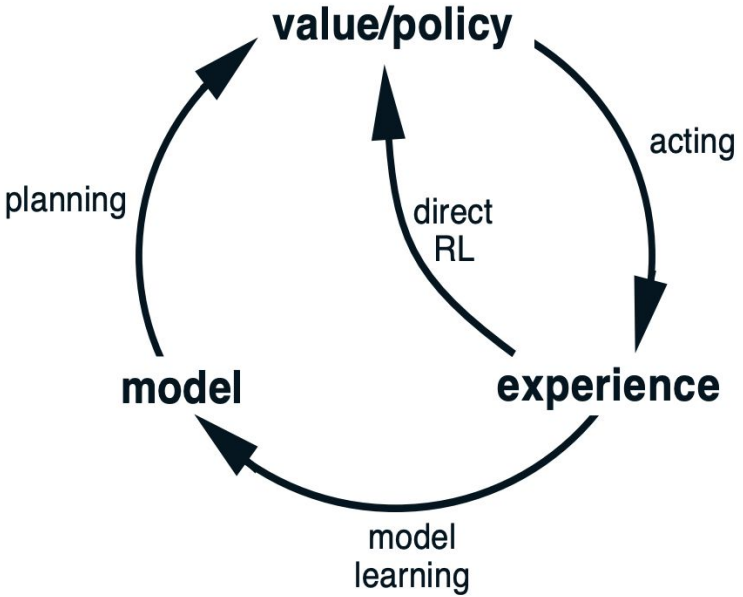
Tabular model-based reinforcement learning

Marlos C. Machado

Dyna: Integrated Planning, Acting, and Learning

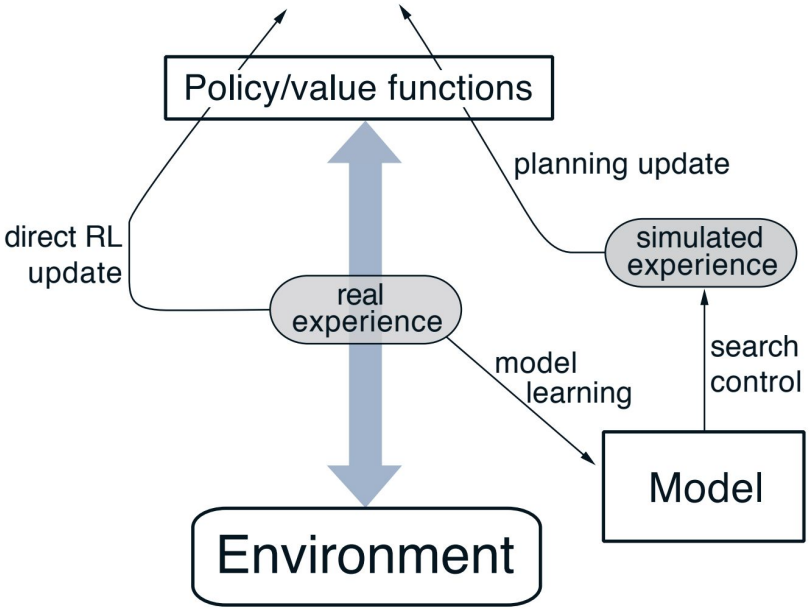
- Online planning, in small, incremental steps.

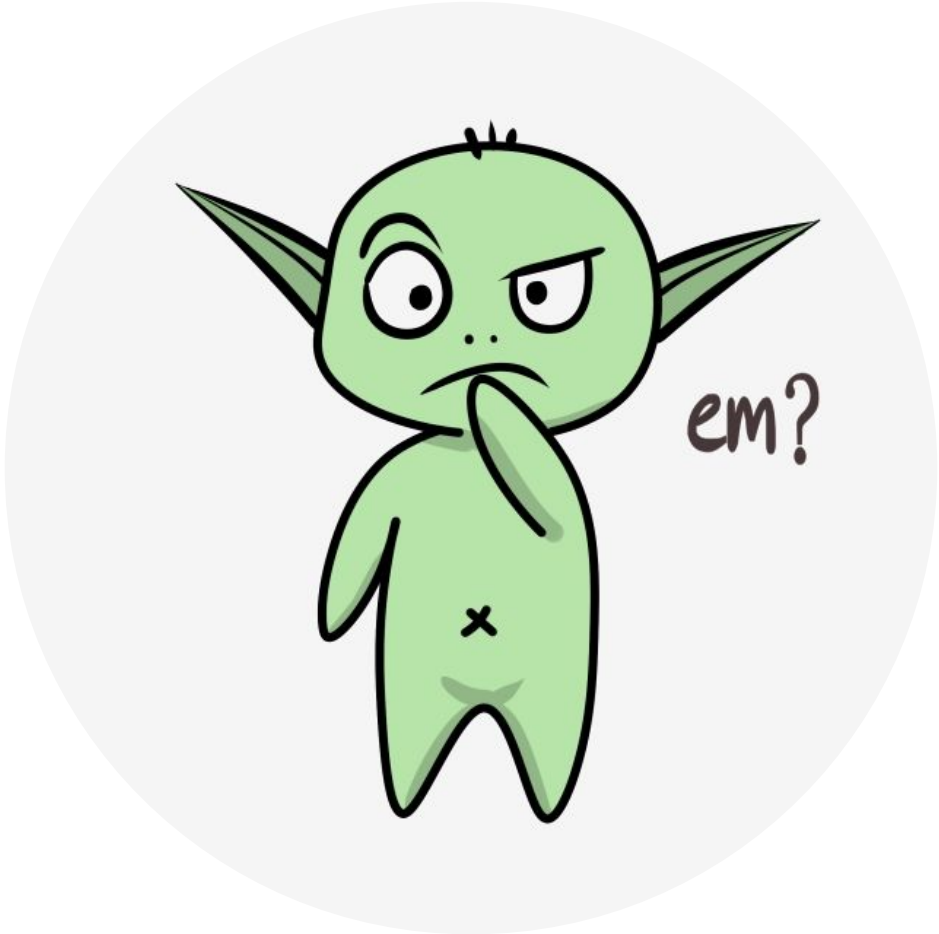


- Indirect methods often make fuller use of a limited amount of experience and thus achieve a better policy with fewer environmental interactions.
- Direct methods are much simpler and are not affected by biases in the design of the model.

