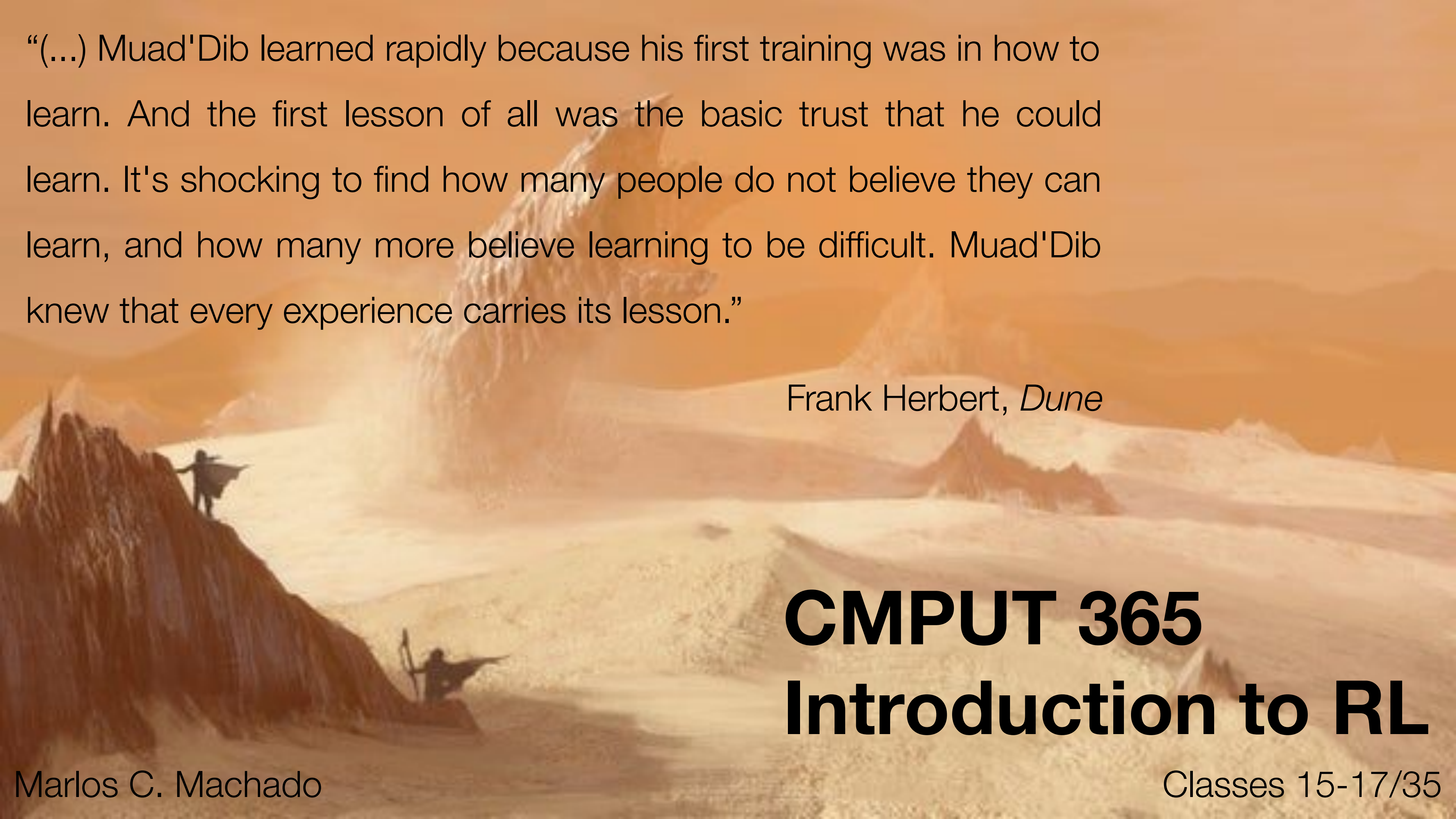


> “(...) Muad'Dib learned rapidly because his first training was in how to learn. And the first lesson of all was the basic trust that he could learn. It's shocking to find how many people do not believe they can learn, and how many more believe learning to be difficult. Muad'Dib knew that every experience carries its lesson.”
>
> Frank Herbert, *Dune*

CMPUT 365 Introduction to RL

Marlos C. Machado

Classes 15-17/35

Coursera Reminder

You **should be enrolled in the private session** we created in Coursera for CMPUT 365.

I **cannot** use marks from the public repository for your course marks. You **need to check**, **every time**, if you are in the private session and if you are submitting quizzes and assignments to the private section.

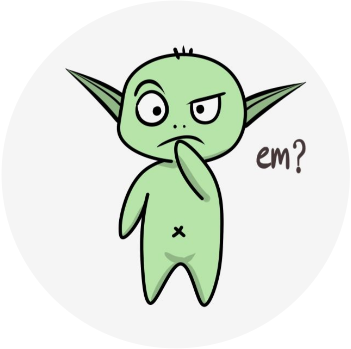
At the end of the term, I **will not port grades** from the public session in Coursera.

If you have any questions or concerns, **talk with the TAs** or email us `cmput365@ualberta.ca`.

Reminders and Notes

- Exam grades have been posted this morning avg: 9.3 (46.5%), max: 20 (100%), min 1 (5%)
- Mid-term Course and Instruction Feedback online evaluation is now available.
- Exam viewing:
  - It will likely happen next week
- Useful information for you:
  - We will post the grades for the Monte Carlo quiz on Canvas soon.
  - The Quiz and Programming Assignment for Temporal Difference Learning is due on Friday.

Please, interrupt me at any time!



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Chapter 6

Temporal-Difference Learning

Prediction

Temporal-difference learning – Why?

"If one had to identify one idea as central and novel to reinforcement learning, it would undoubtedly be temporal-difference (TD) learning."

TD Prediction

A simple every-visit Monte Carlo method is:

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[\underline{G_t} - V(S_t)\Big]$$

What if we don't want to wait until we have a full return (end of episode)!

$$\textit{NewEstimate} \leftarrow \textit{OldEstimate} \ + \ \textit{StepSize}\ \Big[\textit{Target} - \textit{OldEstimate}\Big]$$

What can we use instead of G_{t+1}?

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots$$

$$G_t = R_{t+1} + \gamma G_{t+1}$$

$$v_\pi(s) = \mathbb{E}_\pi \left[G_t \mid S_t = s\right]$$

$$v_\pi(s) = \mathbb{E}_\pi \left[R_{t+1} + \gamma G_{t+1} \mid S_t = s\right]$$

$$v_\pi(s) = \mathbb{E}_\pi \left[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s\right]$$

TD Prediction

A simple every-visit Monte Carlo method is:

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[\underbrace{G_t}_{\text{Target}} - V(S_t)\Big]$$

Temporal-Difference Learning:

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[\underbrace{R_{t+1} + \gamma V(S_{t+1})}_{\text{Target}} - V(S_t)\Big]$$

TD Prediction

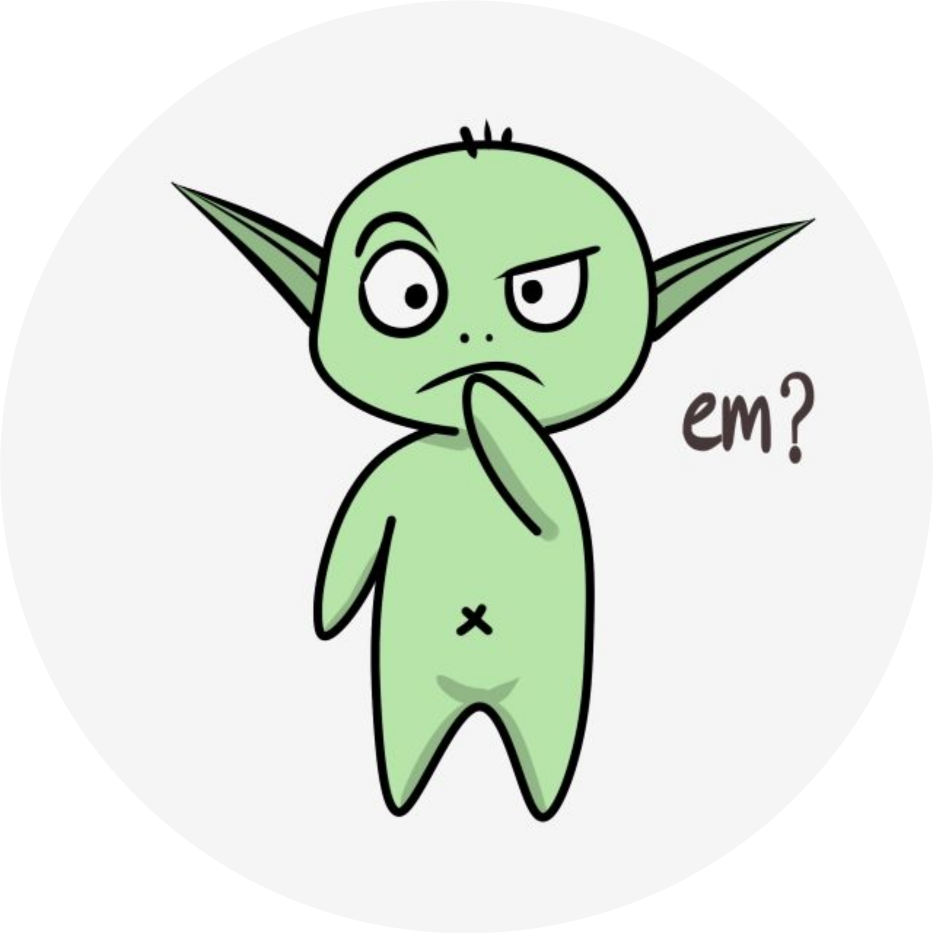
A simple every-visit Monte Carlo method is:

