Image Restoration

C306
Lecture 21
2001
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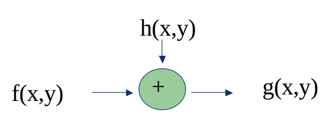
Restoration

- Fixing images degraded by a known degradation function
- Fourier techniques: Cropping freq spectrum
  - Can remove additative artifacts
- Inverse filtering: “Un-blurring”
  - Removes linear blur (from mask convolution)
- Bilinear transforms
  - Fixes geometric distorsions

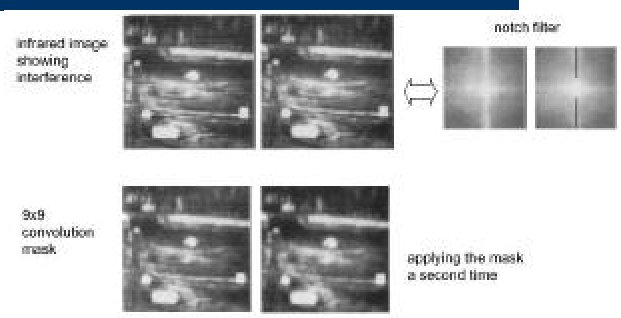
Interactive technique: Cropping in Frequency space

- Useful if degradation is an additative signal with a narrow, well defined spectrum.

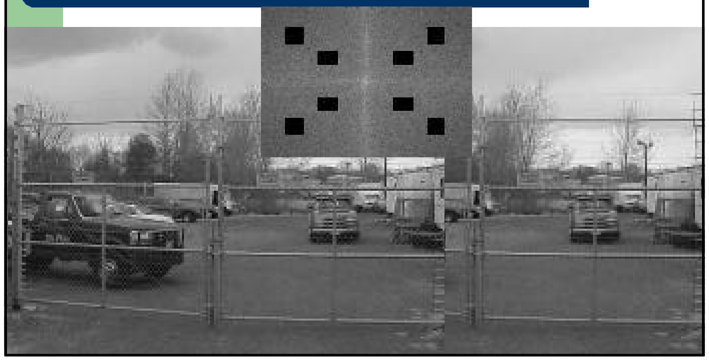


- Example: TV-interference
- Remove by zeroing (cropping) relevant parts of the F-t representation

Example: Restoration by cropping the F-t

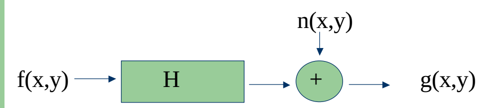


Example 2:
Assignment 3: Remove fence



Degradation Model

- Linear, shift invariant function (filter)



H = Degradation function

n = Additative noise term

Degradation: example terms

H due to:

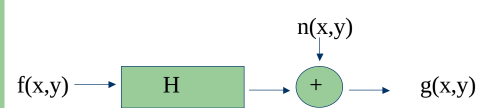
- Motion blur
- Out of focus

n due to:

- Electrical noise in video camera

Degradation as Convolution

- Linear, shift invariant function (filter)



g(x,y) = f(x,y)*h(x,y) +n(x,y) in Spatial Coordinates

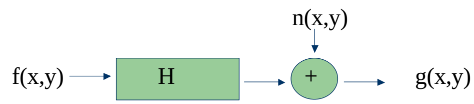
G(u,v) = F(u,v)H(u,v) + N(u,v) in frequency (F-t) coord

(Spatial ad-hoc solution)

- Example: motion blur
- Image f = [a, b, c, d, …..]
- Blur h = [1, 1, 1]
- Blurred g=f*h=[a, a+b, a+b+c, b+c+d, c+d….]

$$f_r = \begin{cases} 0 \\ g(1) \\ g(n) - f_r(n-1) - f_r(n-2) \end{cases}$$

General Solution by Convolution Th

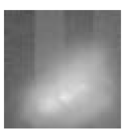


G(u,v) = F(u,v)H(u,v) + N(u,v) in frequency (F-t) coord

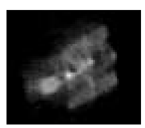
Restored: F_r = G/H= FH/H + N/H = F + N/H ← problem term when H=0

Better: $F_r = \dfrac{G}{H + a}$ a = small constant

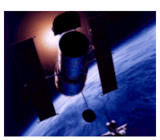
Real applications of Image Restoration

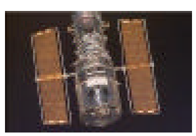


Restore

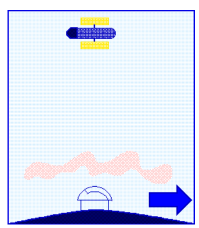


Hubble!!!





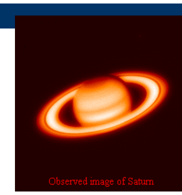
But never as good as original.



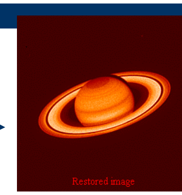
Degradation here:

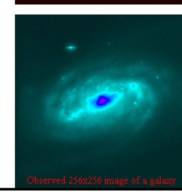
Atmosphere Disturb

Hubble telescope Original and restored images

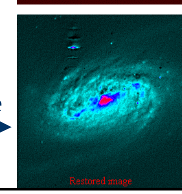


Restore





Restore



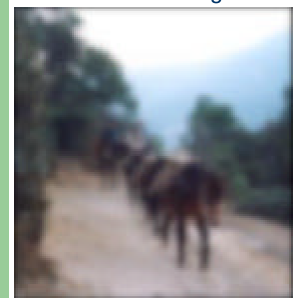
Practical procedure:

