“The rotten tree-trunk, until the very moment when the storm-blast breaks it in two, has all the appearance of might it ever had.”

Isaac Asimov, *Foundation*

CMPUT 365
Introduction to RL

Marlos C. Machado

Class 10/ 35

Plan

- Value Functions and Bellman Equations
  - Non–comprehensive overview
  - We are still not talking about solution methods, we are only formalizing things

Reminder

You **should be enrolled in the private session** we created in Coursera for CMPUT 365.

I **cannot** use marks from the public repository for your course marks.

You **need** to **check**, **every time**, if you are in the private session and if you are submitting quizzes and assignments to the private section.

Some students who are enrolled in Coursera **haven't submitted any quizzes or assignments** in the private session, and that's all I can see.

The deadlines in the public session **do not align** with the deadlines in Coursera.

Please, interrupt me at any time!

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Value Function

- The value function of a state s under a policy π, denoted $v_\pi(s)$ is the expected return when starting in s and following π thereafter.

state-value function for policy π

$$v_\pi(s) \doteq \mathbb{E}_\pi[G_t \mid S_t = s] = \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s\right]$$

$$q_\pi(s,a) \doteq \mathbb{E}_\pi[G_t \mid S_t = s, A_t = a] = \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s, A_t = a\right]$$

action-value function for policy π

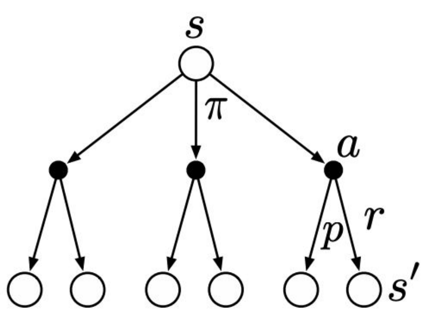
Value Functions Satisfy Recursive Relationships

$$v_\pi(s) \doteq \mathbb{E}_\pi[G_t \mid S_t = s]$$

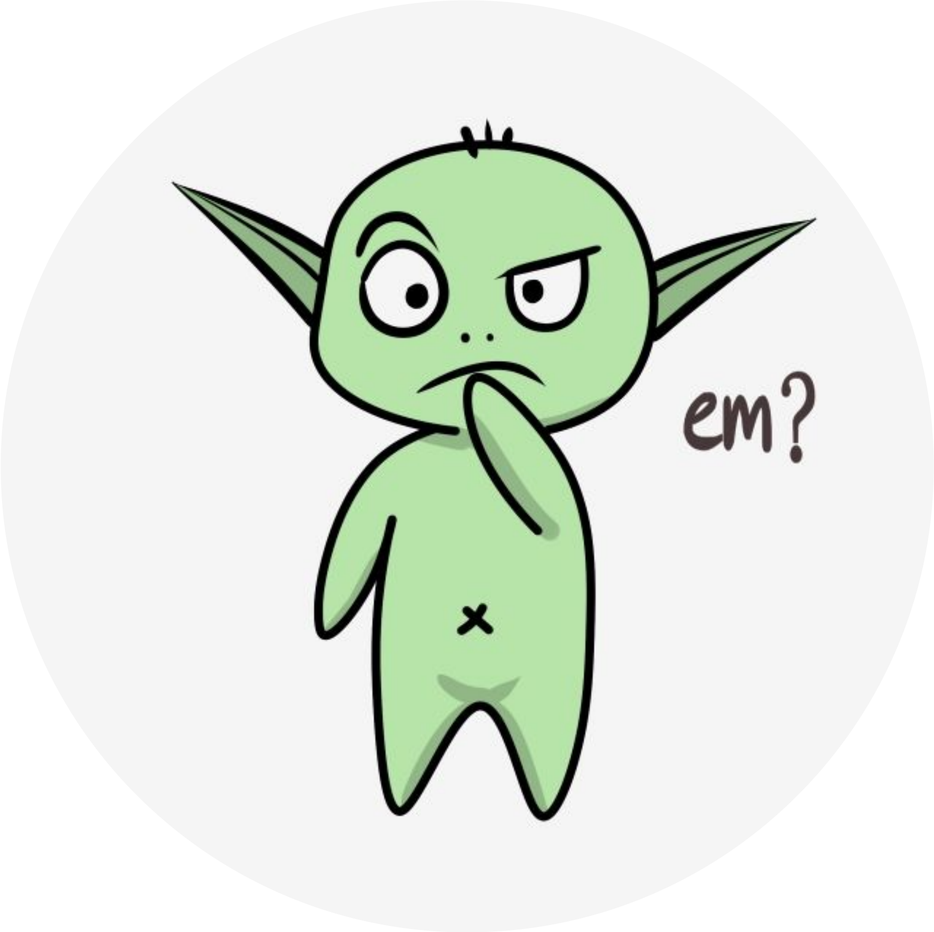
Value Functions Satisfy Recursive Relationships