Dyna-Q

- Dyna-Q includes all of the processes shown in the diagram—planning, acting, model-learning, and direct RL—all occurring continually (and *simultaneously*).
- Planning method: random-sample one-step tabular Q-planning.
- Direct RL method: one-step tabular Q-learning.
- Model-learning method: table-based and assumes the environment is deterministic.





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Model-free value-based reinforcement learning with function approximation

Marlos C. Machado

The Prediction Objective (A Notion of Accuracy)

- In the tabular case we can have equality, but with FA, not anymore.
  - Making one state's estimate more accurate invariably means making others' less accurate.
- Mean Squared Error:

$$\overline{\text{VE}}(\mathbf{w}) \doteq \sum_{s \in \mathcal{S}} \mu(s) \Big[v_\pi(s) - \hat{v}(s,\mathbf{w}) \Big]^2$$

How much do we care about the error in each state s.

Usually, the fraction of time spent in s.
On-policy distribution.

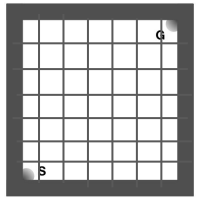
Recipe for Deriving a Concrete Algorithm for SGD

1. Specify a function approximation architecture (parametric form of v_π).
2. Write down your objective function.
3. Take the derivative of the objective function with respect to the weights.
4. Simplify the general gradient expression for your parametric form.
5. Make a weight update rule:

   $w = w - \alpha$ `GRAD`

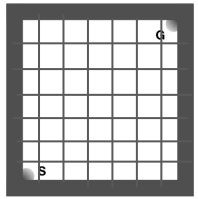
1. Specify a FA architecture (parametric form of v_π)

- We will use *state aggregation with linear function approximation*



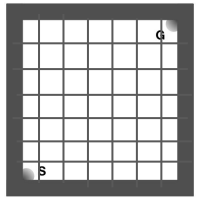
1. Specify a FA architecture (parametric form of v_π)

- We will use *state aggregation with linear function approximation*
- State aggregation
  - The features are always binary with only a single active feature that is not zero



1. Specify a FA architecture (parametric form of v_π)

- We will use *state aggregation with linear function approximation*
- State aggregation
  - The features are always binary with only a single active feature that is not zero
- Value function
  - Linear function

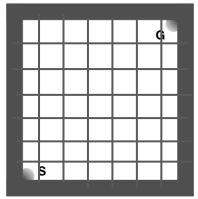


$$v_\pi(s) \approx \hat{v}(s, \mathbf{w}) \doteq \mathbf{w}^\top \mathbf{x}(s) \doteq \sum_{i=1}^{d} w_i \cdot x_i(s)$$

2. Write down your objective function

- We will use the *value error*

$$\begin{aligned}\overline{\text{VE}}(\mathbf{w}) &\doteq \sum_{s\in\mathcal{S}} \mu(s)\Big[v_\pi(s) - \hat{v}(s,\mathbf{w})\Big]^2 \\ &= \sum_{s\in\mathcal{S}} \mu(s)\Big[v_\pi(s) - \mathbf{w}^\top \mathbf{x}(s)\Big]^2\end{aligned}$$



3. Take the derivative of the obj. function w.r.t. the weights

$$
\begin{aligned}
\nabla\overline{\text{VE}}(\mathbf{w}) &= \nabla \sum_{s \in \mathcal{S}} \mu(s) \Big[v_\pi(s) - \mathbf{w}^\top \mathbf{x}(s) \Big]^2 \\
&= \sum_{s \in \mathcal{S}} \mu(s) \nabla \Big[v_\pi(s) - \mathbf{w}^\top \mathbf{x}(s) \Big]^2 \\
&= -\sum_{s \in \mathcal{S}} \mu(s) 2 \Big[v_\pi(s) - \mathbf{w}^\top \mathbf{x}(s) \Big] \nabla \mathbf{w}^\top \mathbf{x}(s)
\end{aligned}
$$

4. Simplify the general gradient expression

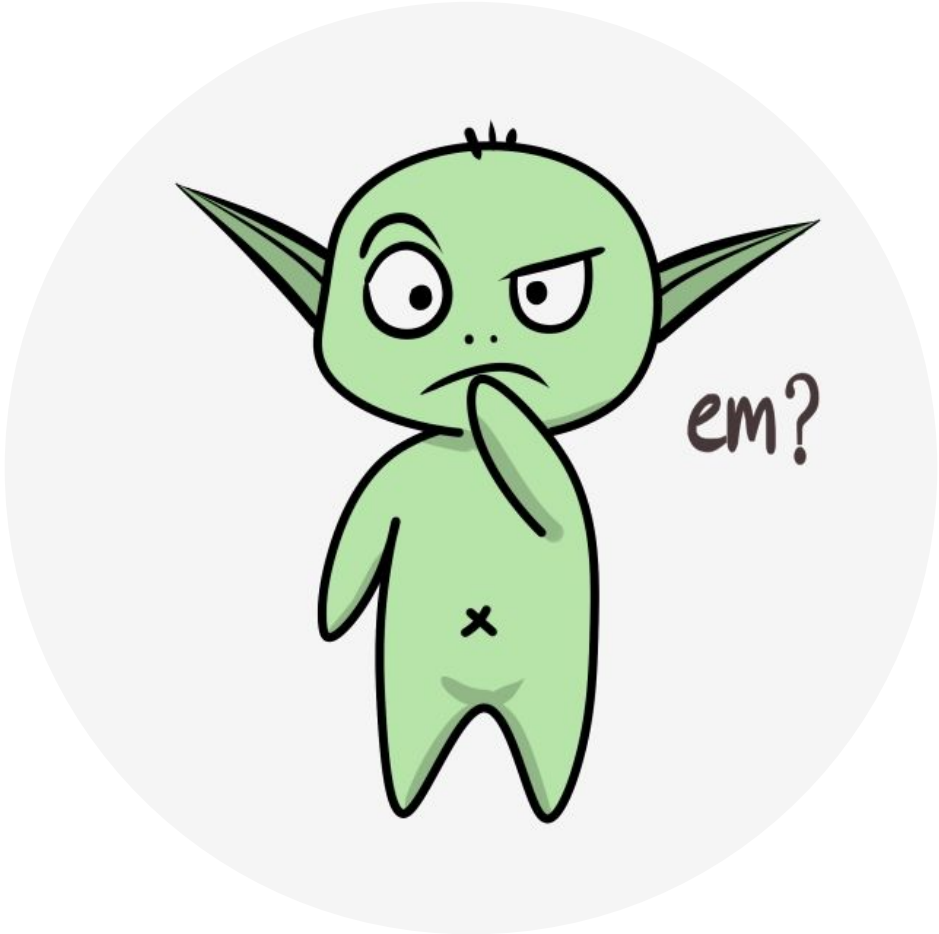
$$\begin{aligned}\nabla\overline{\text{VE}}(\mathbf{w}) &= -\sum_{s\in\mathcal{S}}\mu(s)2\Big[v_\pi(s)-\mathbf{w}^\top\mathbf{x}(s)\Big]\nabla\mathbf{w}^\top\mathbf{x}(s)\\ &= -\sum_{s\in\mathcal{S}}\mu(s)2\Big[v_\pi(s)-\mathbf{w}^\top\mathbf{x}(s)\Big]\mathbf{x}(s)\end{aligned}$$