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[G_t - V(S_t)\Big]$$

Temporal-Difference Learning (specifically, **one-step TD**, or **TD(0)**):

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\Big]$$

These are estimates all the way down…



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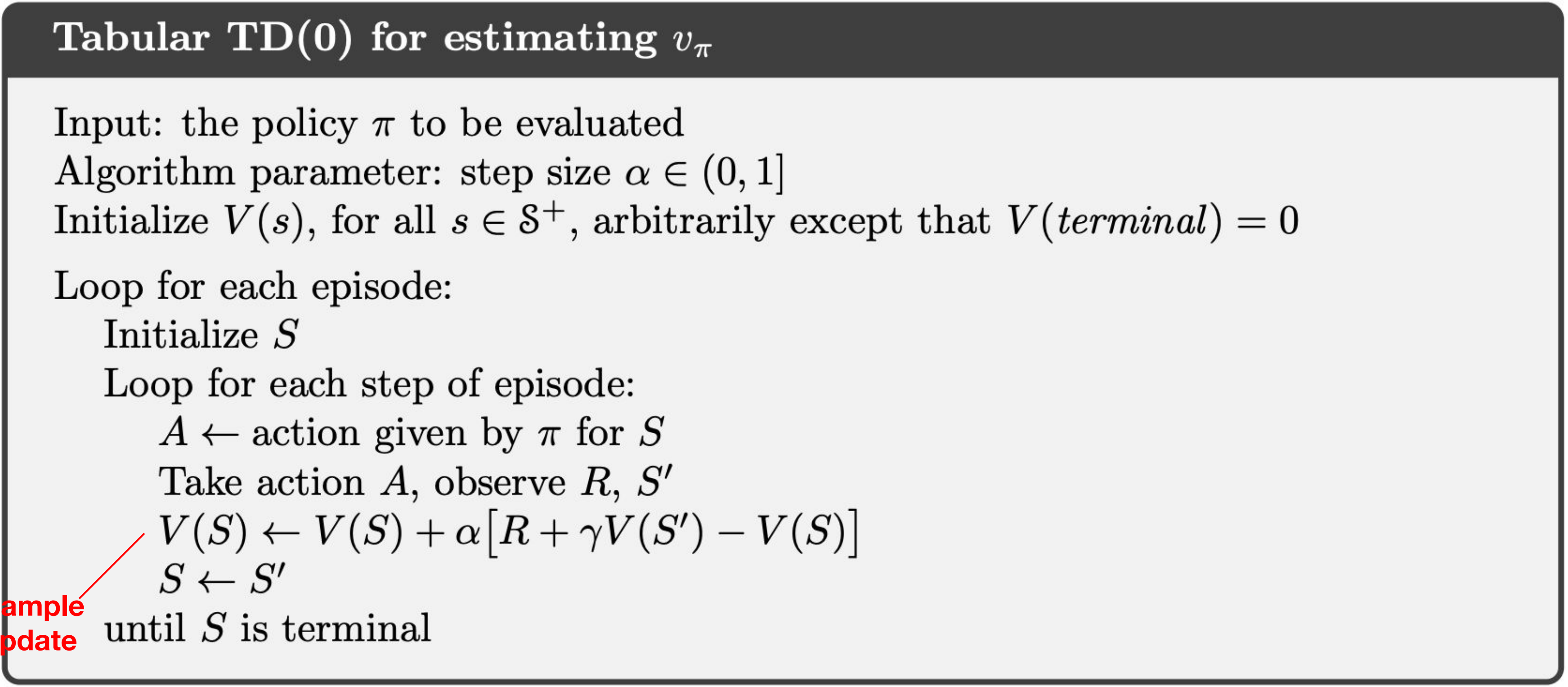
Tabular TD(0)

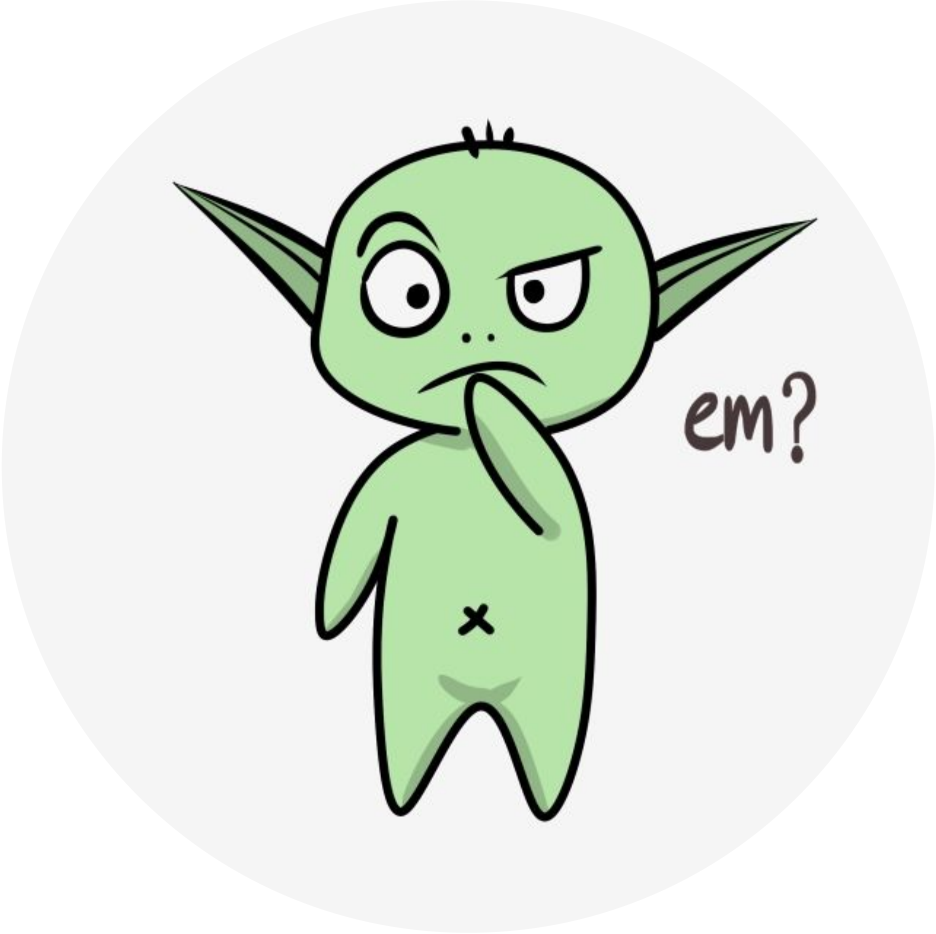


Tabular TD(0) for estimating v_π

```
Input: the policy $\pi$ to be evaluated
Algorithm parameter: step size $\alpha \in (0, 1]$
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(\mathit{terminal}) = 0$

Loop for each episode:
    Initialize $S$
    Loop for each step of episode:
        $A \leftarrow$ action given by $\pi$ for $S$
        Take action $A$, observe $R$, $S'$
        $V(S) \leftarrow V(S) + \alpha\big[R + \gamma V(S') - V(S)\big]$
        $S \leftarrow S'$
    until $S$ is terminal
```

Sample update



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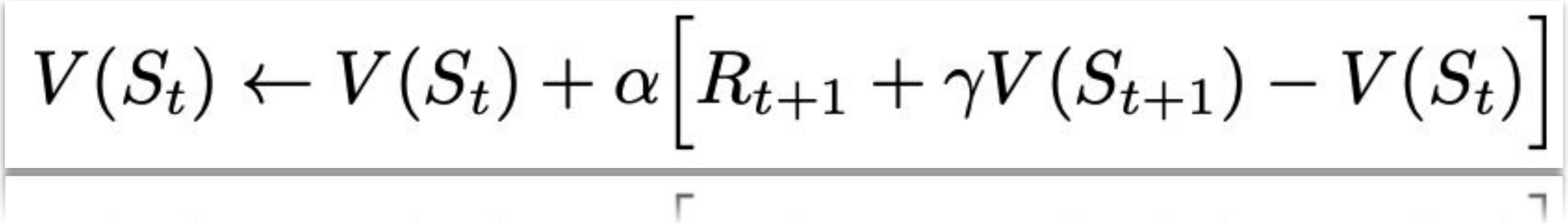
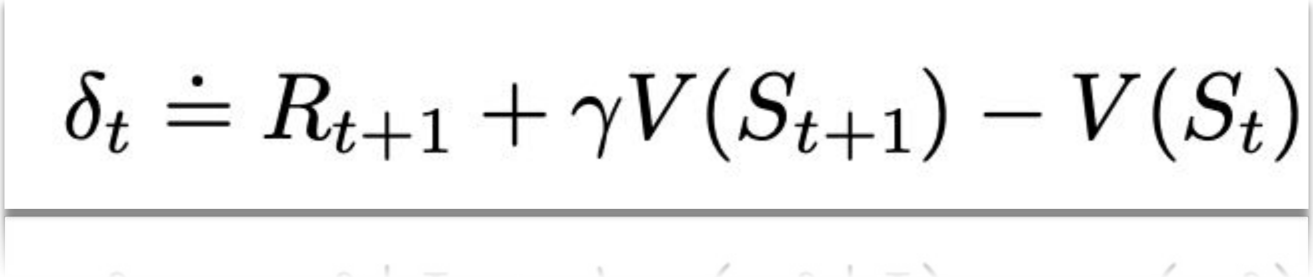
Temporal-Difference Error

$$\delta_t \doteq R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$$

Temporal-Difference Error



$$V(S_t) \leftarrow V(S_t) + \alpha\Big[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\Big]$$