$$
\begin{aligned}
v_\pi(s) &\doteq \mathbb{E}_\pi[G_t \mid S_t = s] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1} \mid S_t = s] \\
&= \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r | s, a) \Big[r + \gamma \mathbb{E}_\pi[G_{t+1} | S_{t+1} = s'] \Big] \\
&= \sum_a \pi(a|s) \sum_{s', r} p(s', r | s, a) \Big[r + \gamma v_\pi(s') \Big]
\end{aligned}
$$

This is a system of linear equations!



Backup diagram for v_π



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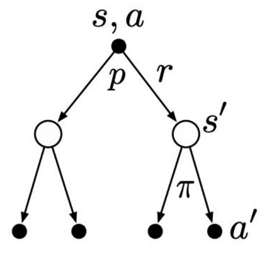
Example

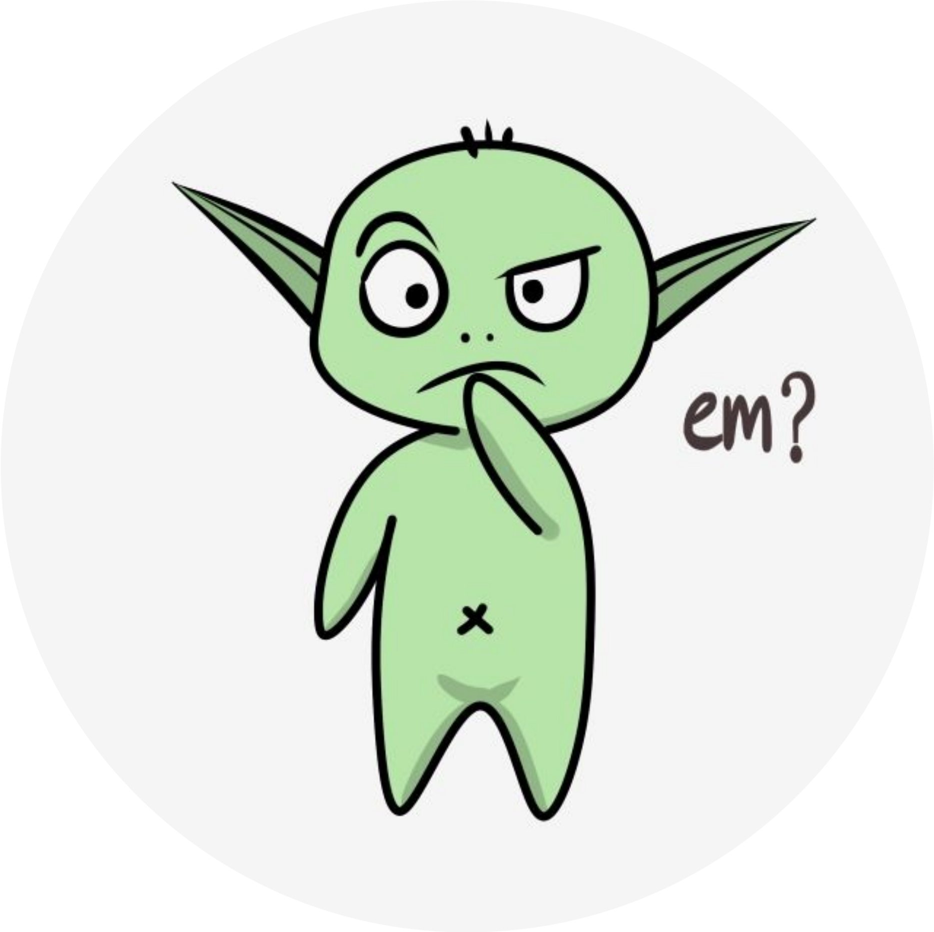
Whiteboard

State-Action Value Functions Satisfy Recursive Relationships

Exercise 3.17 What is the Bellman equation for action values, that is, for q_π? It must give the action value $q_\pi(s, a)$ in terms of the action values, $q_\pi(s', a')$, of possible successors to the state–action pair (s, a). Hint: The backup diagram to the right corresponds to this equation. Show the sequence of equations analogous to (3.14), but for action values. □



q_π backup diagram



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Optimal Policies and Optimal Value Functions