$$\nabla\mathbf{w}^\top\mathbf{x}(s)=\mathbf{x}(s)$$

5. Make a weight update rule

$$\nabla \overline{\text{VE}}(\mathbf{w}) = -\sum_{s \in \mathcal{S}} \mu(s) 2 \Big[v_\pi(s) - \mathbf{w}^\top \mathbf{x}(s) \Big] \mathbf{x}(s)$$

$$\begin{aligned} \mathbf{w}_{t+1} &= \mathbf{w}_t + \alpha 2 \Big[v_\pi(s) - \mathbf{w}^\top \mathbf{x}(s) \Big] \mathbf{x}(s) \\ &= \mathbf{w}_t + \alpha \Big[v_\pi(s) - \mathbf{w}^\top \mathbf{x}(s) \Big] \mathbf{x}(s) \end{aligned}$$



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A More Realistic Update

- Let U_t denote the *t*-th training example, $S_t \mapsto v_\pi(S_t)$, of some (possibly random), approximation to the true value.

$$\mathbf{w}_{t+1} \doteq \mathbf{w}_t + \alpha \Big[U_t - \hat{v}(S_t, \mathbf{w}_t) \Big] \nabla \hat{v}(S_t, \mathbf{w}_t)$$

Gradient Monte Carlo Algorithm for Estimating $\hat{v} \approx v_\pi$

Input: the policy π to be evaluated
Input: a differentiable function $\hat{v} : \mathcal{S} \times \mathbb{R}^d \to \mathbb{R}$
Algorithm parameter: step size $\alpha > 0$
Initialize value-function weights $\mathbf{w} \in \mathbb{R}^d$ arbitrarily (e.g., $\mathbf{w} = \mathbf{0}$)

Loop forever (for each episode):
  Generate an episode $S_0, A_0, R_1, S_1, A_1, \ldots, R_T, S_T$ using π
  Loop for each step of episode, $t = 0, 1, \ldots, T-1$:
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha \big[G_t - \hat{v}(S_t, \mathbf{w}) \big] \nabla \hat{v}(S_t, \mathbf{w})$

Semi-gradient TD

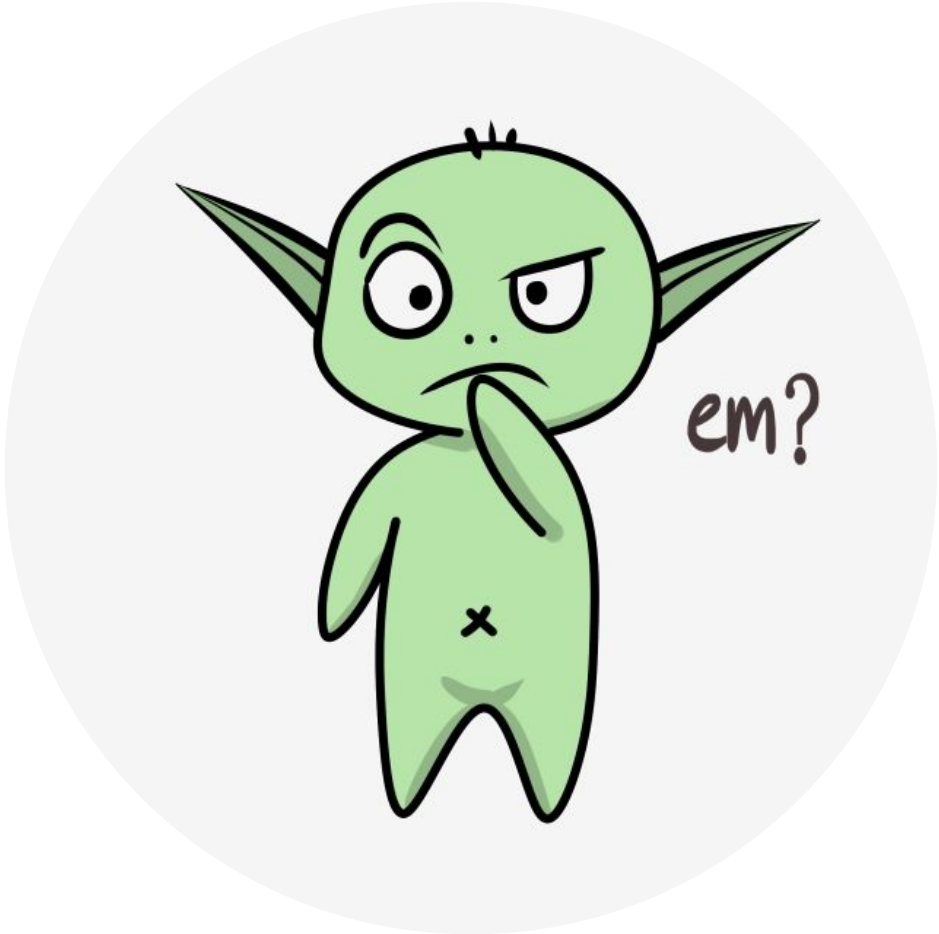
- What if $U_t \doteq R_{t+1} + \gamma\, \hat{v}(S_{t+1}, \mathbf{w}_t)$?
- We lose several guarantees when we use a bootstrapping estimate as target.
  - The target now also depends on the value of $\mathbf{w}_t$, so the target is not independent of $\mathbf{w}_t$.
- Bootstrapping are not instances of true gradient descent. They take into account the effect of changing the weight vector $\mathbf{w}_t$ on the estimate, but ignore its effect on the target. Thus, they are a *semi-gradient method*.
- Regardless of the theoretical guarantees, we use them all the time ¯_(ツ)_/¯

