"All have their worth," said Yavanna,

"and each contributes to the worth of the others".

J.R.R. Tolkien, *The Silmarillion*

CMPUT 365
Introduction to RL

Marlos C. Machado

Class 12/35

Plan

- Dynamic programming
  - Finally, a solution method (albeit limited)!

Reminder I

You **should be enrolled in the private session** we created in Coursera for CMPUT 365.

I **cannot** use marks from the public repository for your course marks.

You **need** to **check**, **every time**, if you are in the private session and if you are submitting quizzes and assignments to the private section.

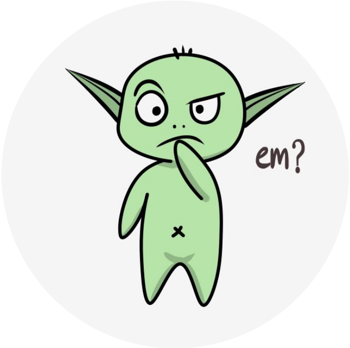
The deadlines in the public session **do not align** with the deadlines in Coursera.

If you have any questions or concerns, **talk with the TAs** or email us `cmput365@ualberta.ca`.

Reminder II

- Practice quiz for Coursera's Dynamic Programming module is due today.
  Fundamentals of RL: Dynamic Programming – Week 4.
- Progr. assign. for Coursera's Dynamic Programming module is due Wednesday.
  Fundamentals of RL: Dynamic Programming – Week 4.
- Midterm is next Wednesday, after Thanksgiving.

Please, interrupt me at any time!



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Dynamic Programming – Why?

- “DP provides an essential foundation for the understanding of the methods presented in the rest of this book”.
- … but “classical DP algorithms are of limited utility in reinforcement learning both because of their assumption of a perfect model and because of their great computational expense”.
- “all of these [RL] methods can be viewed as attempts to achieve much the same effect as DP, only with less computation and without assuming a perfect model of the environment”.

Models and Planning

- How should we think about p(s’, r | s, a)?

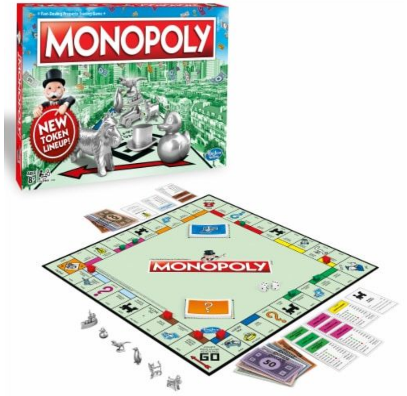
It is a model. It tells us everything that is possible and impossible to happen (and their probability!)

- Is dynamic programming different from what we did in bandits?

Figuring out how to act

Imagine the universe consists of you playing Monopoly against a computer. Your goal is to win the game.

There are two ways you can do so:



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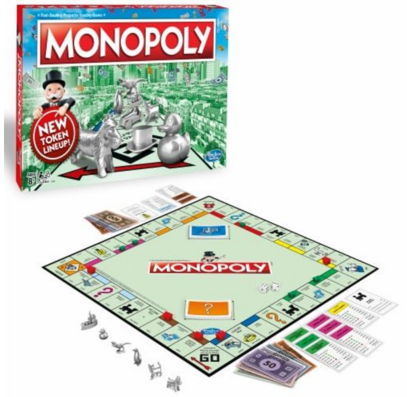
Figuring out how to act

Imagine the universe consists of you playing Monopoly against a computer. Your goal is to win the game.

There are two ways you can do so:

1. **Trial and error learning.** Play against it over and over, figure out the game rules and the computer's strategy.



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Figuring out how to act

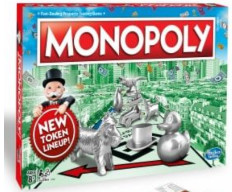
Imagine the universe consists of you playing Monopoly against a computer. Your goal is to win the game.

There are two ways you can do so:

1. **Trial and error learning.** Play against it over and over, figure out the game rules and the computer's strategy.

2. **Planning.** You could be given access to the game's rulebook as well as the code implementing the AI playing against you. You would then sit and there and **think** about how to win. You could **reason** about the rules and the AI, and **plan** how to win.