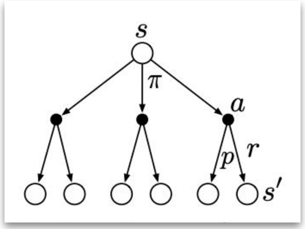


$$\delta_t \doteq R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$$

TD is a sample update with bootstrapping

- Dynamic programming update:

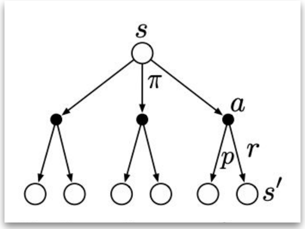
$$\begin{aligned} v_{k+1}(s) &\doteq \mathbb{E}_\pi[R_{t+1} + \gamma v_k(S_{t+1}) \mid S_t = s] \\ &= \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a)\Big[r + \gamma v_k(s')\Big] \end{aligned}$$



TD is a sample update with bootstrapping

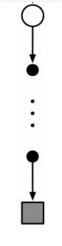
- Dynamic programming update:

$$
\begin{aligned}
v_{k+1}(s) &\doteq \mathbb{E}_\pi[R_{t+1} + \gamma v_k(S_{t+1}) \mid S_t = s] \\
&= \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a)\Big[r + \gamma v_k(s')\Big]
\end{aligned}
$$



- Monte Carlo update:

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[G_t - V(S_t)\Big]$$



TD is a sample update with bootstrapping

- Dynamic programming update:

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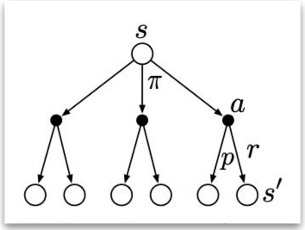
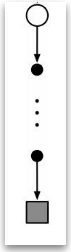


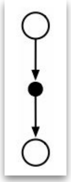
- Monte Carlo update:

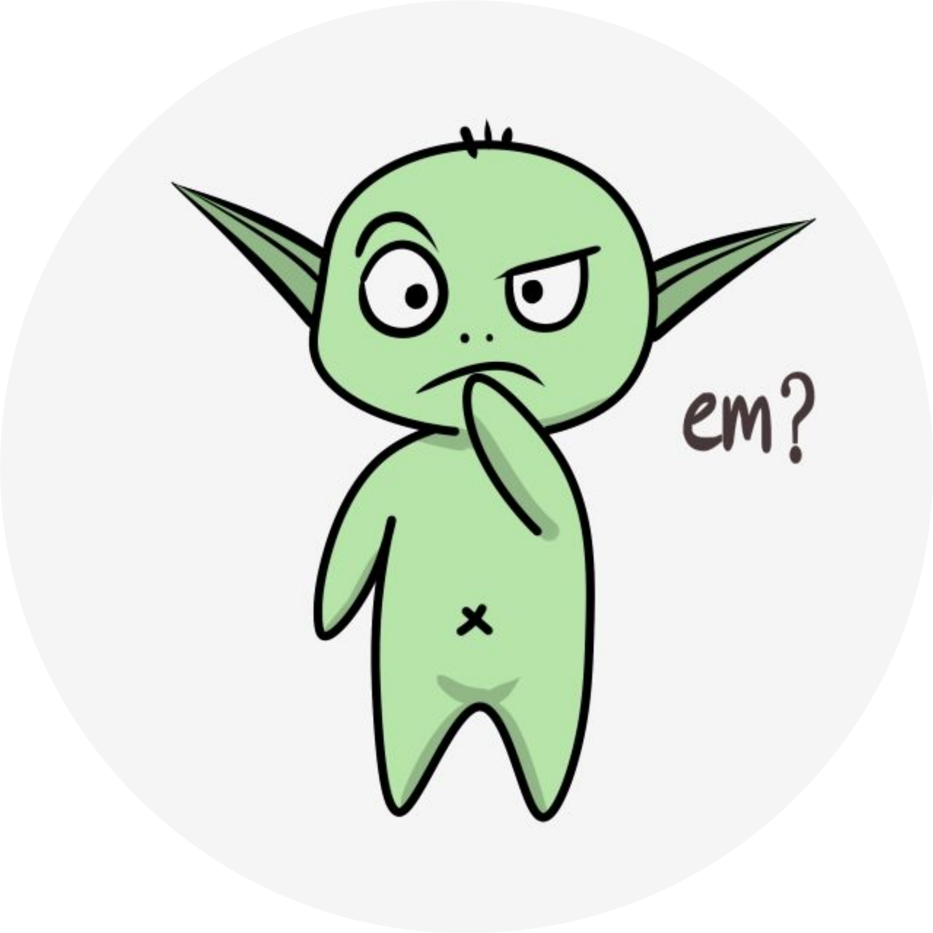
$$V(S_t) \leftarrow V(S_t) + \alpha\Big[G_t - V(S_t)\Big]$$



- TD update:

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\Big]$$





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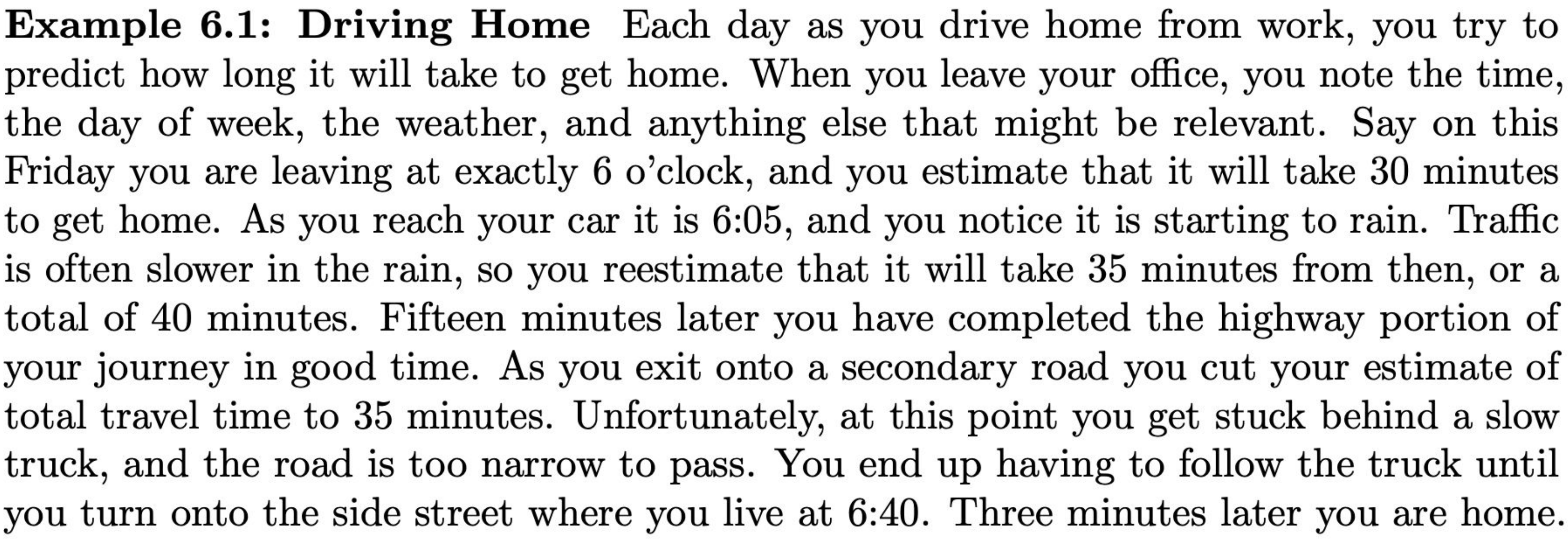
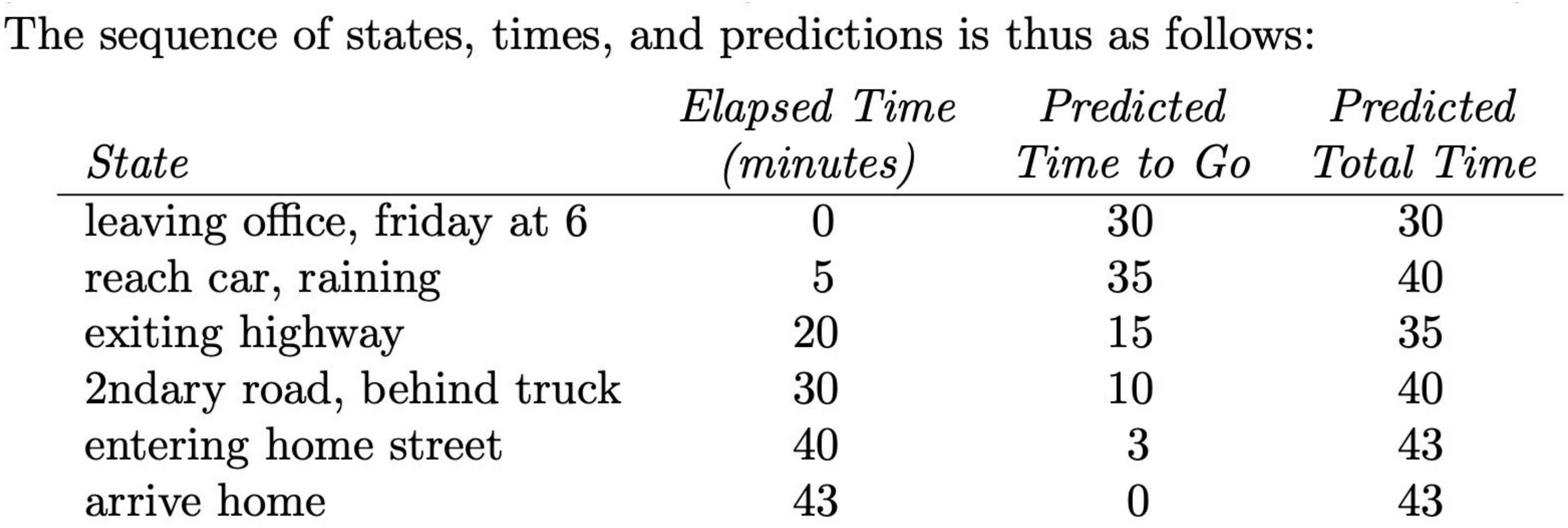
Example – Driving Home