Semi-gradient TD

Semi-gradient TD(0) for estimating $\hat{v} \approx v_\pi$

Input: the policy π to be evaluated
Input: a differentiable function $\hat{v} : \mathcal{S}^+ \times \mathbb{R}^d \to \mathbb{R}$ such that $\hat{v}(\text{terminal},\cdot) = 0$
Algorithm parameter: step size $\alpha > 0$
Initialize value-function weights $\mathbf{w} \in \mathbb{R}^d$ arbitrarily (e.g., $\mathbf{w} = \mathbf{0}$)

Loop for each episode:
    Initialize S
    Loop for each step of episode:
        Choose $A \sim \pi(\cdot|S)$
        Take action A, observe R, S'
        $\mathbf{w} \leftarrow \mathbf{w} + \alpha\big[R + \gamma\hat{v}(S',\mathbf{w}) - \hat{v}(S,\mathbf{w})\big]\nabla\hat{v}(S,\mathbf{w})$
        $S \leftarrow S'$
    until S is terminal



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Episodic Semi-gradient Control

- We need to approximate the action-value function now, $\hat{q} \approx q_\pi$, that is represented as a parameterized function form with weight vector **w**.
- Before (until last class): $S_t \mapsto U_t$.
  Now: S_t, $A_t \mapsto U_t$.
- Action-value prediction:

$$\mathbf{w}_{t+1} \doteq \mathbf{w}_t + \alpha \Big[U_t - \hat{q}(S_t, A_t, \mathbf{w}_t) \Big] \nabla \hat{q}(S_t, A_t, \mathbf{w}_t)$$

- Episodic semi-gradient one-step *Sarsa*:

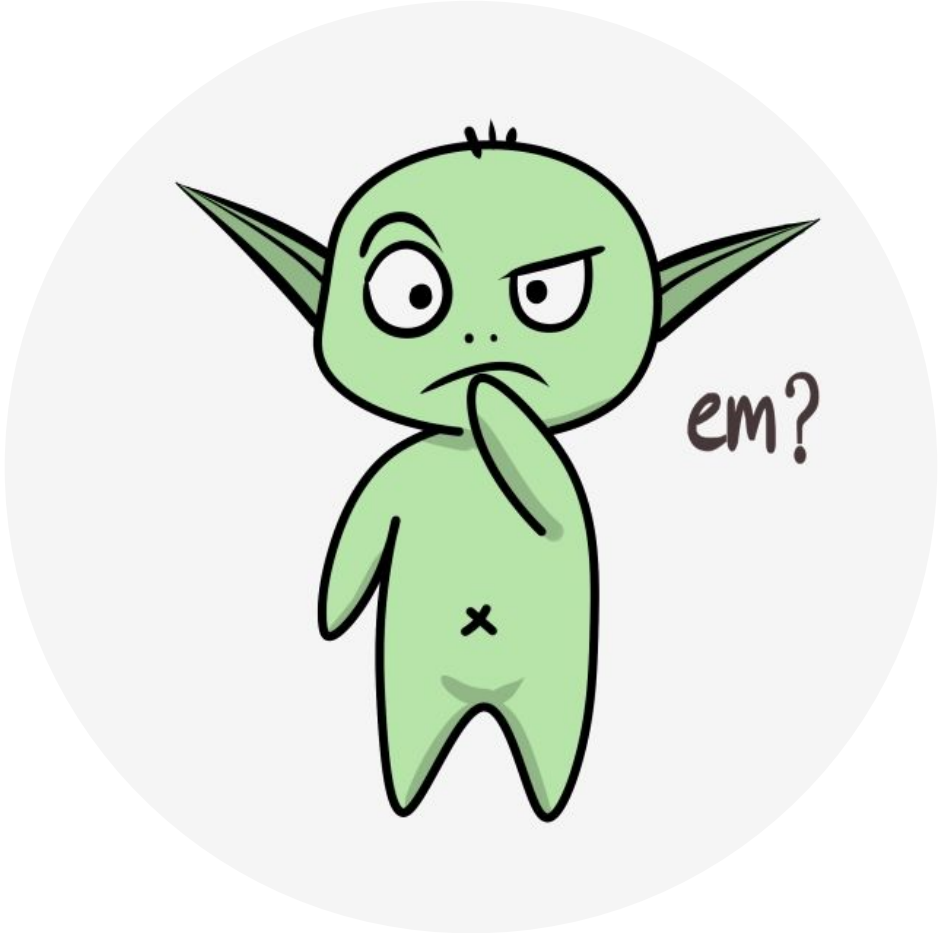
$$\mathbf{w}_{t+1} \doteq \mathbf{w}_t + \alpha \Big[R_{t+1} + \gamma \hat{q}(S_{t+1}, A_{t+1}, \mathbf{w}_t) - \hat{q}(S_t, A_t, \mathbf{w}_t) \Big] \nabla \hat{q}(S_t, A_t, \mathbf{w}_t)$$

Episodic Semi-gradient Sarsa

Episodic Semi-gradient Sarsa for Estimating $\hat{q} \approx q_*$

Input: a differentiable action-value function parameterization $\hat{q} : \mathcal{S} \times \mathcal{A} \times \mathbb{R}^d \to \mathbb{R}$
Algorithm parameters: step size $\alpha > 0$, small $\varepsilon > 0$
Initialize value-function weights $\mathbf{w} \in \mathbb{R}^d$ arbitrarily (e.g., $\mathbf{w} = \mathbf{0}$)

Loop for each episode:
  $S, A \leftarrow$ initial state and action of episode (e.g., ε-greedy)
  Loop for each step of episode:
    Take action A, observe R, S'
    If S' is terminal:
      $\mathbf{w} \leftarrow \mathbf{w} + \alpha\big[R - \hat{q}(S, A, \mathbf{w})\big]\nabla\hat{q}(S, A, \mathbf{w})$
      Go to next episode
    Choose A' as a function of $\hat{q}(S', \cdot, \mathbf{w})$ (e.g., ε-greedy)
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha\big[R + \gamma\hat{q}(S', A', \mathbf{w}) - \hat{q}(S, A, \mathbf{w})\big]\nabla\hat{q}(S, A, \mathbf{w})$
    $S \leftarrow S'$
    $A \leftarrow A'$



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This works!

