More on image transforms

C306
Fall 2001
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Assignment

- Heard concerns about long time between turn in and demo; matlab problems
- Consider:
  - New due date Tue Oct 9, 17:00 at start of lab
  - Everyone has to demo in Tue or Thu section or make individual appointment with TA

Image operations: Point op v.s. Transforms

- So far we have considered operations on one image point, ie contrast stretching, histogram eq. Etc
- By contrast, image transforms are defined over regions of an image or the whole image

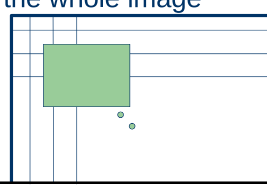


Image transforms

- Image transforms are some of the most important methods in image processing
- Will be used later in:
  1. Filtering
  2. Restoration
  3. Enhancement
  4. Compression
  5. Image analysis

Vector spaces

- We will start with the concepts of
  1. vector spaces and
  2. coordinate changes,

just as used in the camera models:

Examples:

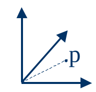
- Euclidean world space $= \Re^3; \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
- Image space, (q by q image): $= \Re^{q^2}; \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1q} \\ a_{21} & & & \\ \vdots & & \ddots & \\ a_{q1} & & & a_{qq} \end{bmatrix} \leftrightarrow \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{q^2} \end{bmatrix}$

Basis (of a vector space)

- A member can be described as a linear comb

$$\mathbf{x} = \mathbf{a}\mathbf{e}_1 + \mathbf{a}\mathbf{e}_2 + \ldots + \mathbf{a}\mathbf{e}_m$$

- Example: 3D vectors



- Example: Image = a +b +c

Vector spaces
Important properties from lin alg

1. Product
   1. With scalar
   2. "dot" product
   3. Vector product (for 3-vectors)
2. vector sum, difference
3. Norm
4. Orthogonality
5. Basis

See Shapiro-Stockman p. 178-181

Finding local image properties:

Roberts basis: $W_1 = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $W_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ $W_3 = \frac{1}{\sqrt{2}}\begin{bmatrix} +1 & 0 \\ 0 & -1 \end{bmatrix}$ $W_4 = \frac{1}{2}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

Constant region: $\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} = \frac{20}{2}(\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}) = 10W_1 \oplus 0W_2 \oplus 0W_3 \oplus 0W_4$

Step Edge: $\begin{bmatrix} -1 & +1 \\ -1 & +1 \end{bmatrix} = 0W_1 \oplus \frac{2}{\sqrt{2}}W_2 \oplus \frac{2}{\sqrt{2}}W_3 \oplus 0W_4$

Step Edge: $\begin{bmatrix} +1 & +1 \\ -3 & +1 \end{bmatrix} = 0W_1 \oplus \frac{4}{\sqrt{2}}W_2 \oplus 0W_3 \oplus \frac{-4}{2}W_4$

Line: $\begin{bmatrix} 0 & 8 \\ 8 & 0 \end{bmatrix} = 8W_1 \oplus 0W_2 \oplus 0W_3 \oplus 8W_4$

Roberts basis in matlab

- Basis:

```
W1 = 1/2*[
1 1
  1 1];

W2 = 1/sqrt(2)*[
0 1
-1 0];

W3 = 1/sqrt(2)*[
1 0
  0 -1];

W4 = 1/2*[
-1 1
   1 -1];
```

- Test region:

```
CR = [
5 5
  5 5];

% Coefficients:
c1 = sum(sum(W1.*CR))
% c1 = 10

c2 = sum(sum(W2.*CR))
% c2 = 0

% Do same for W3 and
W4
```

Better way to implement: Use matrix algebra!

- Construct basis matrix

```
B = [
W1(:)'
W2(:)'
W3(:)'
W4(:)']
```

- Flatten the test area:

```
CRf = CR(:)
```

- Compute coordinate change:

```
NewCoord = B*CRf
% = (10,0,0,0)'
```

- Test2: Step edge:

```
SE = [
-1 1
-1 1]
SEf = SE(:)

NewCoordSE = B*SEf
% =(0,1.41,-1.41,0)'
```

- Verify that we can transform back:

```
SEtestf =B'*NewCoordSE

reshape(SEtestf, 2, 2)
% SEtest =
%    -1.0000     1.0000
%    -1.0000     1.0000
```

Standard basis

- e1 e2 e3 • e8 e9

$$\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}\quad\begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix}\quad\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}\quad\cdots\quad\begin{bmatrix}0&0&0\\0&0&0\\0&1&0\end{bmatrix}\quad\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}$$

Nine "standard" basis vectors for the space of all 3x3 matrices.

$$\begin{bmatrix}9&5&0\\5&0&0\\0&0&0\end{bmatrix} = 9\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix} + 5\begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix} + 5\begin{bmatrix}0&0&0\\1&0&0\\0&0&0\end{bmatrix}$$

= 9*e1 + 5*e2 + 5*e4

Frei-Chen basis

gradient: $\mathbf{W}_1 = 1/\sqrt{8}\begin{bmatrix}1&\sqrt{2}&1\\0&0&0\\-1&-\sqrt{2}&-1\end{bmatrix}$ $\quad\mathbf{W}_2 = 1/\sqrt{8}\begin{bmatrix}1&0&-1\\\sqrt{2}&0&-\sqrt{2}\\1&0&-1\end{bmatrix}$

ripple: $\mathbf{W}_3 = 1/\sqrt{8}\begin{bmatrix}0&-1&\sqrt{2}\\1&0&-1\\-\sqrt{2}&1&0\end{bmatrix}$ $\quad\mathbf{W}_4 = 1/\sqrt{8}\begin{bmatrix}\sqrt{2}&-1&0\\-1&0&1\\0&1&-\sqrt{2}\end{bmatrix}$

line: $\mathbf{W}_5 = 1/2\begin{bmatrix}0&1&0\\-1&0&-1\\0&1&0\end{bmatrix}$ $\quad\mathbf{W}_6 = 1/2\begin{bmatrix}-1&0&1\\0&0&0\\1&0&-1\end{bmatrix}$

Laplacian: $\mathbf{W}_7 = 1/6\begin{bmatrix}1&-2&1\\-2&4&-2\\1&-2&1\end{bmatrix}$ $\quad\mathbf{W}_8 = 1/6\begin{bmatrix}-2&1&-2\\1&4&1\\-2&1&-2\end{bmatrix}$

constant: $\mathbf{W}_9 = 1/3\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$

Frei

Example of representing an intensity neighborhood using the Frei-Chen basis.