1. Find best approximation to degradation that happened
2. Pad or not to pad with 0's, image mean value or other.
3. Compute FFT of degradation
4. FFT real, symmetric? If not maybe force (abs( ))
5. Deconvolute/divide out degradation F_{rest} = G./H
6. Ifft back to get "restored" image

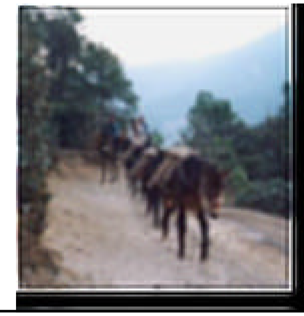
- See matlab example in lecture notes 22

Example: F-t based restoration

- Unfocused image

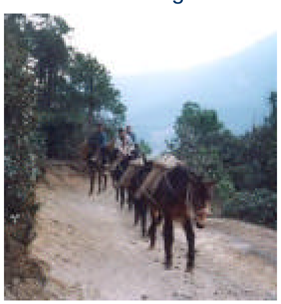


- Restored image

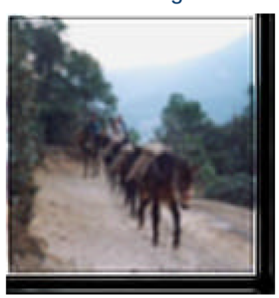


But seldom as good as “new”

- Focused image:



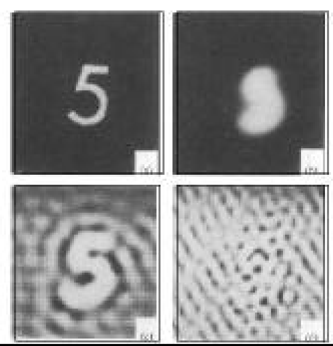
- Restored image



Example: Different heuristic a’s

n(x,y)

f(x,y) → H → + → g(x,y)



G(u,v) = F(u,v)H(u,v) + N(u,v)

in frequency (F-t) coord

Restored: $F_r = \frac{G}{H + a}$

a = small constant

Least Mean Square Filtering

- Problems with inverse filtering:
  - sensitive to the ill-conditioned behavior of the inverse filter
  - main concern is inverse filtering of $\frac{N(u,v)}{H(u,v)}$
  - idea is to make assumption about distribution of noise

Assumptions:

- restored image is f'(x,y)
- f' and f only differ in the noise term
- we adapt stochastic viewpoint: f and f' are random values and we want to minimize difference:

$$\min E\left\{\left[f(x,y)-f'(x,y)\right]^2\right\}$$

Least Mean Square Filtering

- We seek specific point spread function h(x,y), so that

$$f'(u,v) = h_w(x,y) * g(x,y)$$

- and

$$\min E\left\{\left[f(x,y)-f'(x,y)\right]^2\right\}$$

- It can be shown that this can be achieved by

$$H_w(u,v) = \left[\frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{S_n}{S_f}}\right] = \frac{H(u,v)^*}{|H(u,v)|^2 + \frac{S_n}{S_f}}$$

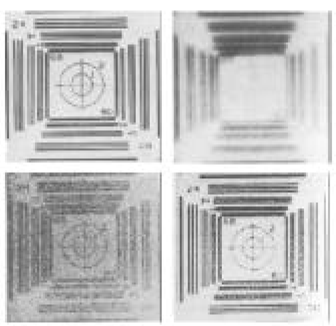
- With * denoting the complex conjugate of H(u,v) and S_n and S_f denoting the power spectrum of noise and signal, resp.

$$|H(u,v)|^2 = H^*(u,v)H(u,v)$$

Example: LMS filt

original image

blurred and additive noise



restoration with inverse filter

restored by method of constrained least squares

Wiener Filter

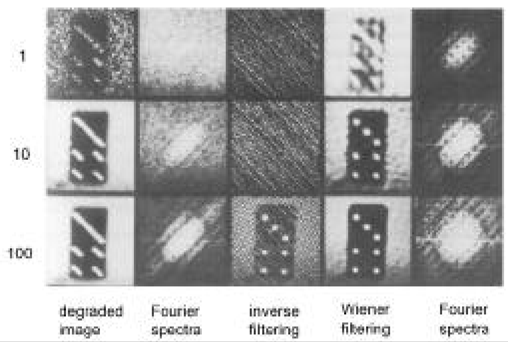
- If S_n and S_f are not known, we approximate the filter by

$$H^*(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + k}\right]$$

- Where k is a constant. k=0 becomes the inverse filter
- Main advantage of Wiener Filter:
  - When any portion of H(u,v) = 0, the S_n / S_f term prevents division by 0.

$$|H(u,v)|^2 = H^*(u,v)H(u,v)$$

Example: Wiener filtering

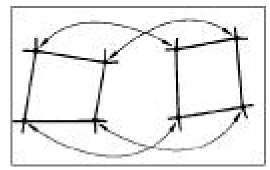


Geometric correction:

- rearrangement of pixel
- original image f and degraded image g with coordinates $(\hat{x}, \hat{y})$

$$\hat{x} = r(x,y) \qquad \hat{y} = s(x,y)$$

- if r and s known -> restoration easy
- using "tie points":

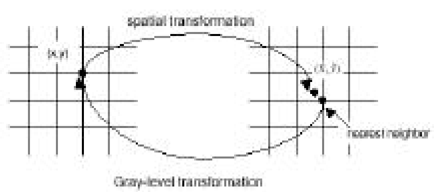


$$\hat{x} = r(x,y) = c_1 x + c_2 y + c_3 xy + c_4$$

$$\hat{y} = s(x,y) = c_5 x + c_6 y + c_7 xy + c_8$$

Resampling for geometric restoration

- nearest neighbor (zero-order interpolation)



- bilinear interpolation
- cubic convolution