State of the Art Control of Atari Games Using Shallow Reinforcement Learning

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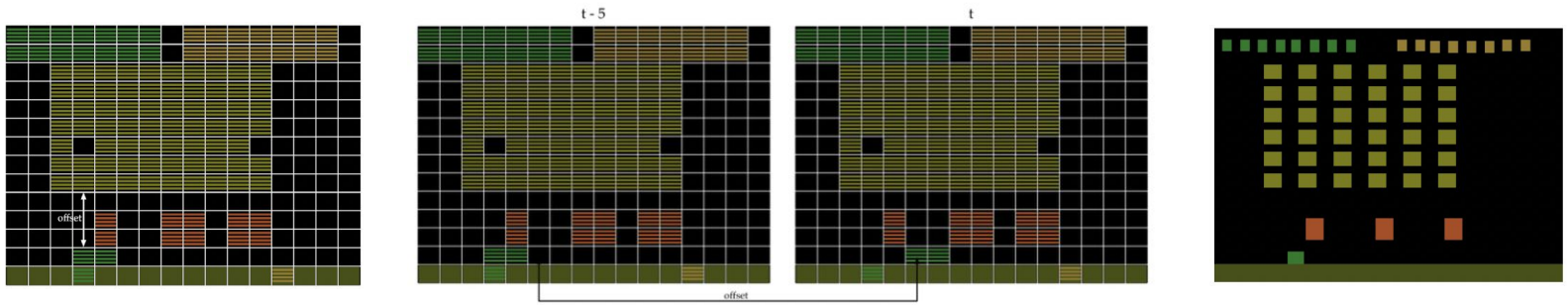
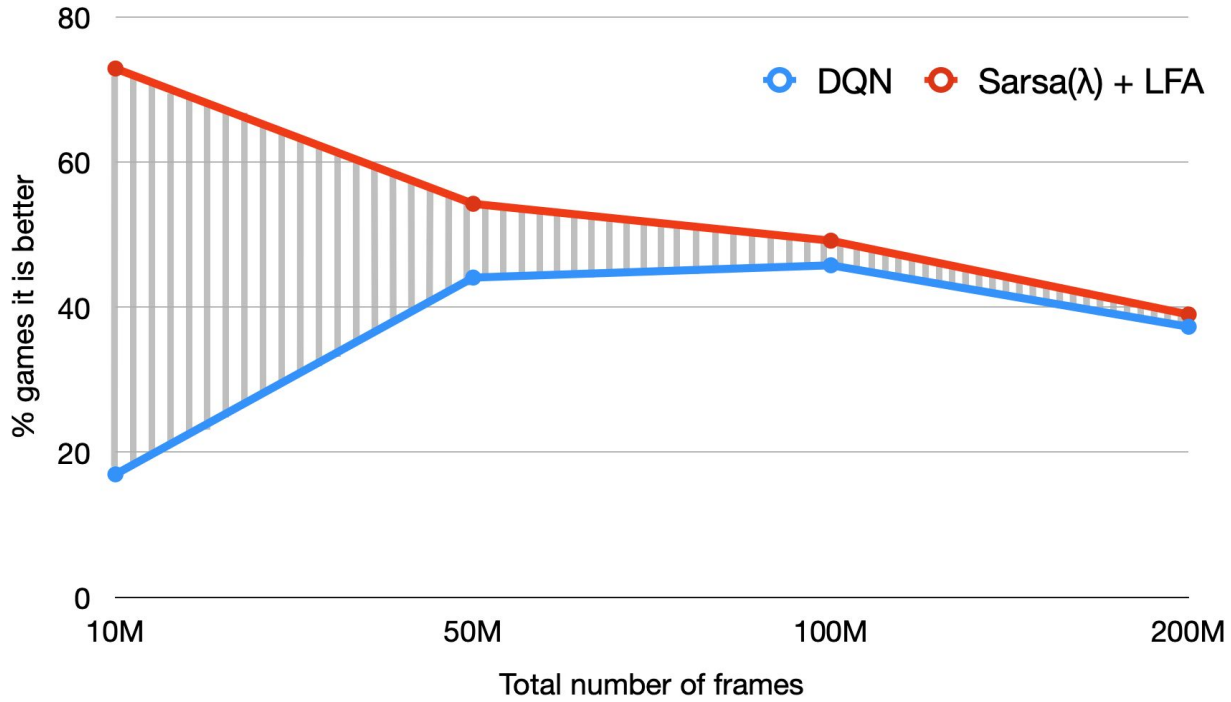
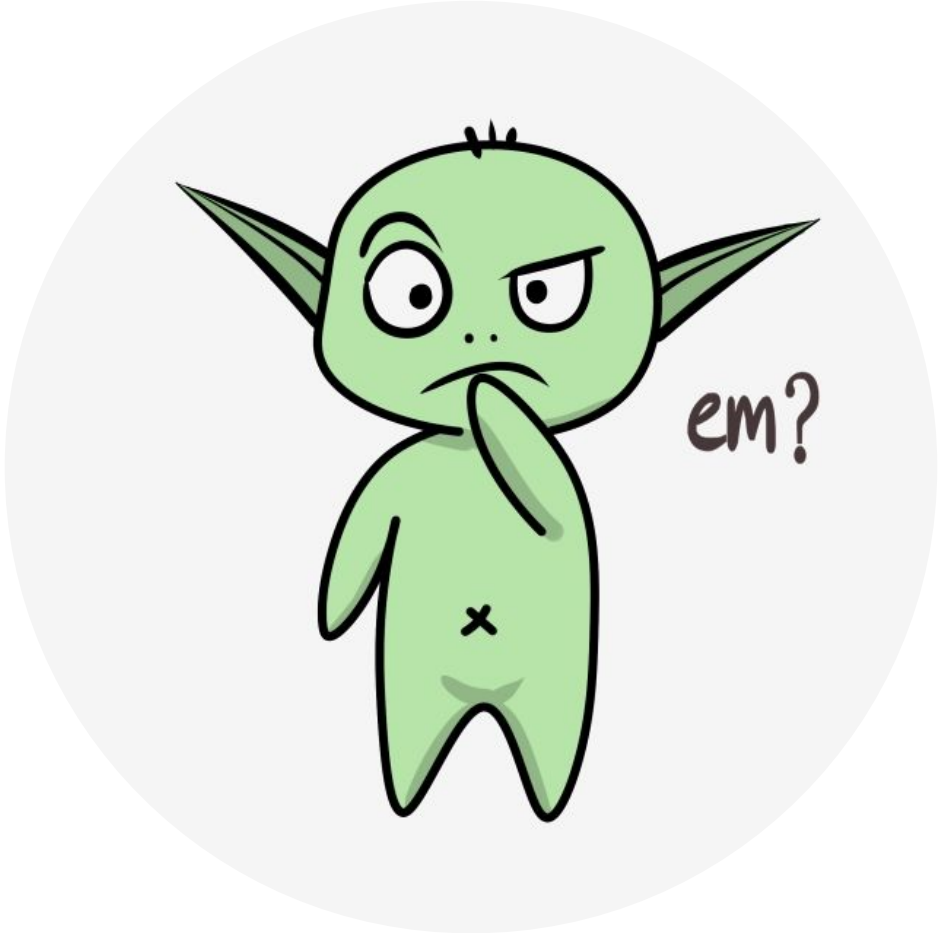


