

## Detecting Motion and Optic Flow

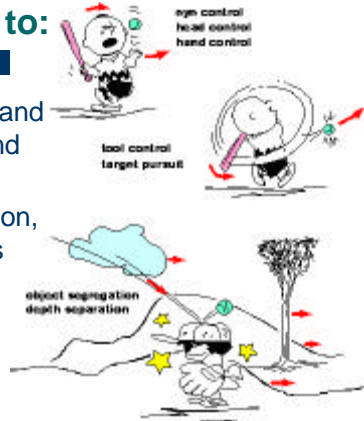
Cmput 306  
Lecture 28  
Martin Jagersand

## Image motion

- Somehow quantify differences in image sequences.
- Image differences.
- Optic flow
- 3-6 dim motion

## Motion is used to:

- **Attention:** Detect and direct using eye and head motions
- **Control:** Locomotion, manipulation, tools
- **Vision:** Segment, depth, trajectory



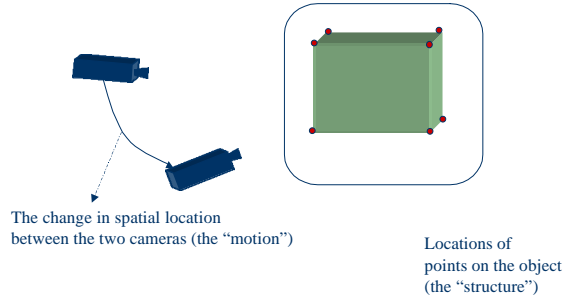
## Small camera re-orientation



Note: Almost all pixels change!

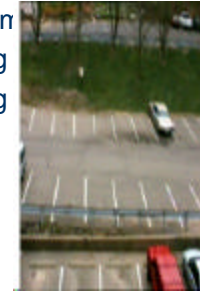


## MOVING CAMERAS ARE LIKE STEREO



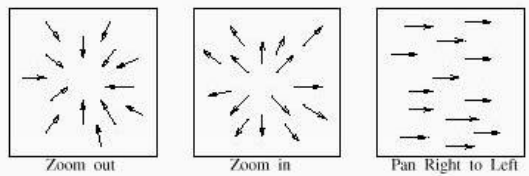
## Classes of motion

- Still camera, single moving object
- Still cam
- Moving
- Moving



## The motion field

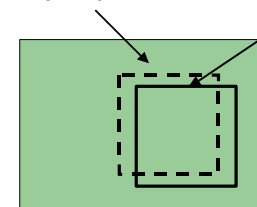
- Vector field over the image:  
 $[u, v] = f(x, y)$ ,  $u, v$  = Vel vector,  $x, y$  = Im pos
- **FOE, FOC** Focus of Expansion, Contraction



## Optic/image flow

- Assumption: Image intensities from object points remain constant over time.

$$Im(x + \hat{t}x; y + \hat{t}y; t + \hat{t}t) = Im(x; y; t)$$



## Taylor expansion of intensity variation

$$I_m(x + \hat{x}; y + \hat{y}; t + \hat{t}) = I_m(x; y; t) + \frac{\partial I_m}{\partial x} \hat{x} + \frac{\partial I_m}{\partial y} \hat{y} + \frac{\partial I_m}{\partial t} \hat{t} + h.o$$

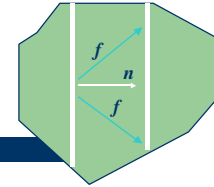
Keep linear terms

- Use constancy assumption and rewrite:

$$0 = \frac{\partial I_m}{\partial x} \hat{x} + \frac{\partial I_m}{\partial y} \hat{y} + \frac{\partial I_m}{\partial t} \hat{t}$$

- Notice: Linear constraint, but no unique solution

## Aperture problem

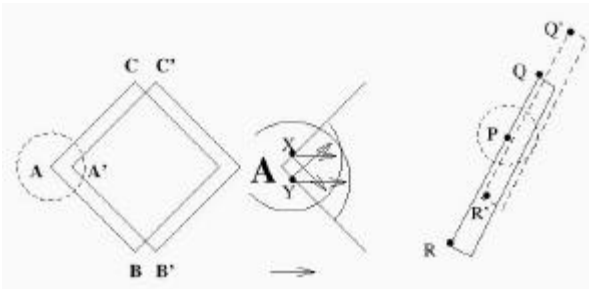


- Rewrite as dot product

$$\hat{a} \frac{\partial I_m}{\partial t} = \frac{\partial I_m}{\partial x} \hat{a} \hat{x} + \frac{\partial I_m}{\partial y} \hat{a} \hat{y} = r \frac{\partial I_m}{\partial n} \hat{a} \hat{n}$$

