Camera geometry

C306
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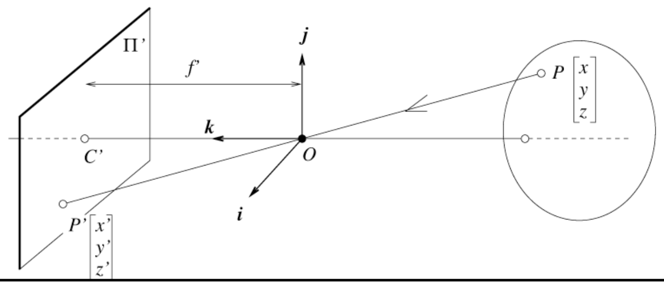
The equation of projection

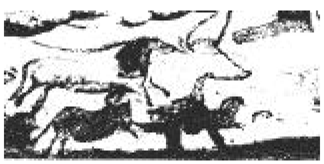
- Cartesian coordinates:
  - We have, by similar triangles, that (x, y, z) ->
    (f x/z, f y/z, -f)
  - Ignore the third coordinate, and get

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

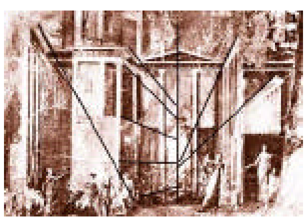


History of Perspective

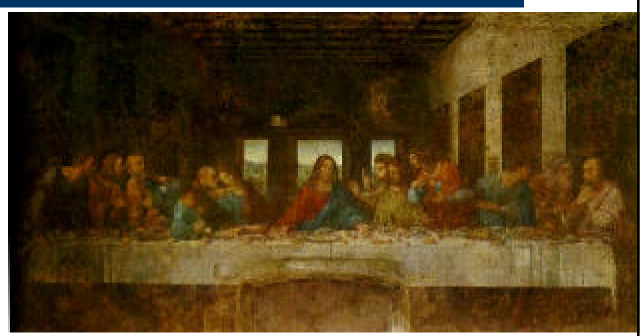
Prehistoric:



Roman



Renaissance
Leonardo da Vinci: The last supper



The camera matrix

- Homogenous coordinates for 3D
  - four coordinates for 3D point
  - equivalence relation (X,Y,Z,T) is the same as (k X, k Y, k Z,k T)
- Turn previous expression into HC's
  - HC's for 3D point are (X,Y,Z,T)
  - HC's for point in image are (U,V,W)

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix} \qquad (U,V,W) \to \left(\frac{U}{W}, \frac{V}{W}\right) = (u,v)$$

Camera parameters

- Issue
  - camera may not be at the origin, looking down the z-axis
    - extrinsic parameters
  - one unit in camera coordinates may not be the same as one unit in world coordinates
    - intrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{intrinsic parameters} \end{pmatrix} \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{projection model} \end{pmatrix} \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{extrinsic parameters} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

Camera Model Structure

Assume R and T express camera *in world coordinates, then*

$$^{c}p = \begin{pmatrix} R' & -R'T \\ 0 \;\; 0 \;\; 0 & 1 \end{pmatrix} {}^{w}p$$

Combining with a perspective model (and neglecting internal parameters) yields

$$^{c}u = M\, ^{w}p = \begin{pmatrix} -R'_x & R'_x T \\ -R'_y & R'_y T \\ \frac{R_z}{f} & \frac{-R_z T}{f} \end{pmatrix} {}^{w}p$$

Note the M is defined only up to a scale factor at this point! If M is viewed as a 3x4 matrix defined up to scale, it is called the *projection matrix.*

Intrinsic Parameters

Intrinsic Parameters describe the conversion from metric to pixel coordinates (and the reverse)

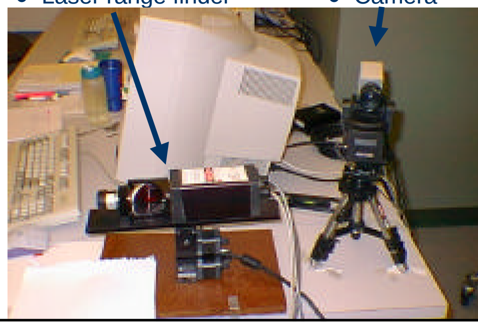
$$x_{mm} = -(x_{pix} - o_x)\, s_x$$
$$y_{mm} = -(y_{pix} - o_y)\, s_y$$

or

$$\begin{pmatrix} x \\ y \\ w \end{pmatrix}_{pix} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix}_{mm} = M_{\text{int}}\, p$$

