# Filtering Part II

C306
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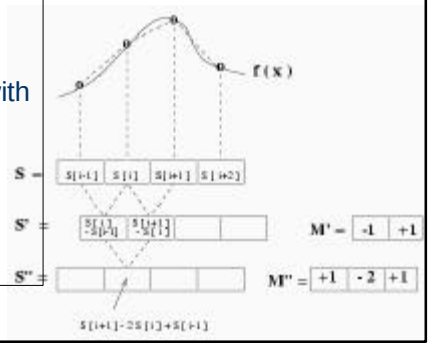
## Today

- High Pass filtering
- Edge Detection
- Image Enhancement

## Derivatives of signals

- Discrete differences
- Convolution with mask M', M''

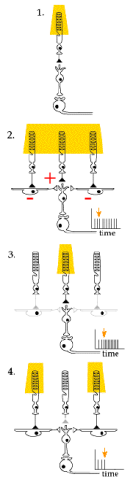



## Human signal processing:

Center-surround organization:

1. Light hyperpolarizes the rod and excites the bipolar cell below it
2. But inhibitory connections through horizontal cells suppress signals
3. Best response to localized "dot"
4. While stimulating surround only lowers firing rate

- What is this???
  Convolution!!! Im*[-1 2 –1]



## High Pass Filtering

- Enhance detail in images
- Unfortunately also enhances noise
- Convolution in spatial coordinates:

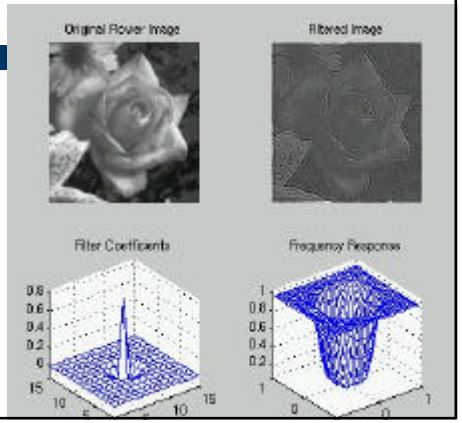
$$\bar{I} = A \tilde{a} I; \quad \text{for } A = \begin{bmatrix} a_{11} & \acute{a}\acute{a}\acute{a} & a_{n1} \\ \vdots & \ddots & \vdots \\ a_{m1} & \acute{a}\acute{a}\acute{a} & a_{mn} \end{bmatrix}; \text{e:g } A = \frac{\grave{a}1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \grave{a}8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Note: sum(A)=0

- Using Conv Theorem we can design filters with any frequency characteristic

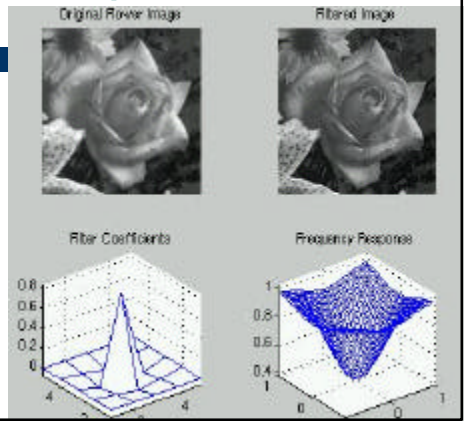
## High Pass Example 1

- Sharp cutoff frequency
- Why is filtered image so dark??



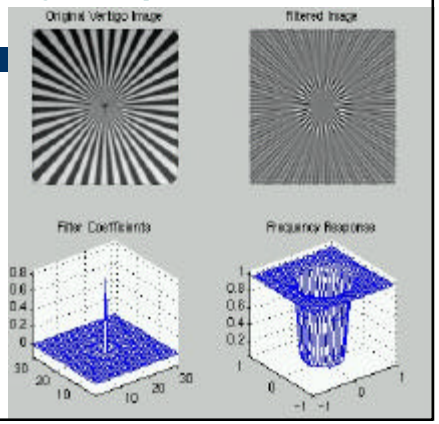

## High Pass example 2

- Less sharp cutoff
- More low frequency "slips" through
- Smaller mask



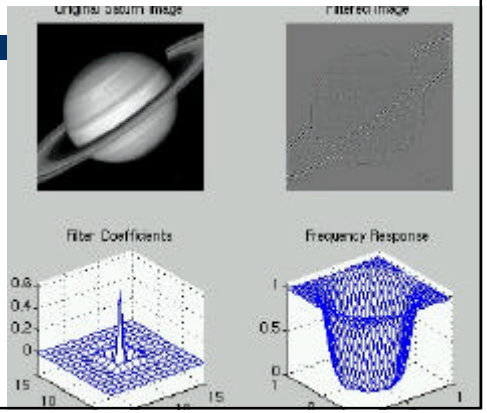

## Ex3 Frequency test picture

- Middle (high frequency) region enhanced
- Why can we still see the thick stripes??