There's no interaction!



The game's rulebook and the code implementing the AI would allow you to compute $p(s', r \mid s, a)$.

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Key Idea Behind Dynamic Programming

"To use value functions to organize and structure the search for good policies."

We use the same equations as before, but we replace an = by a ←, that's it (we turn Bellman equations into assignments).

There's lots to decide

- What should we compute? v_π, q_π, v_*, q_*, π^*?
- How should we select states to imagine about? And in what order?
- How much computation do we need to figure out the optimal policy, π^*, using the function p: $\mathcal{S} \times \mathfrak{R} \times \mathcal{S} \times \mathcal{A} \rightarrow [0, 1]$?
- How many times do we need to iterate over this imagining / planning process?

Obtaining value functions and π^* from π and p (with no interaction) is called ***Dynamic Programming****.*

Policy Evaluation (Prediction)

Given a policy and an MDP, what's the corresponding value function?

$$
\begin{aligned}
v_\pi(s) &= \mathbb{E}_\pi[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s] \\
&= \sum_a \pi(a|s) \sum_{s',r} p(s', r | s, a) \Big[r + \gamma v_\pi(s') \Big]
\end{aligned}
$$

$$\downarrow$$

$$
\begin{aligned}
v_{k+1}(s) &\doteq \mathbb{E}_\pi[R_{t+1} + \gamma v_k(S_{t+1}) \mid S_t = s] \\
&= \sum_a \pi(a|s) \sum_{s',r} p(s', r | s, a) \Big[r + \gamma v_k(s') \Big]
\end{aligned}
$$

expected update

Policy Evaluation (Prediction)

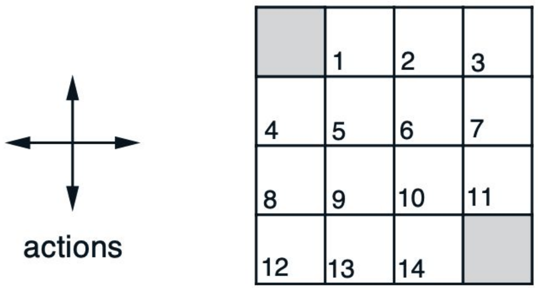
Iterative Policy Evaluation, for estimating $V \approx v_\pi$

Input π, the policy to be evaluated
Algorithm parameter: a small threshold $\theta > 0$ determining accuracy of estimation
Initialize $V(s)$ arbitrarily, for $s \in \mathcal{S}$, and $V(\mathit{terminal})$ to 0

Loop:
  $\Delta \leftarrow 0$
  Loop for each $s \in \mathcal{S}$:
    $v \leftarrow V(s)$
    $V(s) \leftarrow \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)\big[r + \gamma V(s')\big]$
    $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
until $\Delta < \theta$