There are many trade-offs, we need to understand them

[Liang et al., 2016; Machado et al. 2018]





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Policy gradient methods

Marlos C. Machado

The Policy Gradient Theorem

- We have stronger convergence guarantees for policy gradient methods, in part because the policy changes more smoothly than, say, ε-greedy policies.

- Let's do gradient ascent, in the full RL problem. To do that we need to define J(**θ**):

$$J(\boldsymbol{\theta}) \doteq v_{\pi_{\boldsymbol{\theta}}}(s_0)$$

- It seems tricky though. "The problem is that performance depends on both the action selections and the distribution of states in which those selections are made, and that both of these are affected by the policy parameter."
    - The effect of a change in the policy on the state distribution is typically unknown.

How can we estimate the performance gradient w.r.t the policy parameter when the gradient depends on the unknown effect of policy changes on the state distribution?

The Policy Gradient Theorem

- The Policy Gradient Theorem [Marbach and Tsitsiklis, 1998, 2001; Sutton et al. 2000] provides an analytic expression for the gradient of performance w.r.t. the policy parameter that does not involve the derivative of the state distribution.

$$\nabla J(\boldsymbol{\theta}) \propto \sum_s \mu(s) \sum_a q_\pi(s,a) \nabla \pi(a|s,\boldsymbol{\theta})$$

Proportional to

On-policy distribution

A First Policy Gradient Method

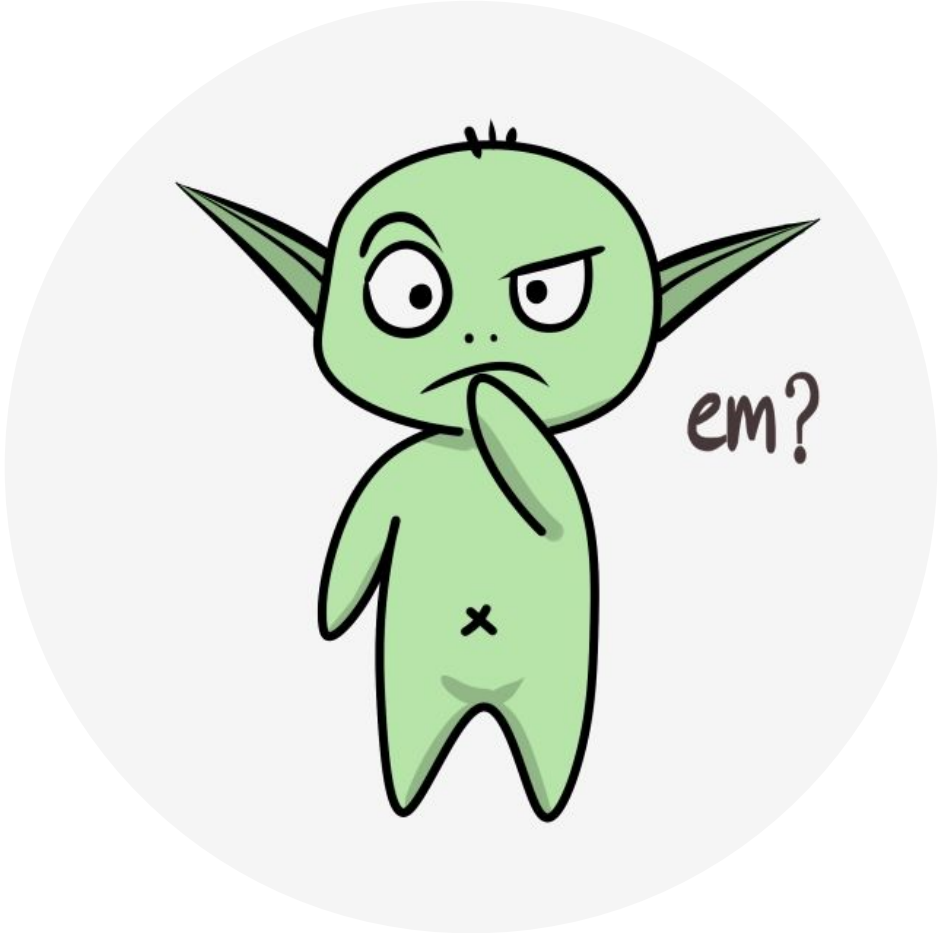
$$\nabla J(\boldsymbol{\theta}) \propto \sum_s \mu(s) \sum_a q_\pi(s,a) \nabla \pi(a|s,\boldsymbol{\theta})$$

$$= \mathbb{E}_\pi \left[\sum_a q_\pi(S_t,a) \nabla \pi(a|S_t,\boldsymbol{\theta}) \right].$$