Example 6.1: Driving Home Each day as you drive home from work, you try to predict how long it will take to get home. When you leave your office, you note the time, the day of week, the weather, and anything else that might be relevant. Say on this Friday you are leaving at exactly 6 o'clock, and you estimate that it will take 30 minutes to get home. As you reach your car it is 6:05, and you notice it is starting to rain. Traffic is often slower in the rain, so you reestimate that it will take 35 minutes from then, or a total of 40 minutes. Fifteen minutes later you have completed the highway portion of your journey in good time. As you exit onto a secondary road you cut your estimate of total travel time to 35 minutes. Unfortunately, at this point you get stuck behind a slow truck, and the road is too narrow to pass. You end up having to follow the truck until you turn onto the side street where you live at 6:40. Three minutes later you are home.

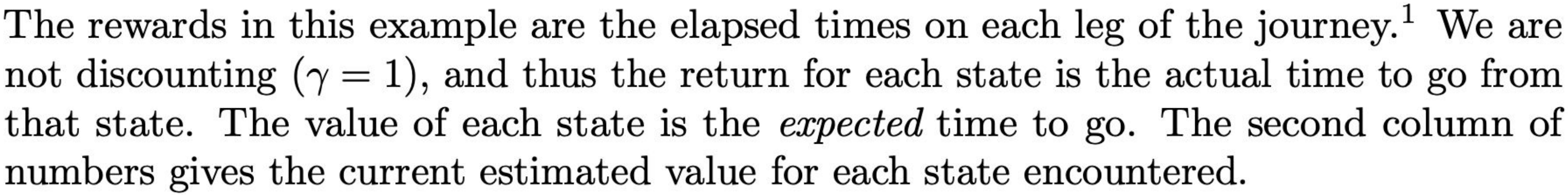
Example – Driving Home



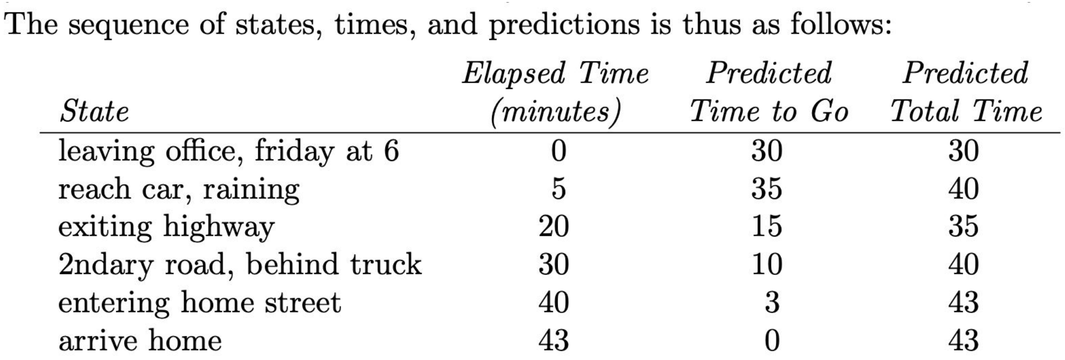
The sequence of states, times, and predictions is thus as follows:

State	*Elapsed Time (minutes)*	*Predicted Time to Go*	*Predicted Total Time*
leaving office, friday at 6	0	30	30
reach car, raining	5	35	40
exiting highway	20	15	35
2ndary road, behind truck	30	10	40
entering home street	40	3	43
arrive home	43	0	43

Example – Driving Home



The rewards in this example are the elapsed times on each leg of the journey.[1] We are not discounting ($\gamma = 1$), and thus the return for each state is the actual time to go from that state. The value of each state is the *expected* time to go. The second column of numbers gives the current estimated value for each state encountered.



The sequence of states, times, and predictions is thus as follows:

State	*Elapsed Time (minutes)*	*Predicted Time to Go*	*Predicted Total Time*
leaving office, friday at 6	0	30	30
reach car, raining	5	35	40
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2ndary road, behind truck	30	10	40
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arrive home	43	0	43

Example – Driving Home

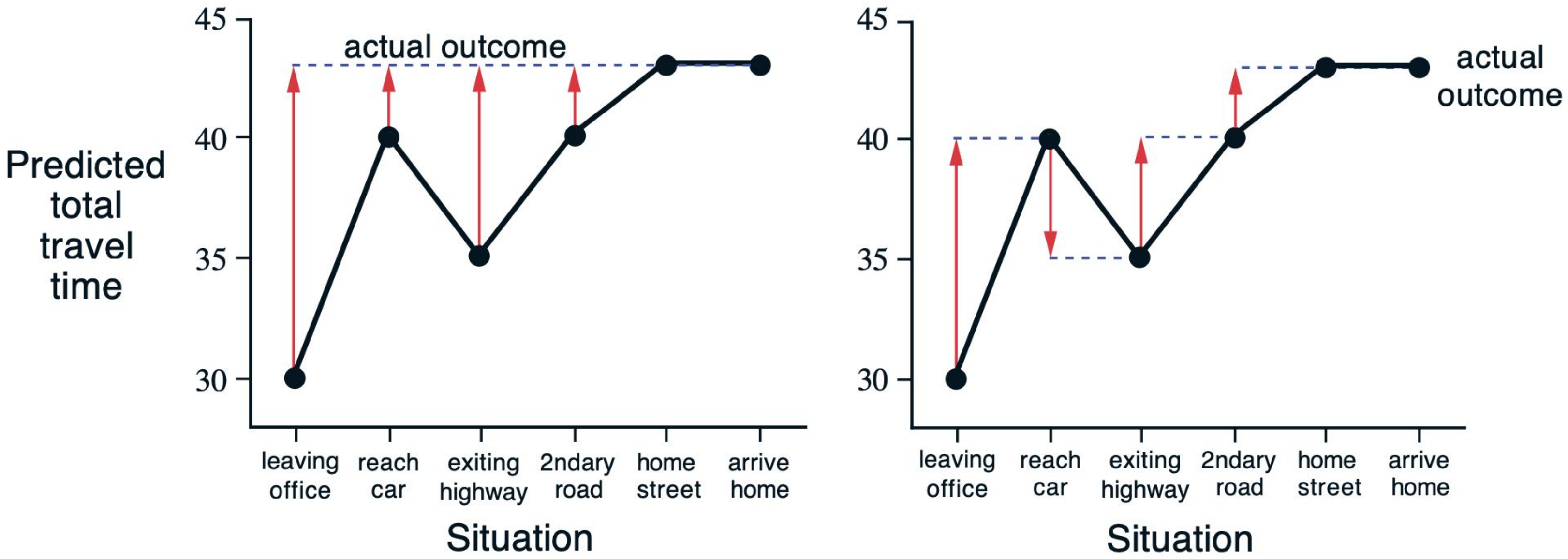
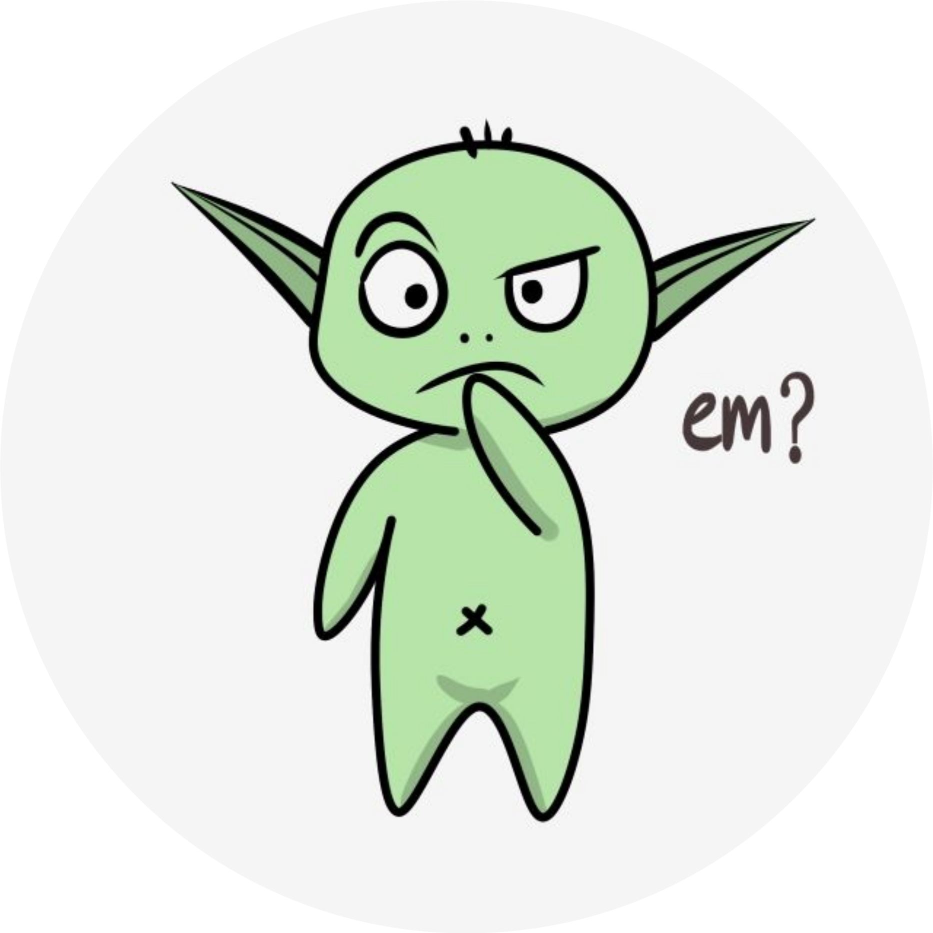