"in-place" update

Policy Evaluation – Example



$R_t = -1$
on all transitions

v_k for the random policy

0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0

→

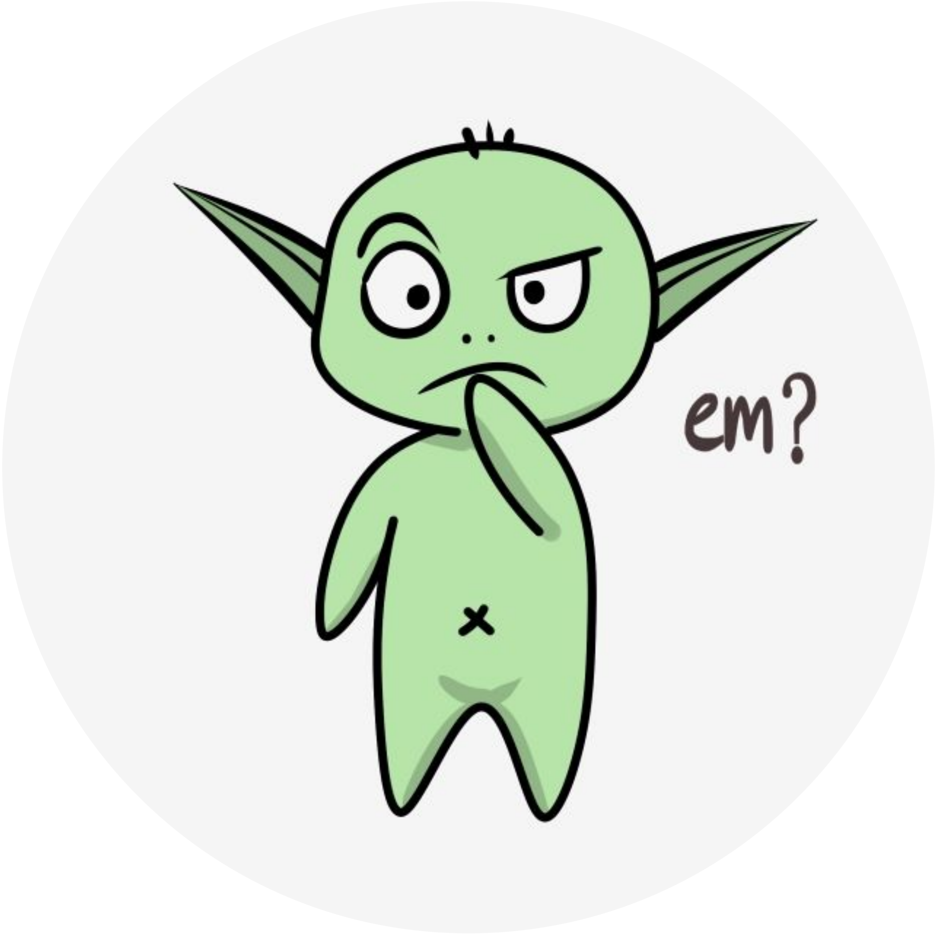
0.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	0.0

→

0.0	-1.7	-2.0	-2.0
-1.7	-2.0	-2.0	-2.0
-2.0	-2.0	-2.0	-1.7
-2.0	-2.0	-1.7	0.0

→ … →

0.0	-14.	-20.	-22.
-14.	-18.	-20.	-20.
-20.	-20.	-18.	-14.
-22.	-20.	-14.	0.0



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Policy Improvement

Given a value function for a policy π, how can we get a better policy π'?

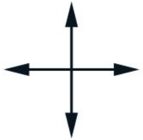
We already know how good policy π is, what if we acted differently now? What if instead of selecting action π(s) we selected action a ≠ π(s), but then we followed π?

We know the value of doing that!

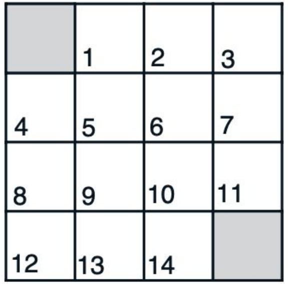
$$
\begin{aligned}
q_\pi(s,a) &\doteq \mathbb{E}[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s, A_t = a] \\
&= \sum_{s',r} p(s', r \mid s, a)\Big[r + \gamma v_\pi(s')\Big].
\end{aligned}
$$

If this new action is better, in general this new policy is better overall

Policy Improvement – Intuition



actions

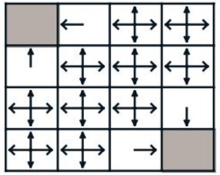


$R_t = -1$
on all transitions

v_k for the
random policy

$k = 1$

0.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	0.0



Policy Improvement Theorem

That this is true is a special case of a general result called the *policy improvement theorem*. Let π and π' be any pair of deterministic policies such that, for all $s \in \mathcal{S}$,

$$q_\pi(s, \pi'(s)) \geq v_\pi(s). \tag{4.7}$$

Then the policy π' must be as good as, or better than, π. That is, it must obtain greater or equal expected return from all states $s \in \mathcal{S}$:

$$v_{\pi'}(s) \geq v_\pi(s). \tag{4.8}$$

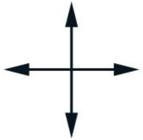
A more aggressive update

Instead of doing it for a particular action in a single state, we can consider changes at *all* states and to *all* possible actions.

