20 Edge Detection and Linking

C306
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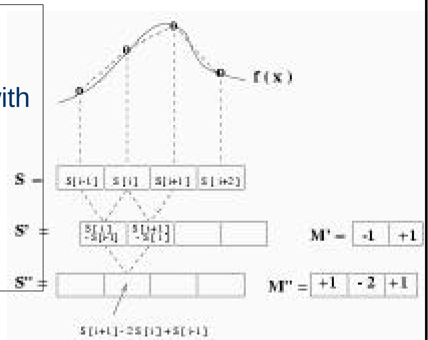
Today

- Edge Detection
- Edge linking

Derivatives of signals

- Discrete differences
- Convolution with mask M', M''



High Pass Filtering

- Enhance detail in images
- Unfortunately also enhances noise
- Convolution in spatial coordinates:

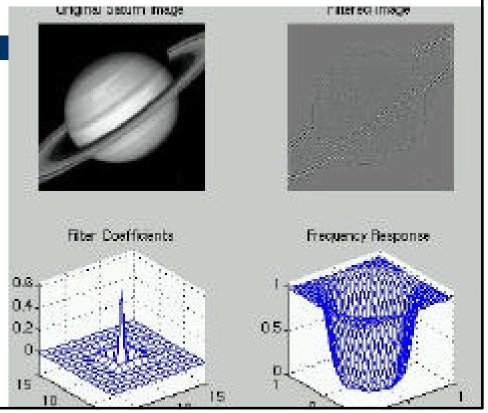
$$\bar{I} = A \otimes I; \quad \text{for } A = \begin{pmatrix} a_{11} & \cdots & a_{n1} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}; \text{e.g. } A = -\frac{1}{9}\begin{pmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Note: sum(A)=0

- Using Conv Theorem we can design filters with any frequency characteristic

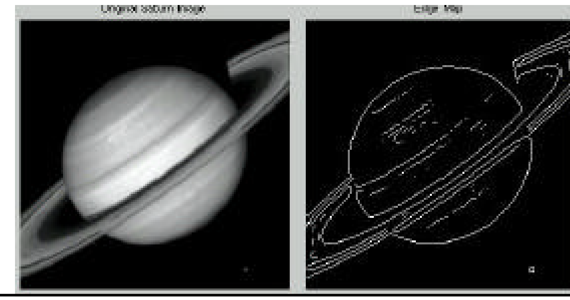
High pass extracts lines

- Only the highest freq
- Lines are intensity discontinuities



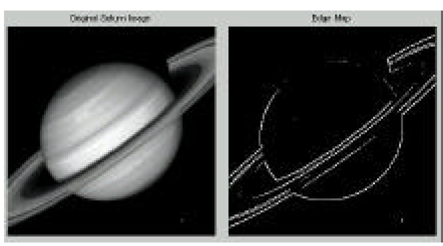
Threshold the HP filter

- Not “ideal” linefinder



Robert’s line finder

- Not ideal either, Why??



$$R_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Prewitt’s linefinder

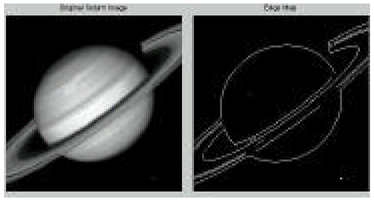
- A bit better.

$$P_v = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$P_h = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Sobel's linefinder

- Most common simple linefinder



$$S_v = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$S_h = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

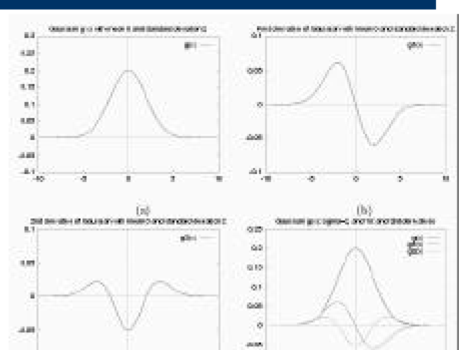
- But still not perfect!!
- Linefinding is unsolved problem

Derivatives of Gaussian

- Gaussian (the pdf of a normal distr RV)
- Smoth in both spatial and frequency coordinates

$$\begin{aligned} g(x) &= \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}} \\ g'(x) &= \frac{-1}{\sqrt{2\pi}\,\sigma^3} x e^{-\frac{x^2}{2\sigma^2}} \\ &= \frac{-x}{\sigma^2}\, g(x) \\ g''(x) &= \left(\frac{x^2}{\sqrt{2\pi}\,\sigma^5} - \frac{1}{\sqrt{2\pi}\,\sigma^3}\right) e^{-\frac{x^2}{2\sigma^2}} \\ &= \frac{x^2}{\sigma^4}\, g(x) - \frac{1}{\sigma^2}\, g(x) \\ &= \left(\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}\right) g(x) \end{aligned}$$