Figure 6.1: Changes recommended in the driving home example by Monte Carlo methods (left) and TD methods (right).



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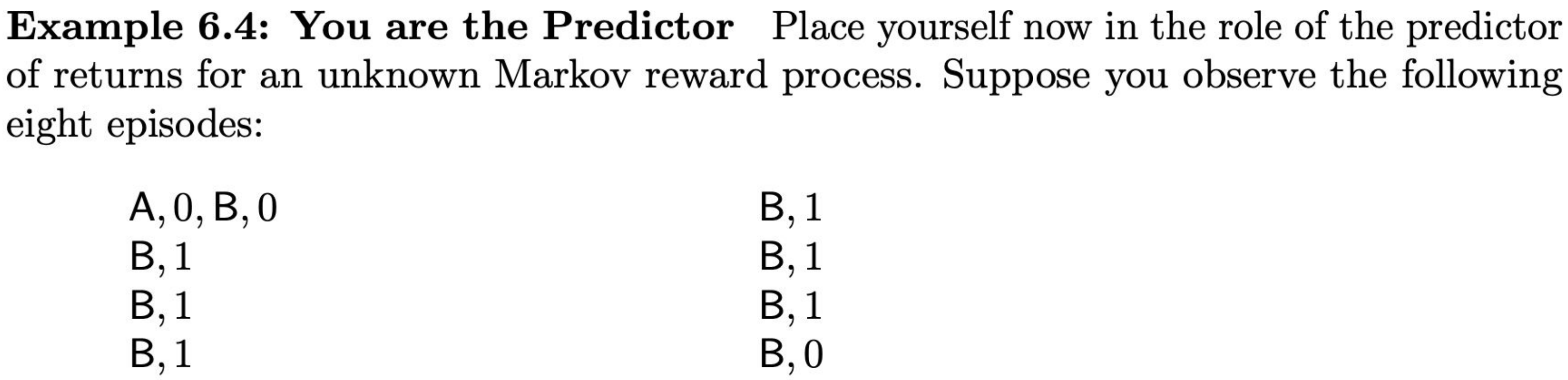
Optimality of TD(0)

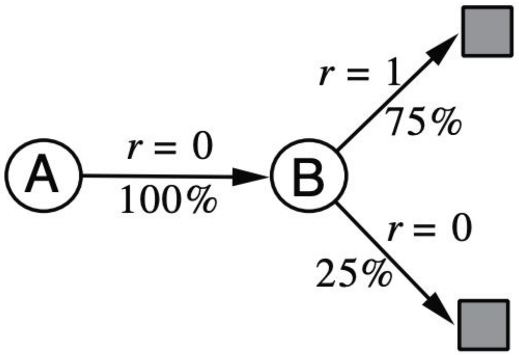


Example 6.4: You are the Predictor Place yourself now in the role of the predictor of returns for an unknown Markov reward process. Suppose you observe the following eight episodes:

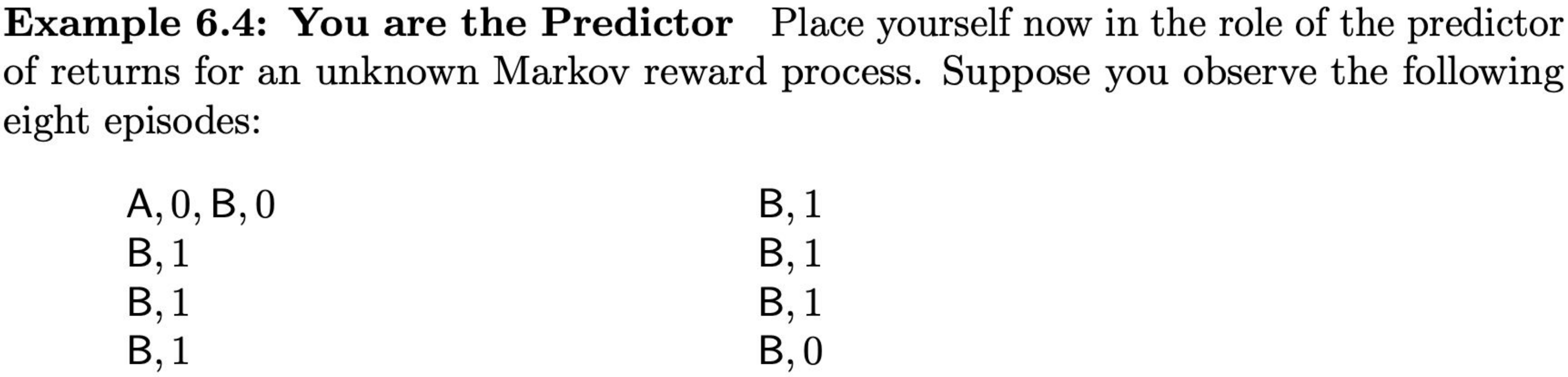
A, 0, B, 0
B, 1
B, 1
B, 1
B, 1
B, 1
B, 1
B, 0

V(A) = ?

V(B) = ?



Optimality of TD(0)



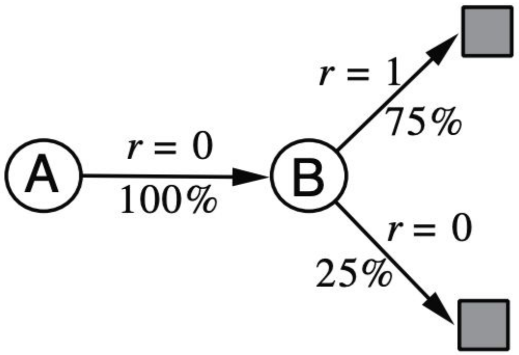
Example 6.4: You are the Predictor Place yourself now in the role of the predictor of returns for an unknown Markov reward process. Suppose you observe the following eight episodes:

A, 0, B, 0
B, 1
B, 1
B, 1
B, 1
B, 1
B, 1
B, 0

TD **MC**

V(A) = ? ¾ or 0?

V(B) = ¾



Optimality of TD(0)

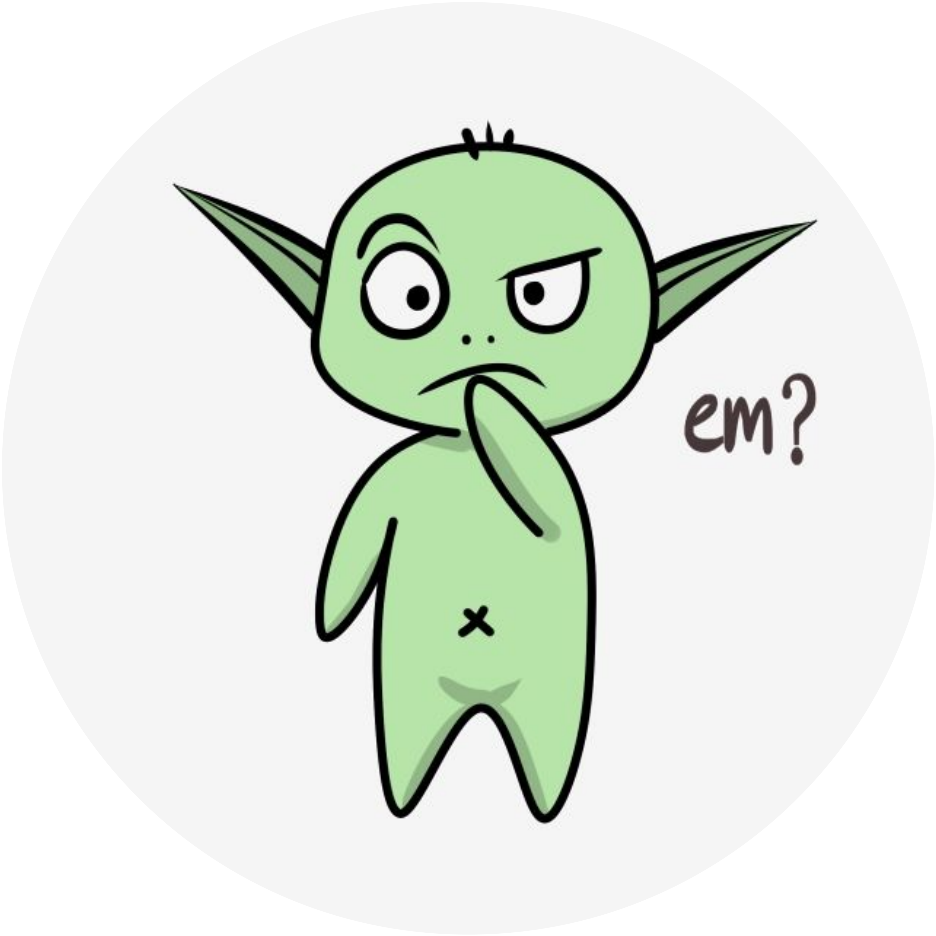
- Under batch training, constant-α MC converges to values, V(s), that are sample averages of the actual returns experienced after visiting each state s. These are optimal estimates in the sense that they minimize the mean square error from the actual returns in the training set.