Consider the intensity neighborhood $\mathbf{N} = \begin{array}{|c|c|c|}\hline 10 & 10 & 10 \\ \hline 10 & 10 & 5 \\ \hline 10 & 5 & 5 \\ \hline \end{array}$

We find the component of this vector along each basis vector using the dot product as before. Since the basis is orthonormal, the total image energy is just the sum of the component energies and the structure of N can be interpreted in terms of the components.

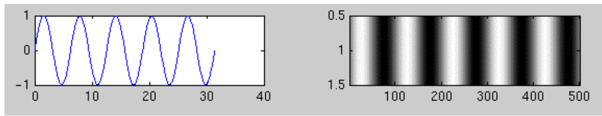
$$\begin{aligned}
N \circ W_1 &= \frac{5+5\sqrt{2}}{\sqrt{8}} \approx 4.3; \; energy \approx 18 \\
N \circ W_2 &= \frac{5+5\sqrt{2}}{\sqrt{8}} \approx 4.3; \; energy \approx 18 \\
N \circ W_3 &= 0; \; energy = 0 \\
N \circ W_4 &= \frac{5\sqrt{2}-10}{\sqrt{8}} \approx -1; \; energy \approx 1 \\
N \circ W_5 &= 0; \; energy = 0 \\
N \circ W_6 &= 2.5; \; energy \approx 6 \\
N \circ W_7 &= 2.5; \; energy \approx 6 \\
N \circ W_8 &= 0; \; energy = 0 \\
N \circ W_9 &= 25; \; energy = 625
\end{aligned}$$

The total energy in N is $N \circ N = 675$, 625 of which is explained just by the

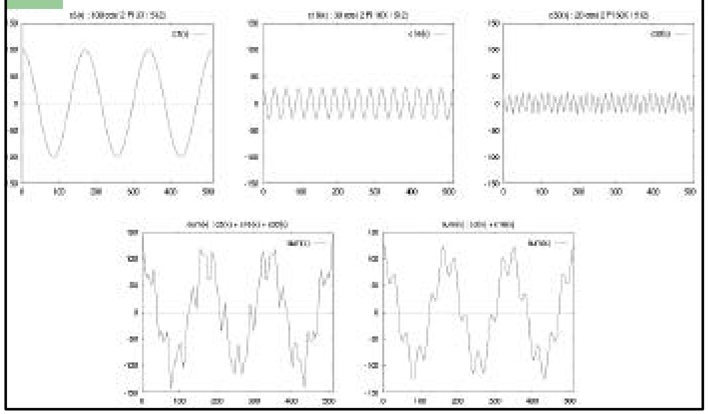
Sine function

```
t = 0:pi/50:10*pi
I = sin(t)
subplot(221)
plot(t,I)
```

- `Image(I) ???`
- `subplot(222)`
  `image(64*(I+1)/2)`
  `colormap gray`



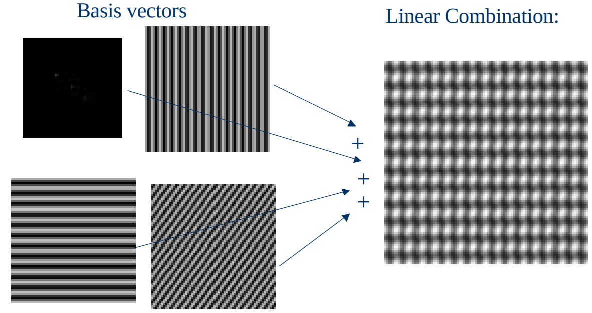
Adding sines



Constructing an image

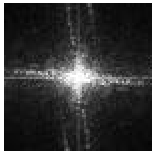
Basis vectors

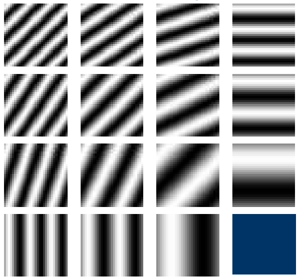
Linear Combination:



Analyzing an image

All basis images

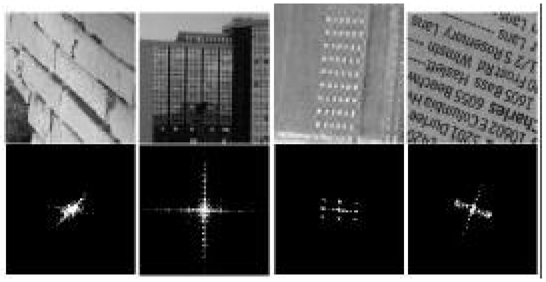




... ...

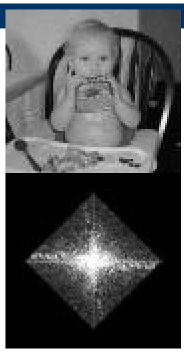
intensity ~ that frequency's coefficient

Manmade object images:



- What does the F-t express?

Cropping in Fourier space



Data Reduction: only use *some* of the existing frequencies

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