Any constant of proportionality can be absorbed into the step size α

It weighs the sum by how often the states occur under the target policy π

$$\boldsymbol{\theta}_{t+1} \doteq \boldsymbol{\theta}_t + \alpha \sum_a \hat{q}(S_t,a,\mathbf{w}) \nabla \pi(a|S_t,\boldsymbol{\theta})$$



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REINFORCE: Monte Carlo Policy Gradient [Willliams, 1992]

$$\boldsymbol{\theta}_{t+1} \doteq \boldsymbol{\theta}_t + \alpha \sum_a \hat{q}(S_t, a, \mathbf{w}) \nabla \pi(a|S_t, \boldsymbol{\theta})$$

We want an update that at time t involves just A_t. We need to replace a sum over the RV's possible values by an expectation under π, and then sampling the expectation.

$$\begin{aligned}
\nabla J(\boldsymbol{\theta}) &\propto \mathbb{E}_\pi \left[\sum_a q_\pi(S_t, a) \nabla \pi(a|S_t, \boldsymbol{\theta}) \right] \\
&= \mathbb{E}_\pi \left[\sum_a \pi(a|S_t, \boldsymbol{\theta}) q_\pi(S_t, a) \frac{\nabla \pi(a|S_t, \boldsymbol{\theta})}{\pi(a|S_t, \boldsymbol{\theta})} \right] \\
&= \mathbb{E}_\pi \left[q_\pi(S_t, A_t) \frac{\nabla \pi(A_t|S_t, \boldsymbol{\theta})}{\pi(A_t|S_t, \boldsymbol{\theta})} \right] \quad = \mathbb{E}_\pi \left[G_t \frac{\nabla \pi(A_t|S_t, \boldsymbol{\theta})}{\pi(A_t|S_t, \boldsymbol{\theta})} \right]
\end{aligned}$$

REINFORCE: Monte Carlo Policy Gradient [Willliams, 1992]

$$\nabla J(\boldsymbol{\theta}) \propto \mathbb{E}_{\pi}\left[G_t \frac{\nabla \pi(A_t|S_t,\boldsymbol{\theta})}{\pi(A_t|S_t,\boldsymbol{\theta})}\right]$$

$$\boldsymbol{\theta}_{t+1} \doteq \boldsymbol{\theta}_t + \alpha G_t \frac{\nabla \pi(A_t|S_t,\boldsymbol{\theta}_t)}{\pi(A_t|S_t,\boldsymbol{\theta}_t)}$$

REINFORCE uses the full return,
thus it is a Monte Carlo method.

REINFORCE: Monte Carlo Policy Gradient [Willliams, 1992]

$$\boldsymbol{\theta}_{t+1} \doteq \boldsymbol{\theta}_t + \alpha G_t \frac{\nabla \pi(A_t|S_t,\boldsymbol{\theta}_t)}{\pi(A_t|S_t,\boldsymbol{\theta}_t)}$$

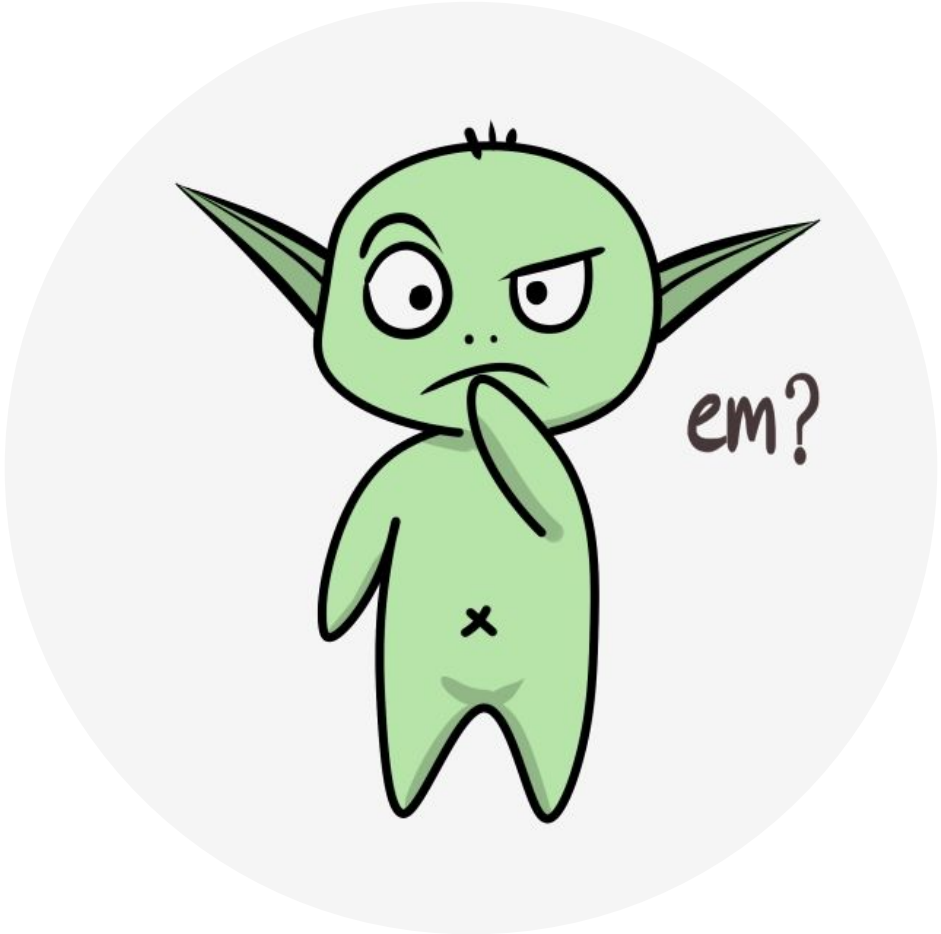
REINFORCE: Monte-Carlo Policy-Gradient Control (episodic) for π_*

Input: a differentiable policy parameterization $\pi(a|s,\boldsymbol{\theta})$
Algorithm parameter: step size $\alpha > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ (e.g., to $\mathbf{0}$)

Loop forever (for each episode):
  Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot,\boldsymbol{\theta})$
  Loop for each step of the episode $t = 0, 1, \ldots, T-1$:
    $G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$ (G_t)
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \gamma^t G \nabla \boxed{\ln \pi(A_t|S_t,\boldsymbol{\theta})}$

Recall:

$$\nabla \ln x = \frac{\nabla x}{x}$$



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Actor-Critic Methods

- In REINFORCE with baseline, the learned state-value function estimates the value of the first state of each state transition.
- In actor-critic methods, the state-value function is applied also to the second state of the transition.
- When the state-value function is used to assess actions in this way it is called a critic, and the overall policy-gradient method is termed an actor–critic method.