- But TD(0) gives us the answer that it is based on first modeling the Markov process and then computing the correct estimates given the model (the *certainty-equivalence estimate*).

TD vs Monte Carlo

“Batch Monte Carlo methods always find the estimates that minimize mean square error on the training set, whereas batch TD(0) always finds the estimates that would be exactly correct for the maximum-likelihood model of the Markov process.”

In general, the *maximum-likelihood estimate* of a parameter is the parameter value whose probability of generating the data is greatest.



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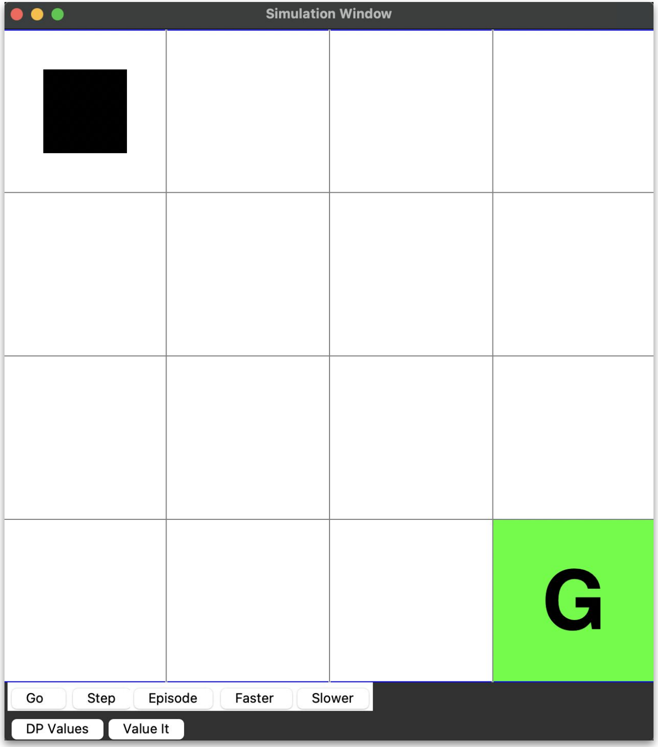
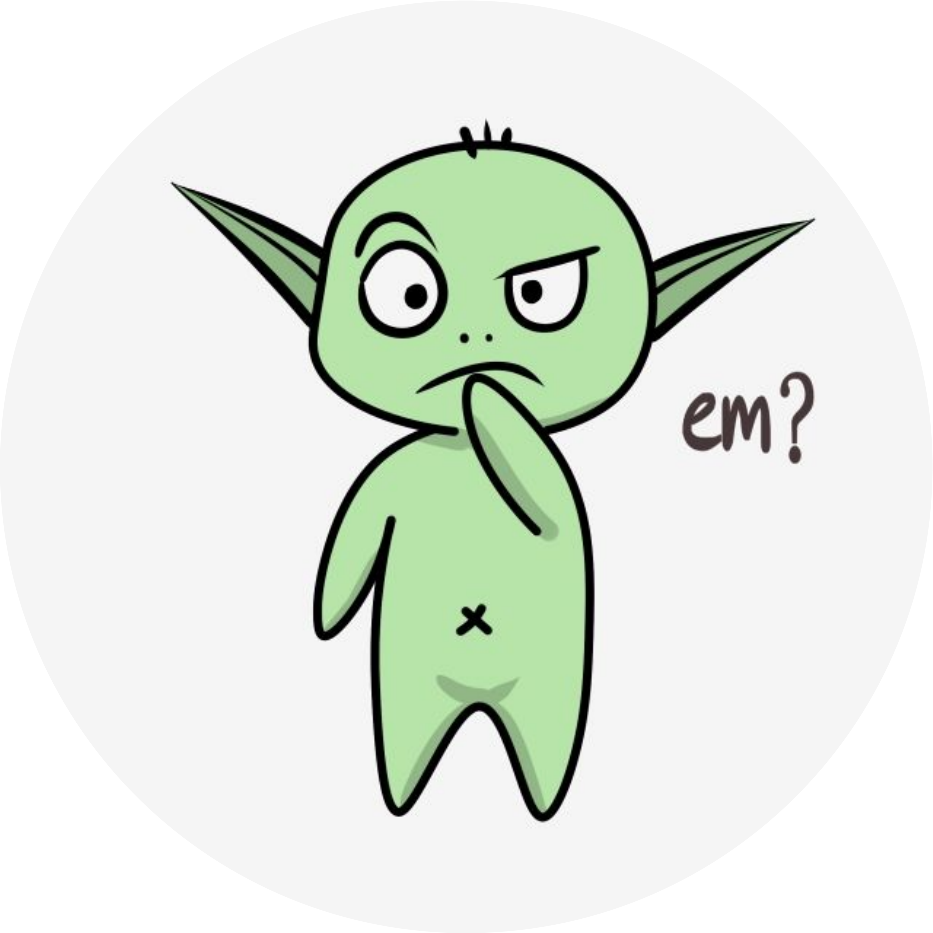
Demo


Simulation Window
G
Go
Step
Episode
Faster
Slower
DP Values
Value It



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Reward Prediction Error Hypothesis in Neuroscience

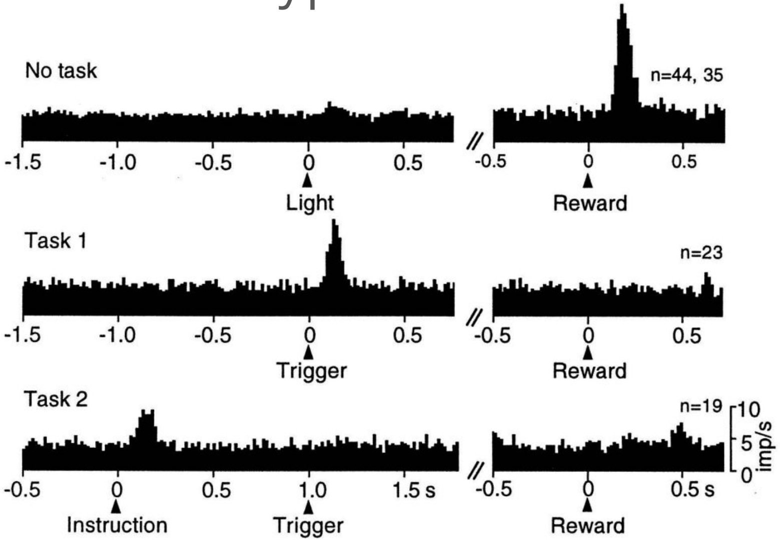


Figure 15.2: The response of dopamine neurons shifts from initial responses to primary reward to earlier predictive stimuli. These are plots of the number of action potentials produced by monitored dopamine neurons within small time intervals, averaged over all the monitored dopamine neurons (ranging from 23 to 44 neurons for these data). Top: dopamine neurons are activated by the unpredicted delivery of drop of apple juice. Middle: with learning, dopamine neurons developed responses to the reward-predicting trigger cue and lost responsiveness to the delivery of reward. Bottom: with the addition of an instruction cue preceding the trigger cue by 1 second, dopamine neurons shifted their responses from the trigger cue to the earlier instruction cue. From Schultz et al. (1995), MIT Press.