- Value functions define a partial ordering over policies.
  - $\pi \geq \pi'$ iff $v_\pi(s) \geq v_{\pi'}(s)$ for all $s \in \mathscr{S}$.
  - There is always at least one policy that is better than or equal to all other policies. The *optimal policy*.

$$v_*(s) \doteq \max_\pi v_\pi(s)$$

$$q_*(s,a) \doteq \max_\pi q_\pi(s,a)$$

$$q_*(s,a) = \mathbb{E}[R_{t+1} + \gamma v_*(S_{t+1}) \mid S_t = s, A_t = a]$$

Optimal Policies and Optimal Value Functions

- Because v_* is the value function for a policy, it must satisfy the self-consistency condition given by the Bellman equation for state values.

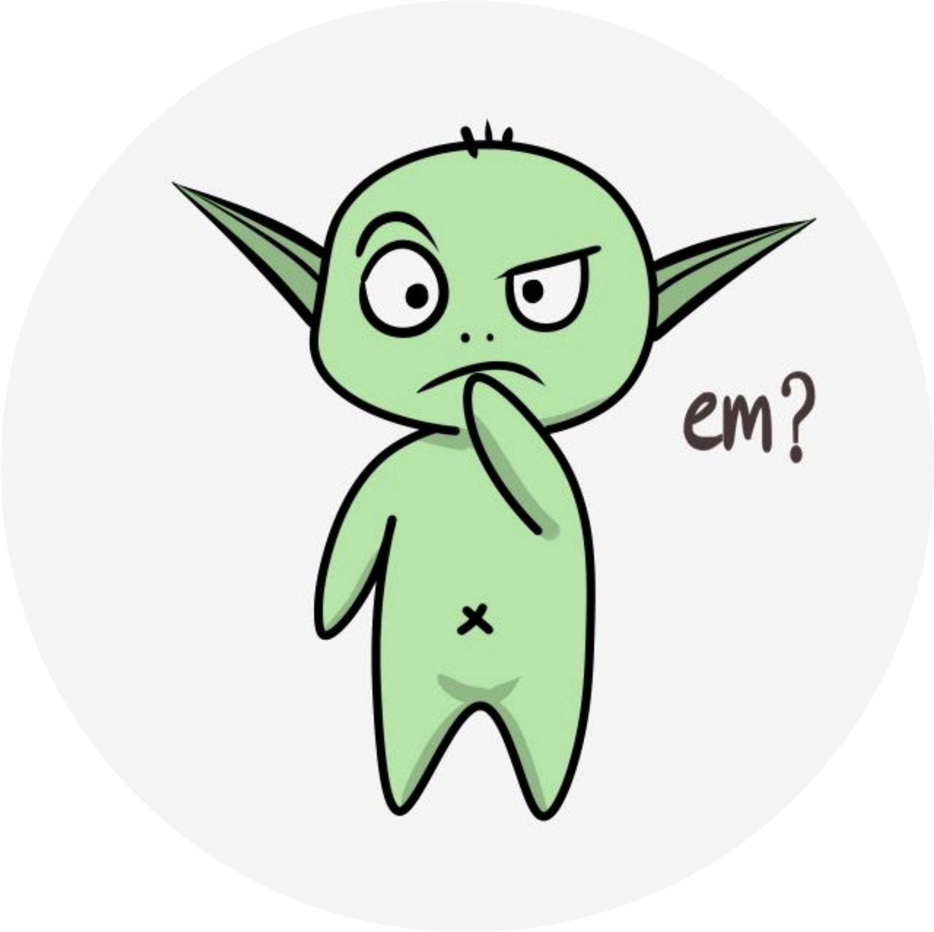
$$v_*(s) = \max_{a \in \mathcal{A}(s)} q_{\pi_*}(s, a)$$

Optimal Policies and Optimal Value Functions

- Because v_* is the value function for a policy, it must satisfy the self-consistency condition given by the Bellman equation for state values.