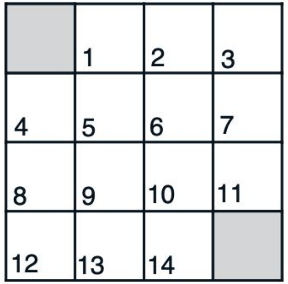
$$
\begin{aligned}
\pi'(s) &\doteq \arg\max_a q_\pi(s, a) \\
&= \arg\max_a \mathbb{E}[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s, A_t = a] \\
&= \arg\max_a \sum_{s', r} p(s', r \mid s, a)\Big[r + \gamma v_\pi(s')\Big],
\end{aligned}
$$

This is called *policy improvement*. And eventually it converges to the optimal policy.

Policy Improvement – Intuition



actions

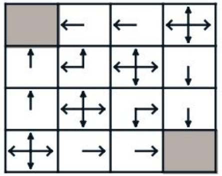


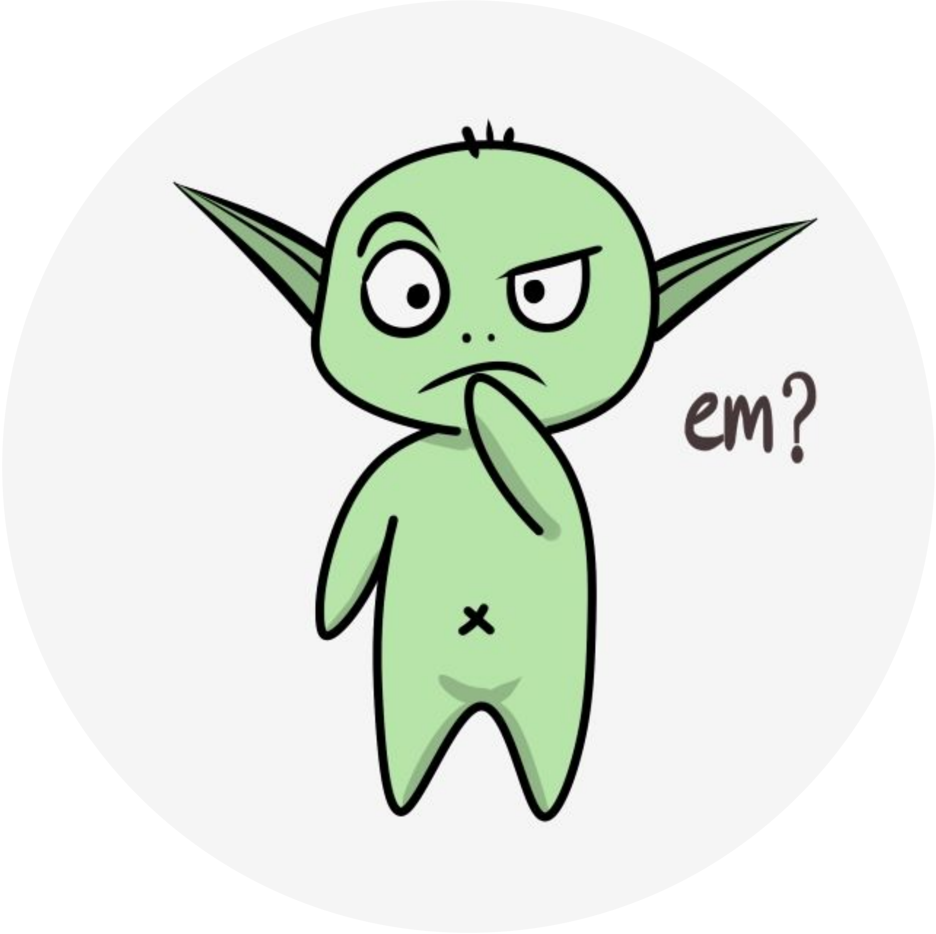
$R_t = -1$
on all transitions

v_k for the random policy

$k = 2$

0.0	-1.7	-2.0	-2.0
-1.7	-2.0	-2.0	-2.0
-2.0	-2.0	-2.0	-1.7
-2.0	-2.0	-1.7	0.0





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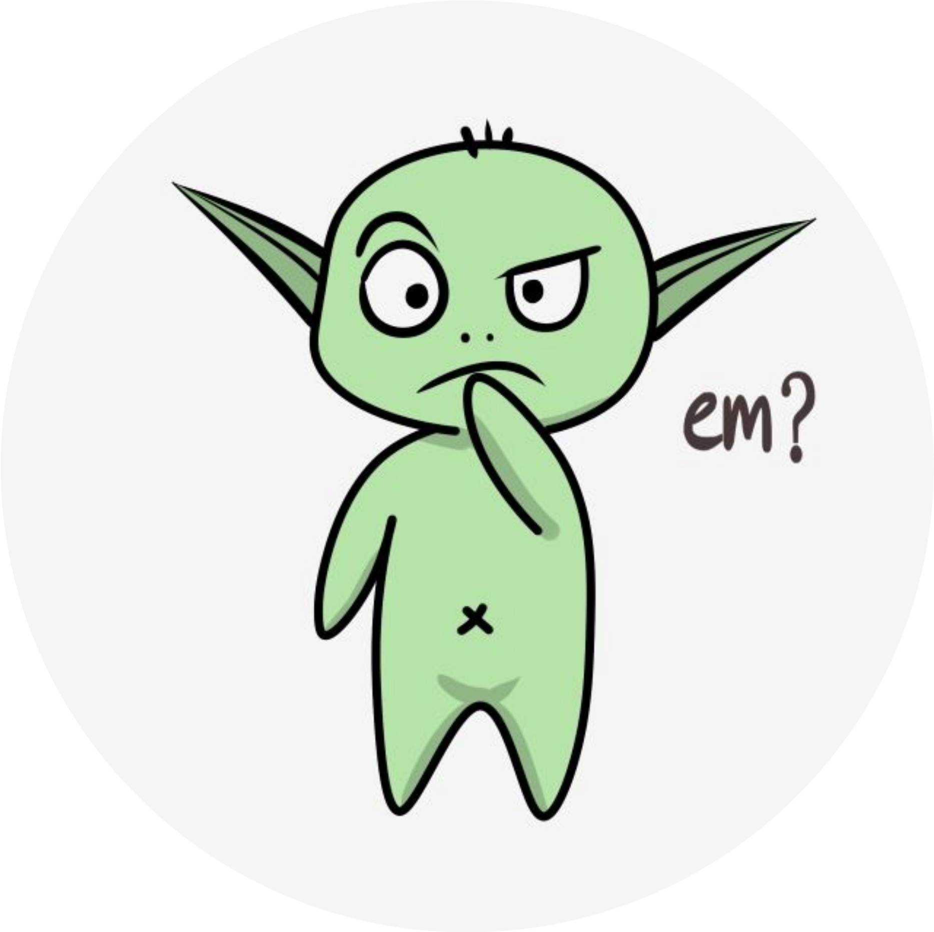
Policy Iteration: Interleaving Policy Eval. and Improvement

Policy Iteration (using iterative policy evaluation) for estimating $\pi \approx \pi_*$

1. Initialization
   $V(s) \in \mathbb{R}$ and $\pi(s) \in \mathcal{A}(s)$ arbitrarily for all $s \in \mathcal{S}$; $V(terminal) \doteq 0$

2. Policy Evaluation
   Loop:
      $\Delta \leftarrow 0$
      Loop for each $s \in \mathcal{S}$:
         $v \leftarrow V(s)$
         $V(s) \leftarrow \sum_{s',r} p(s', r \mid s, \pi(s))\big[r + \gamma V(s')\big]$
         $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
   until $\Delta < \theta$ (a small positive number determining the accuracy of estimation)

3. Policy Improvement
   policy-stable $\leftarrow$ *true*
   For each $s \in \mathcal{S}$:
      old-action $\leftarrow \pi(s)$
      $\pi(s) \leftarrow \arg\max_a \sum_{s',r} p(s', r \mid s, a)\big[r + \gamma V(s')\big]$
      If *old-action* $\neq \pi(s)$, then *policy-stable* $\leftarrow$ *false*
   If *policy-stable*, then stop and return $V \approx v_*$ and $\pi \approx \pi_*$; else go to 2