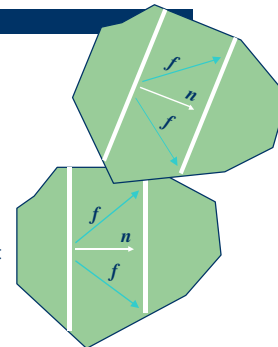
- One equation, two unknowns:  
 $f \cdot n = k$
- Notice: Can only detect vectors normal to gradient direction
- The motion of a line cannot be recovered using only local information

## Aperture problem 2



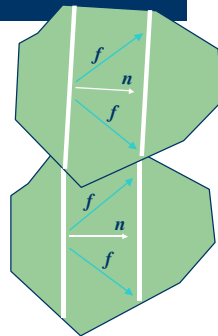
## The flow continuity constraint

- Flows of nearby patches are nearly equal
- Two equations, two unknowns:  
 $f \cdot n_1 = k_1$   
 $f \cdot n_2 = k_2$
- Solution exists, provided  $n_1$  and  $n_2$  not parallel
- Inverse optics gets expensive fast

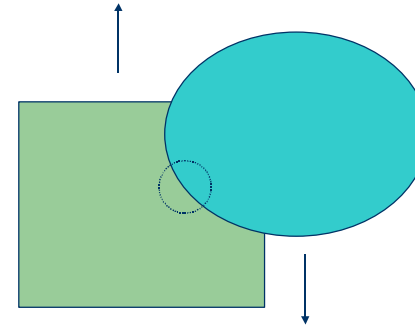


## Sensitivity to error

- $n_1$  and  $n_2$  might be *almost* parallel
- Tiny errors in estimates of  $k$ 's or  $n$ 's can lead to huge errors in the estimate of  $f$



## Sometimes the continuity constraint is false



## Using several points

- Over determined equation system

$$\frac{\partial}{\partial t} \frac{\partial \text{Im}}{\partial t} \mathbf{A} = \frac{\partial}{\partial x} \frac{\partial \text{Im}}{\partial x} \mathbf{A} \quad \frac{\partial}{\partial y} \frac{\partial \text{Im}}{\partial y} \mathbf{A} \quad \hat{x} \quad \hat{y}$$

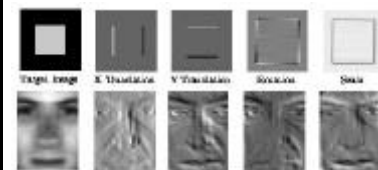
$$\text{Im} = \mathbf{M} \mathbf{u}$$

- Can be solved in e.g. least squares sense using matlab  $\mathbf{u} = \mathbf{M} \backslash \text{Im}$

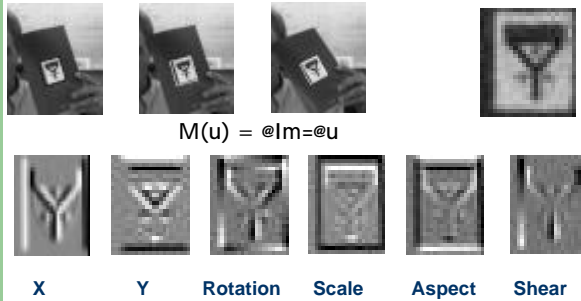
## 3-6D Optic flow

- Generalize to many freedoms (DOFs)

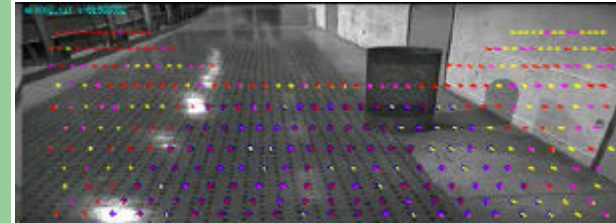
$$\begin{aligned} \delta I &= \|I - I_s\| \\ \delta I &= M \delta \mu \\ M &= [I_x \ I_y \ I_z \ I_s] \\ I_x &= I(x, y) - I(x-1, y) \\ I_y &= I(x, y) - I(x, y-1) \\ I_z &= -y I_x + x I_y \\ I_s &= \frac{1}{\sqrt{x^2 + y^2}} (x I_x + y I_y) \end{aligned}$$



## All 6 freedoms

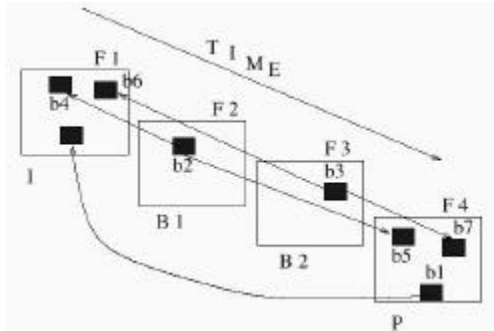


## Flow vectors



- Norbert's trick: Use an mpeg-card to speed up motion computation

## Application: mpeg compression



## Other applications:

- Recursive depth recovery: Kostas and Jane
- Motion control (we will cover)
- Segmentation
- Tracking