$$\begin{aligned} v_*(s) &= \max_{a \in \mathcal{A}(s)} q_{\pi_*}(s,a) \\ &= \max_a \mathbb{E}_{\pi_*}[G_t \mid S_t = s, A_t = a] \\ &= \max_a \mathbb{E}_{\pi_*}[R_{t+1} + \gamma G_{t+1} \mid S_t = s, A_t = a] \\ &= \max_a \mathbb{E}[R_{t+1} + \gamma v_*(S_{t+1}) \mid S_t = s, A_t = a] \\ &= \max_a \sum_{s',r} p(s',r|s,a)\big[r + \gamma v_*(s')\big]. \end{aligned}$$

$$\begin{aligned} q_*(s,a) &= \mathbb{E}\Big[R_{t+1} + \gamma \max_{a'} q_*(S_{t+1},a') \Bigm| S_t = s, A_t = a\Big] \\ &= \sum_{s',r} p(s',r|s,a)\Big[r + \gamma \max_{a'} q_*(s',a')\Big]. \end{aligned}$$



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Reinforcement learning is very related to search algorithms

"Heuristic search methods can be viewed as expanding the right-hand side of the equation below several times, up to some depth, forming a "tree" of possibilities, and then using a heuristic evaluation function to approximate v_*, at the "leaf" nodes."

$$v_*(s) = \max_a \sum_{s',r} p(s', r | s, a) \left[r + \gamma v_*(s') \right].$$

Yay! We solved sequential decision-making problems

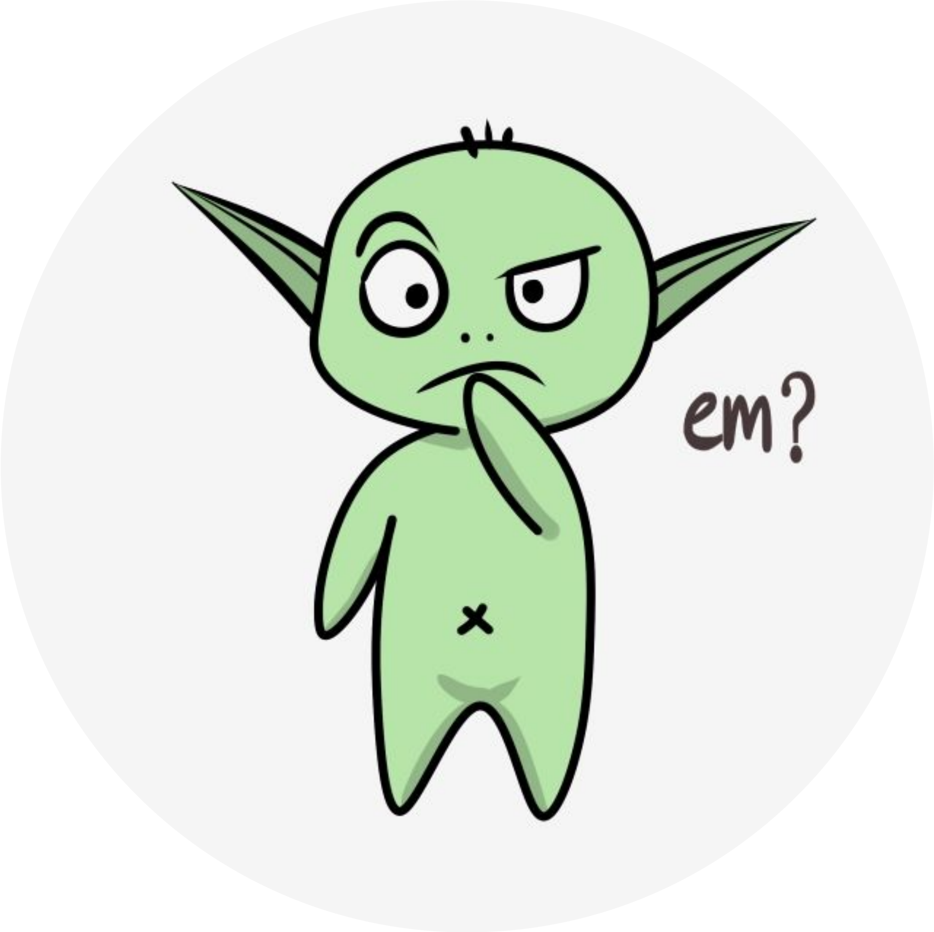
Except…

1.
2.
3.

Yay! We solved sequential decision-making problems

Except…

1. we need to know the dynamics of the environment
2. we have enough computational resources to solve the system of linear eq.
3. the Markov property



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Next class

- What I plan to do:
    - Talk about optimality.
    - Exercises and examples.