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Value Iteration

Value Iteration, for estimating $\pi \approx \pi_*$

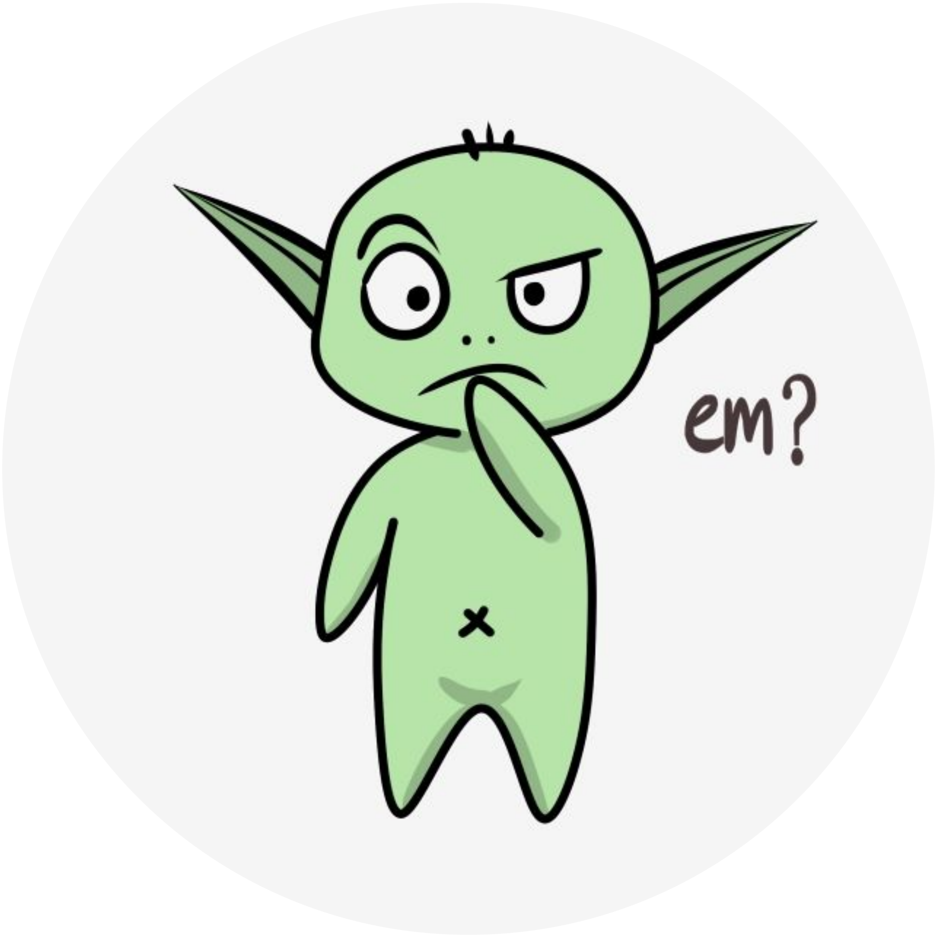
Algorithm parameter: a small threshold $\theta > 0$ determining accuracy of estimation
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(terminal) = 0$

Loop:
| $\Delta \leftarrow 0$
| Loop for each $s \in \mathcal{S}$:
| $v \leftarrow V(s)$
| $V(s) \leftarrow \max_a \sum_{s',r} p(s', r | s, a)\big[r + \gamma V(s')\big]$
| $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
until $\Delta < \theta$

Output a deterministic policy, $\pi \approx \pi_*$, such that
  $\pi(s) = \arg\max_a \sum_{s',r} p(s', r | s, a)\big[r + \gamma V(s')\big]$

It doesn't need to be so synchronous

We just turned the Bellman optimality equation into an update rule!