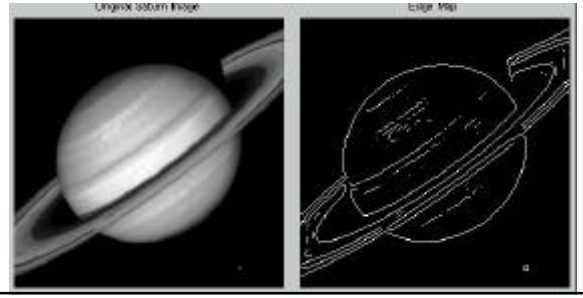
## High pass extracts lines

- Only the highest freq
- Lines are intensity discontinuities



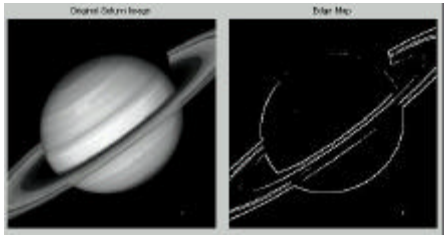

## Threshold the HP filter

- Not “ideal” linefinder



## Robert’s line finder

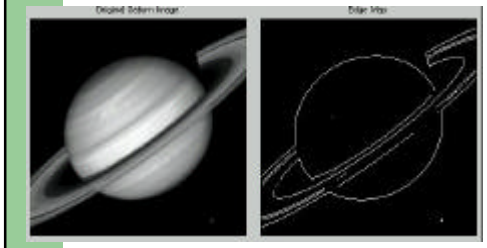
- Not ideal either, Why??



$$R_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad R_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

## Prewitt’s linefinder

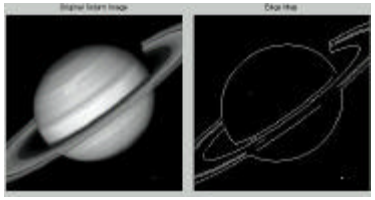
- A bit better.



$$P_v = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \qquad P_h = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

## Sobel's linefinder

- Most common simple linefinder



$$S_v = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad S_h = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

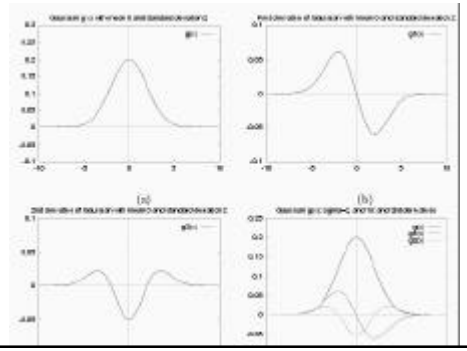
- But still not perfect!!
- Linefinding is unsolved problem

## Derivatives of Gaussian

- Gaussian (the pdf of a normal distr RV)
- Smoth in both spatial and frequency coordinates

$$\begin{aligned} g(x) &= \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}} \\ g'(x) &= \frac{-1}{\sqrt{2\pi}\,\sigma^3} x e^{-\frac{x^2}{2\sigma^2}} \\ &= \frac{-x}{\sigma^2}\, g(x) \\ g''(x) &= \left(\frac{x^2}{\sqrt{2\pi}\,\sigma^5} - \frac{1}{\sqrt{2\pi}\,\sigma^3}\right) e^{-\frac{x^2}{2\sigma^2}} \\ &= \frac{x^2}{\sigma^4}\, g(x) - \frac{1}{\sigma^2}\, g(x) \\ &= \left(\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}\right) g(x) \end{aligned}$$

## Gaussian



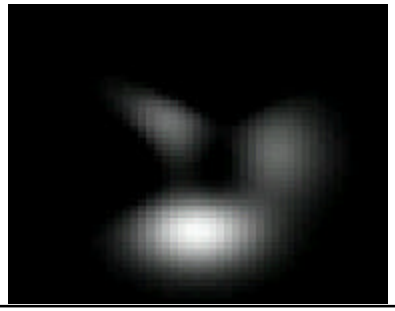
## 2D Gaussian masks

$$G_{3\times3} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}; \qquad G_{7\times7} = \begin{bmatrix} 1 & 3 & 7 & 9 & 7 & 3 & 1 \\ 3 & 12 & 26 & 33 & 26 & 12 & 3 \\ 7 & 26 & 55 & 70 & 55 & 26 & 7 \\ 9 & 33 & 70 & 90 & 70 & 33 & 9 \\ 7 & 26 & 55 & 70 & 55 & 26 & 7 \\ 3 & 12 & 26 & 33 & 26 & 12 & 3 \\ 1 & 3 & 7 & 9 & 7 & 3 & 1 \end{bmatrix}$$

original smoothed $\sigma = 4$ smoothed $\sigma = 1$

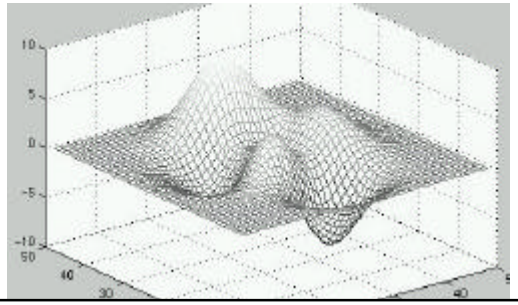
## Alternatives to intensity

- What is this?



## Answer:

- Just a matlab function (peaks)



## Pseudocolor

- More precise illustration of magnitude