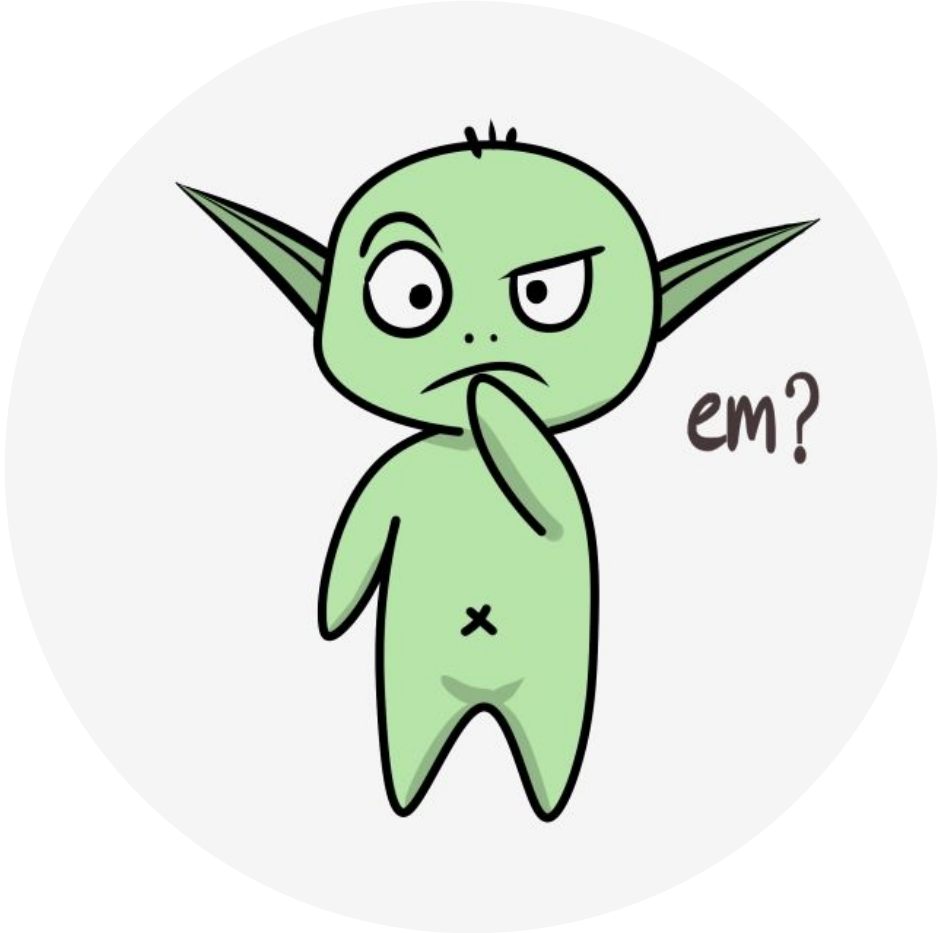
One-Step Actor-Critic

$$
\begin{aligned}
\boldsymbol{\theta}_{t+1} &\doteq \boldsymbol{\theta}_t + \alpha\Big(G_{t:t+1} - \hat{v}(S_t,\mathbf{w})\Big)\frac{\nabla\pi(A_t|S_t,\boldsymbol{\theta}_t)}{\pi(A_t|S_t,\boldsymbol{\theta}_t)} \\
&= \boldsymbol{\theta}_t + \alpha\Big(R_{t+1} + \gamma\hat{v}(S_{t+1},\mathbf{w}) - \hat{v}(S_t,\mathbf{w})\Big)\frac{\nabla\pi(A_t|S_t,\boldsymbol{\theta}_t)}{\pi(A_t|S_t,\boldsymbol{\theta}_t)} \\
&= \boldsymbol{\theta}_t + \alpha\delta_t\frac{\nabla\pi(A_t|S_t,\boldsymbol{\theta}_t)}{\pi(A_t|S_t,\boldsymbol{\theta}_t)}.
\end{aligned}
$$

One-Step Actor-Critic

One-step Actor–Critic (episodic), for estimating $\pi_{\boldsymbol{\theta}} \approx \pi_*$

Input: a differentiable policy parameterization $\pi(a|s,\boldsymbol{\theta})$
Input: a differentiable state-value function parameterization $\hat{v}(s,\mathbf{w})$
Parameters: step sizes $\alpha^{\boldsymbol{\theta}} > 0$, $\alpha^{\mathbf{w}} > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ and state-value weights $\mathbf{w} \in \mathbb{R}^{d}$ (e.g., to $\mathbf{0}$)
Loop forever (for each episode):
  Initialize S (first state of episode)
  $I \leftarrow 1$
  Loop while S is not terminal (for each time step):
    $A \sim \pi(\cdot|S,\boldsymbol{\theta})$
    Take action A, observe S', R
    $\delta \leftarrow R + \gamma \hat{v}(S',\mathbf{w}) - \hat{v}(S,\mathbf{w})$ (if S' is terminal, then $\hat{v}(S',\mathbf{w}) \doteq 0$)
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha^{\mathbf{w}} \delta \nabla \hat{v}(S,\mathbf{w})$
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha^{\boldsymbol{\theta}} I \delta \nabla \ln \pi(A|S,\boldsymbol{\theta})$
    $I \leftarrow \gamma I$
    $S \leftarrow S'$



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Next class

- What I plan to do:
  - Finish this overview on reinforcement learning.
  - Start an overview of neural networks / deep learning.

- What I recommend YOU to do for next class:
  - *Brush-up* on the basics of deep learning if you don't remember.

    Specifically, Goodfellow, Bengio & Courville (2016)'s chapters 6–10.
  - Start looking at Assignment 1.