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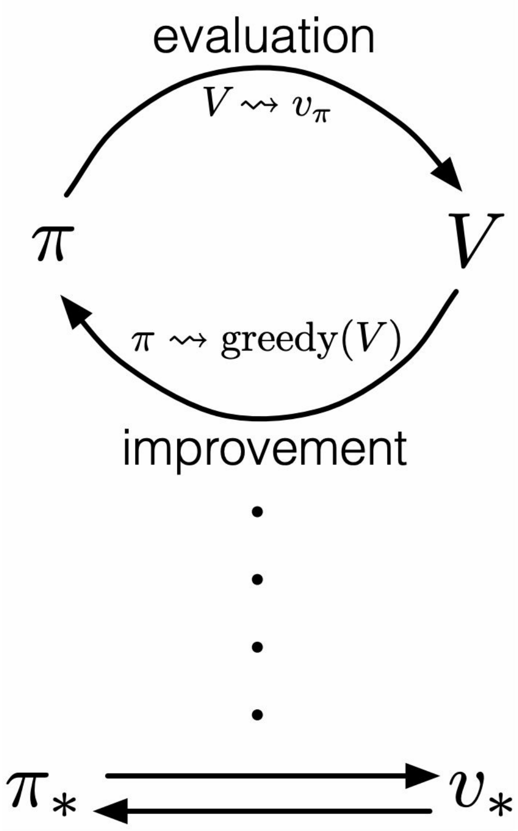
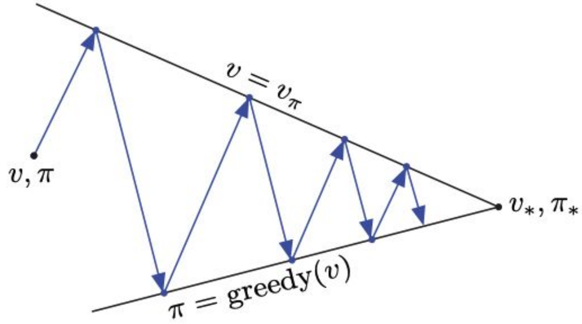
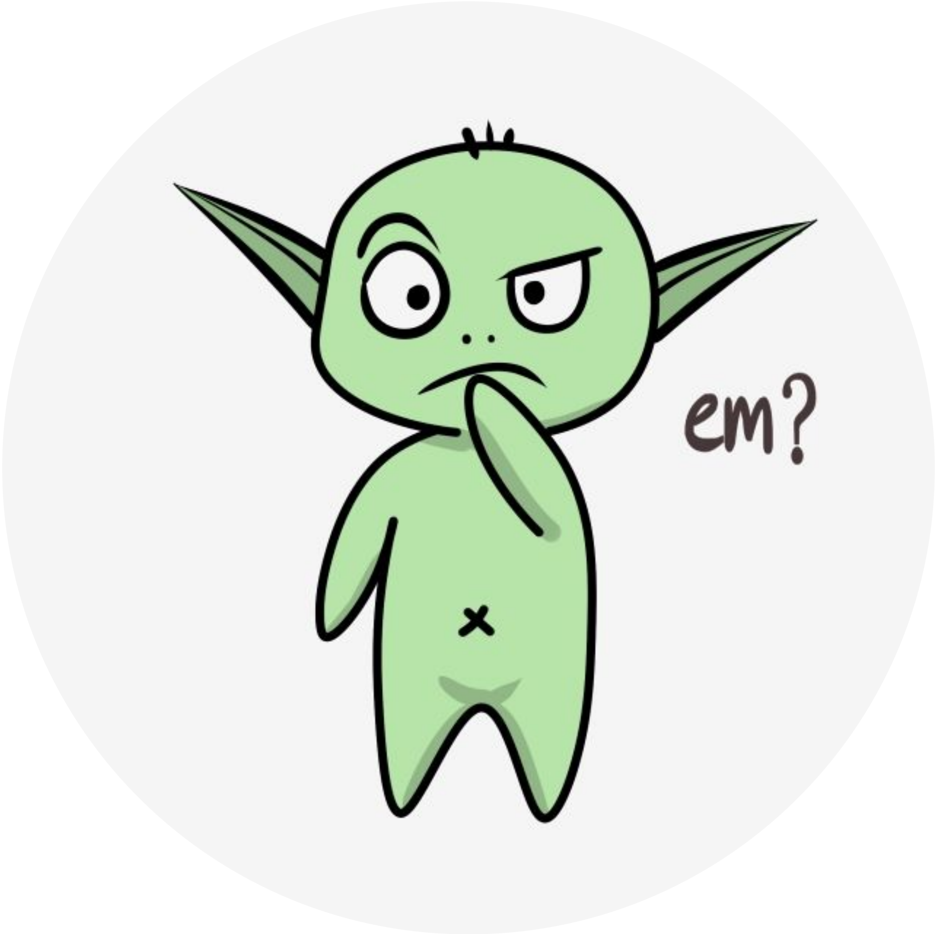
Generalized Policy Iteration







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