Reward Prediction Error Hypothesis in Neuroscience

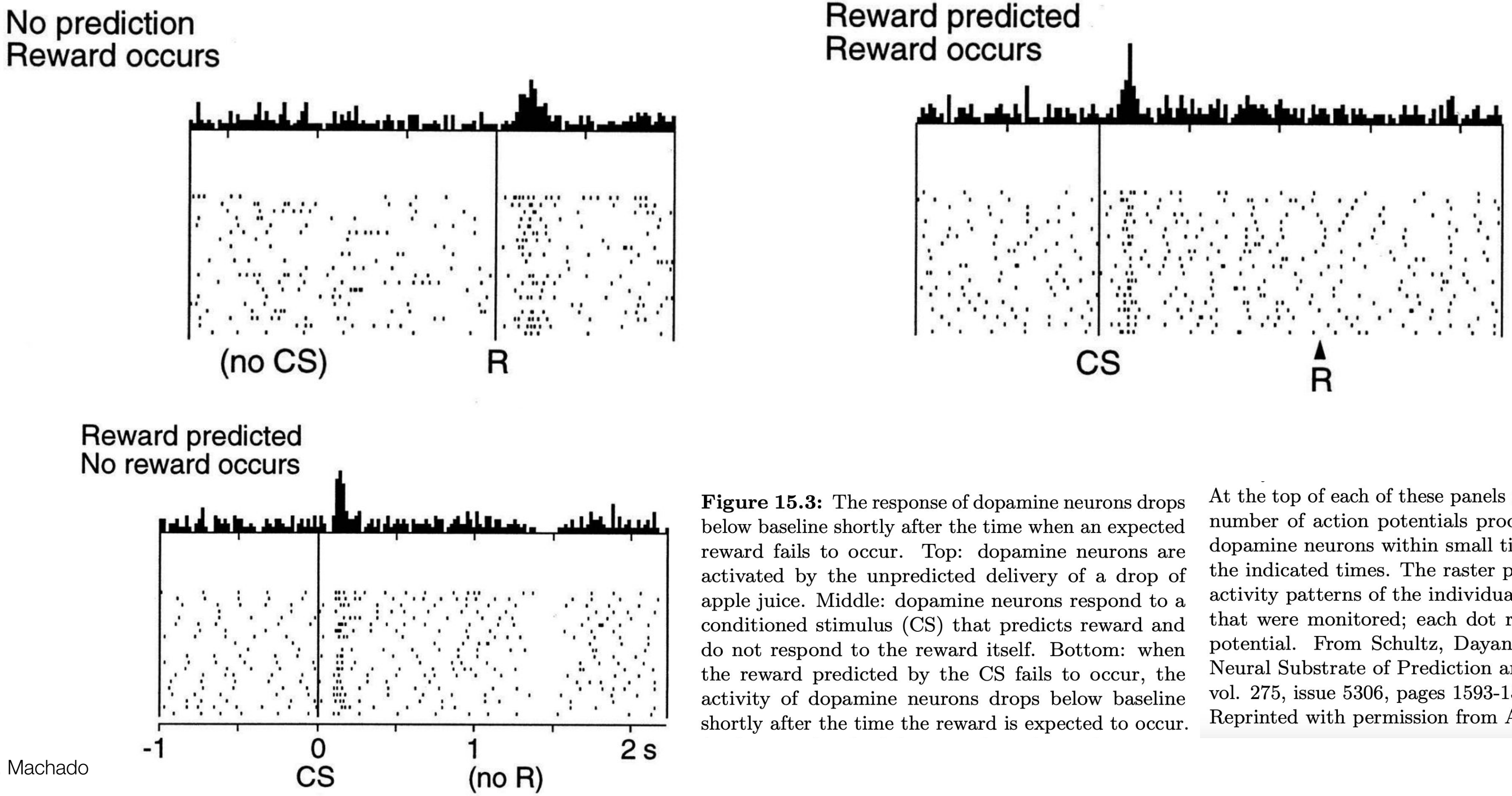
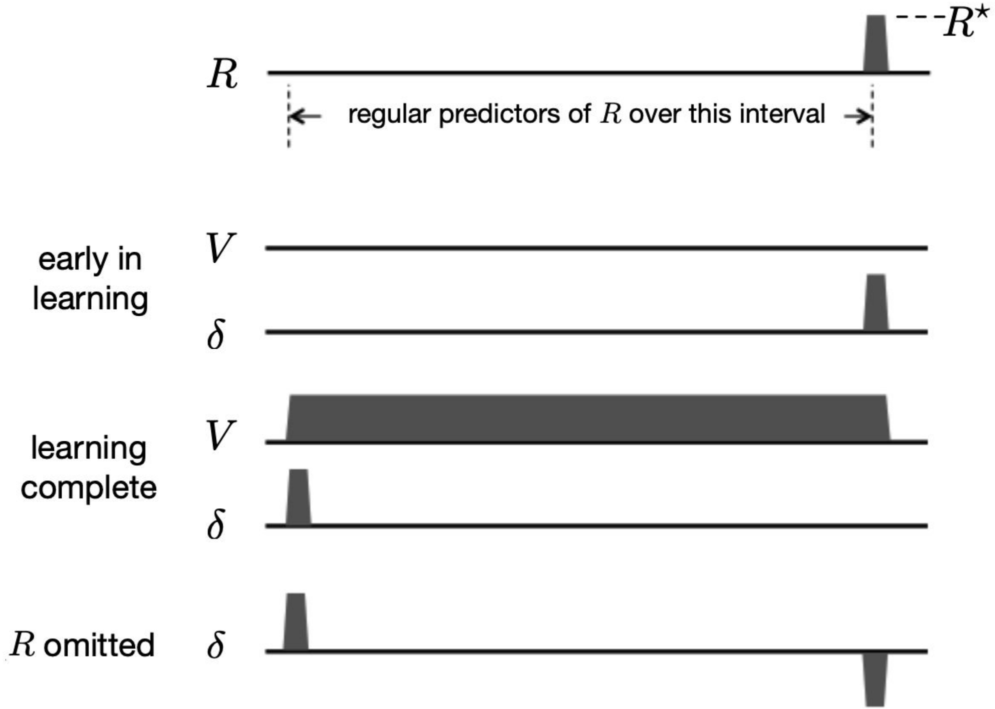
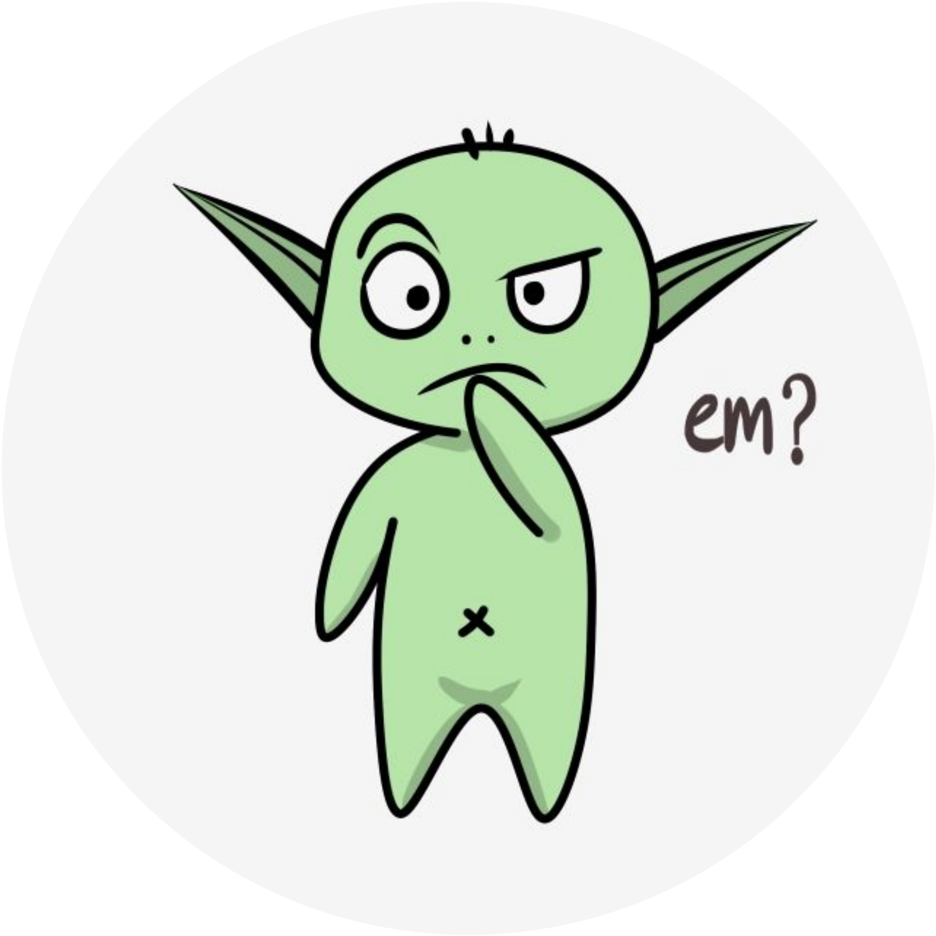


Figure 15.3: The response of dopamine neurons drops below baseline shortly after the time when an expected reward fails to occur. Top: dopamine neurons are activated by the unpredicted delivery of a drop of apple juice. Middle: dopamine neurons respond to a conditioned stimulus (CS) that predicts reward and do not respond to the reward itself. Bottom: when the reward predicted by the CS fails to occur, the activity of dopamine neurons drops below baseline shortly after the time the reward is expected to occur. At the top of each of these panels is shown the average number of action potentials produced by monitored dopamine neurons within small time intervals around the indicated times. The raster plots below show the activity patterns of the individual dopamine neurons that were monitored; each dot represents an action potential. From Schultz, Dayan, and Montague, A Neural Substrate of Prediction and Reward, *Science*, vol. 275, issue 5306, pages 1593-1598, March 14, 1997. Reprinted with permission from AAAS.