Gaussian



2D Gaussian masks

$G_{3\times3} =$

1	2	1
2	4	2
1	2	1

$G_{7\times7} =$

1	3	7	9	7	3	1
3	12	26	33	26	12	3
7	26	55	70	55	26	7
9	33	70	90	70	33	9
7	26	55	70	55	26	7
3	12	26	33	26	12	3
1	3	7	9	7	3	1

original smoothed $\sigma = 4$ smoothed $\sigma = 1$

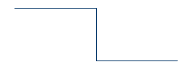
From Pixels to Edges

- Various operators can be used to *enhance* rapid contrast changes
- Detecting these contrast changes involves *thresholding* to separate noise from signal
- *Edges* are a result of *grouping* pixels (sometimes called “edgels”) into groups forming continuous curves.

Definitions:

Edge normal: Unit vector in direction of maximum intensity variation
Edge direction: Perpendicular to edge normal
Edge position: Image position of pixels of edge
Edge strength: Change in contrast along normal

Edge Types



Step

Ridge

Roof

Types of Edge Operators

1. Operators approximating derivatives using differences.
   - directional: Roberts, Prewitt, DoG, etc.
   - Rotationally invariant: Laplacian (sum of second derivatives)
2. Operators based on the zero crossing of the second derivative (e.g. Canny).
3. Operators that attempt to match a specific image profile.

Canny Edge Detector

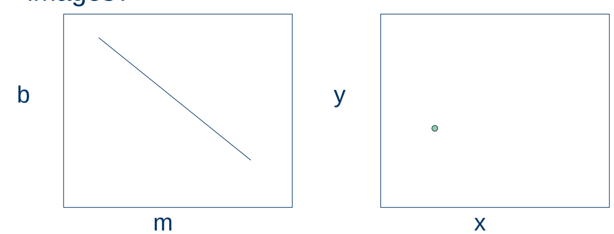
- The Plan:
  - Formulate an optimization problem for detection on 1-D signals
  - Generalize to 2D signals
  - Apply thresholding with hysteresis
  - Apply this operator at various scales
- The Assumptions:
  - Edge enhancement is linear
  - The edge model is step edges with amplitude A
  - Noise is additive, white and Gaussian

The Procedure

- Enhancement:
  - compute x and y derivatives using DoG's.
  - compute direction and magnitude of gradient (two images)
- Nonmaximal Suppression:
  - Sample along the gradient direction
  - If given pixel is smaller than neighbor, set it to zero
- Hysteresis Thresholding:
  - Starting from upper left, visit pixels until one exceeds t_{upper}
  - Follow chains of maxima in edge directions until value drops below t_{lower}
  - Mark and save all visited values as a connected contour

The Hough Transform

- How can we detect extended line structures in images?



$b = -x\,m + y$

Hough Transform Idea

- Each edge point in an image is a constraint on line parameters:
  - constraint is a line
  - each unique point adds another constraint
- Algorithm:
  - Initialize a 2-D array of counters to zero.
  - For each edge point (x,y), increment any counter which contains a parameter point (b,m) satisfying $b = -x\,m + y$
  - Threshold counters
  - Group edgels that belong to above threshold counters into contours
  - Optional: Refit lines to get higher precision.

Hough Transform Variation

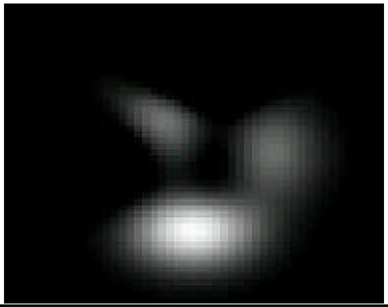
- Problem:
  - m and b are, in principle, unbounded
    - rewrite as $\sin(t)\,x + \cos(t)\,y = d$
    - For each discrete value of t from 0 to pi, increment counter with nominal d minimal distance from exact value
  - coarse grid leads to grouping of distinct lines
    - in post-fitting stage, re-hough at higher resolution

Hough Transform Variation

- Generalizations
  - Any linear in parameters model: e.g a x^2 + b y^2 = 1 can use the same algorithm
  - For f(x,a) = 0, choose any cell a_c s.t. $f(x,a_c) < t$ for some threshold t.
- Limitations:
  - the curse of dimensionality

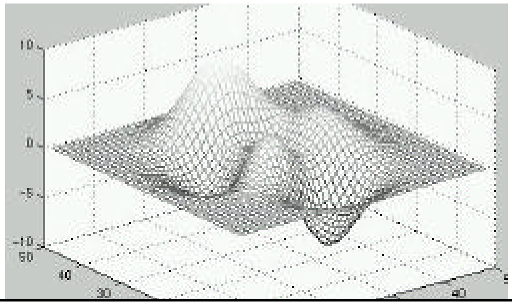
Alternatives to intensity

- What is this?



Answer:

- Just a matlab function (peaks)



Pseudocolor

- More precise illustration of magnitude