TD Error/Dopamine Correspondence





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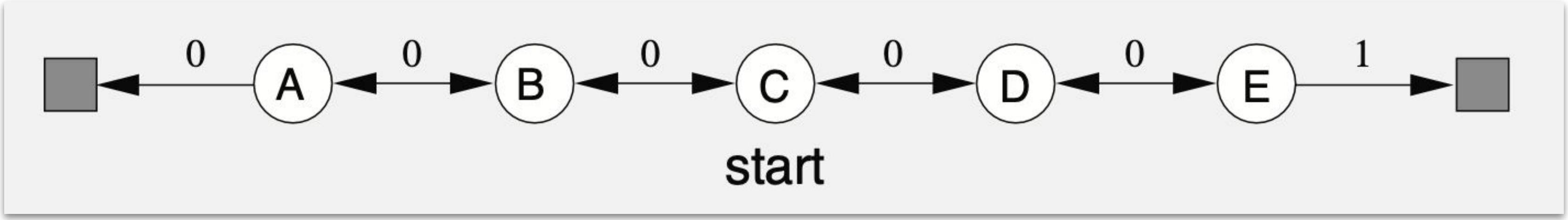
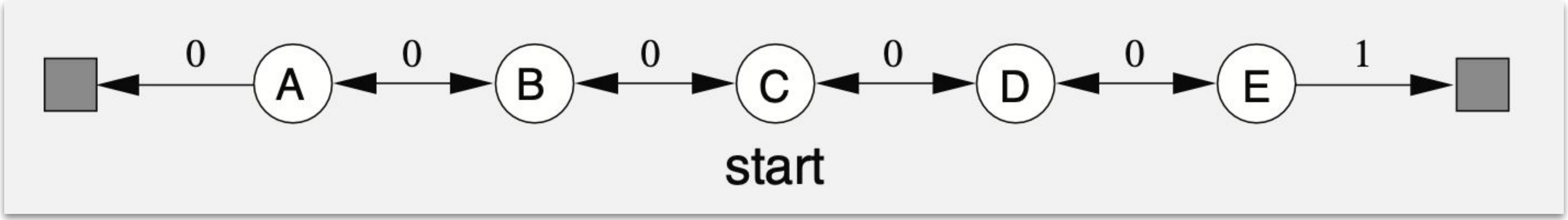
Example / Exercise: Random Walk



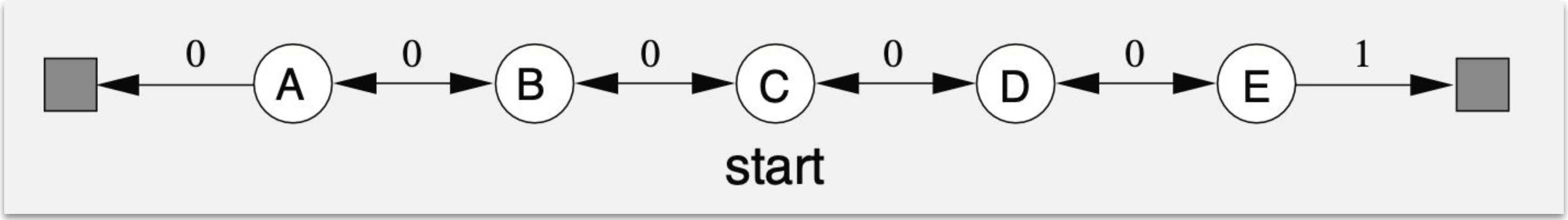
- *Markov reward process* (MRP), an MDP without actions.
- Start state: C
- One can go left or right, on each step, with equal probability.
- Reward: +1 on right exit, 0 otherwise.
- $\gamma = 1.0$
- All values are initialized with 0.5, that is V(s) = 0.5 for all s.

Example / Exercise: Random Walk

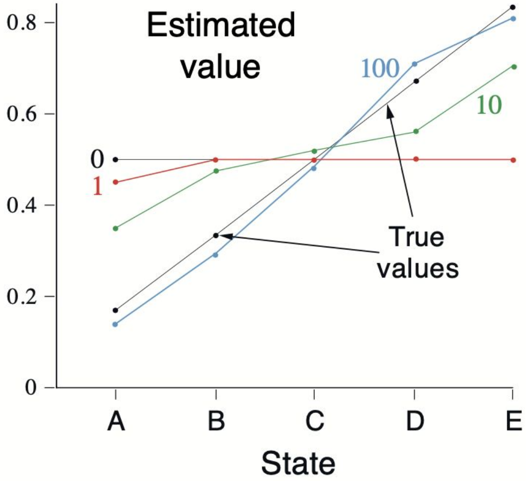


Q1. What does v_π encode in this problem?

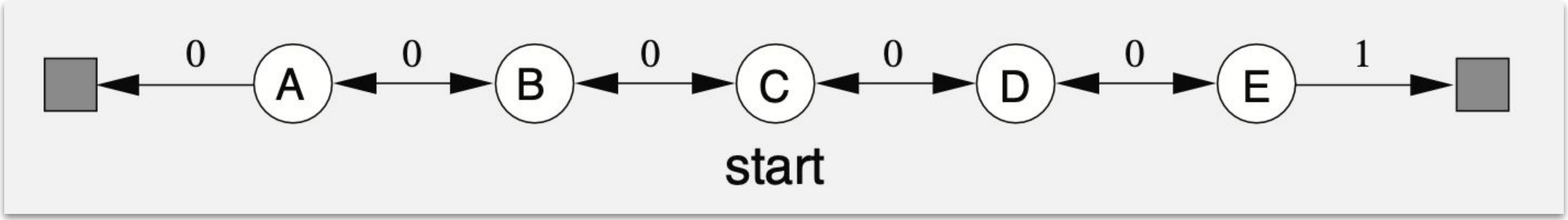
Example / Exercise: Random Walk



Q1. What does v_π encode in this problem?



Example / Exercise: Random Walk

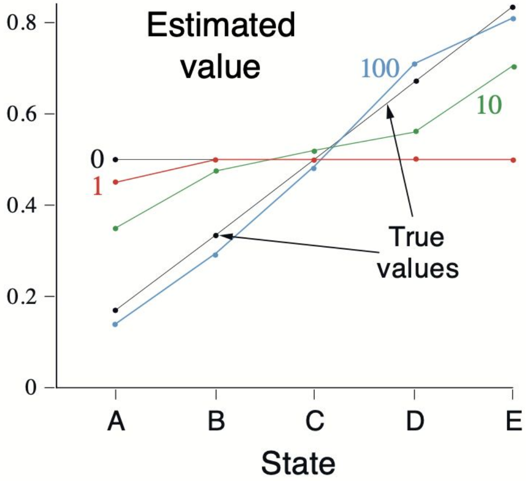


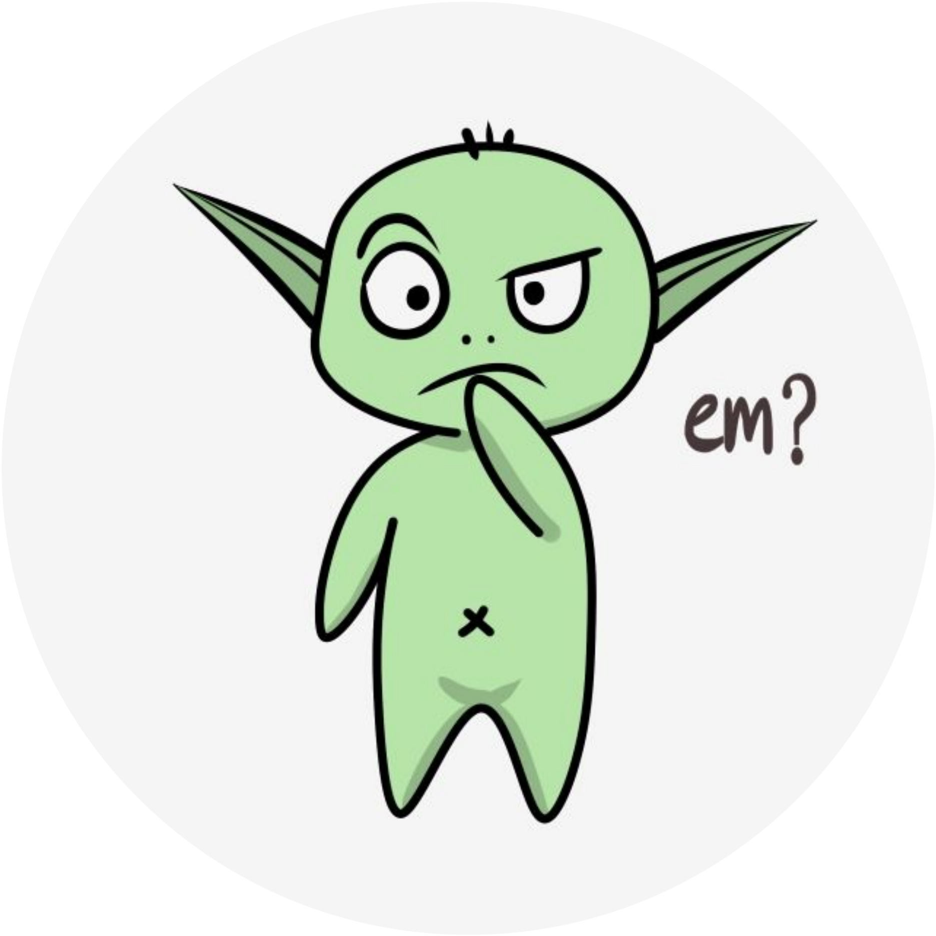
Q1. What does v_π encode in this problem?

Q2. The first episode results in a change in only V(A). What does this tell you about what happened on the first episode?

Q3. Why was only the estimate for this one state changed?

Q4. By exactly how much was it changed? (Assume α=0.1)





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