Example: A real camera

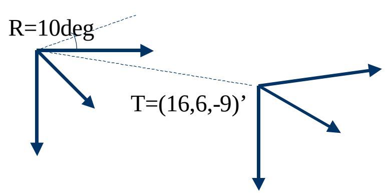
- Laser range finder
- Camera



- Dana

Relative location Camera-Laser

- Camera
- Laser



In homogeneous coordinates

- Rotation:

$$\mathbf{R} = \begin{bmatrix} \cos -10 & 0 & \sin -10 \\ 0 & 1 & 0 \\ -\sin -10 & 0 & \cos -10 \end{bmatrix}$$

- Translation

$$T = \begin{pmatrix} 1 & 0 & 0 & 16 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & -9 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Full projection model

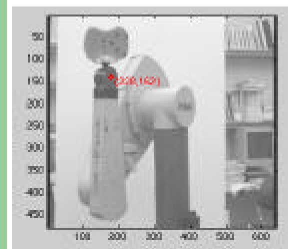
- Camera internal parameters
- Camera projection

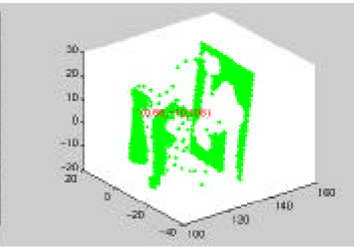
$$p_{camera} = \begin{pmatrix} 1278.6657 & 0 & 256 \\ 0 & 1659.5688 & 240 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0.985 & 0 & -0.174 & 0 \\ 0 & 1 & 0 & 0 \\ 0.174 & 0 & 0.985 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 16 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & -9 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.6612 \\ -10.55 \\ 108.0 \\ 1 \end{pmatrix} = \begin{pmatrix} 22262 \\ 16755 \\ 97.47 \end{pmatrix}$$

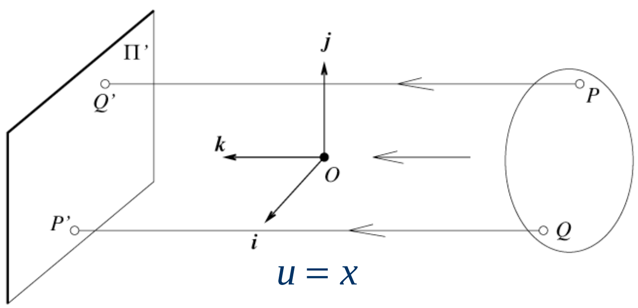
Result

- Camera image
- Laser measured 3D structure





Orthographic projection



$$u = x$$

$$v = y$$

The fundamental model for orthographic projection

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

Perspective and Orthographic Projection

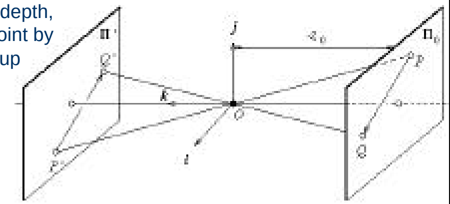
perspective

Orthographic (parallel)

Weak perspective

$$u = Tx$$
$$v = Ty$$
$$T = f / Z$$

- Issue
  - perspective effects, but not over the scale of individual objects
  - collect points into a group at about the same depth, then divide each point by the depth of its group
  - Adv: easy
  - Disadv: wrong



The fundamental model for weak perspective projection

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & f/Z^* \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

Camera Model Structure

Assume R and T express camera *in world coordinates, then*

$${}^c p = \begin{pmatrix} R' & & & -R'T \\ 0 & 0 & 0 & 1 \end{pmatrix} {}^w p$$

Combining with a weak perspective model (and neglecting internal parameters) yields

$${}^c u = M\, {}^w p = \begin{pmatrix} -R'_x & R'_x T \\ -R'_y & R'_y T \\ 0 & \frac{R_z(\overline{P} - T)}{f} \end{pmatrix} {}^w p$$

Where $\overline{P}$ is the nominal distance to the viewed object

Other Models

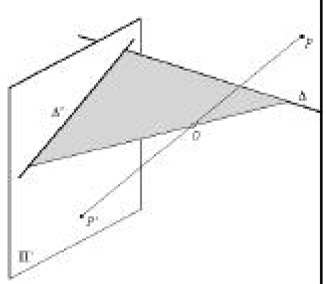
- The *affine camera* is a generalization of weak perspective.
- The *projective camera* is a generalization of the perspective camera.
- Both have the advantage of being linear models on real and projective spaces, respectively.

Camera calibration

- Issues:
  - what are intrinsic parameters of the camera?
  - what is the camera matrix? (intrinsic+extrinsic)
- General strategy:
  - view calibration object
  - identify image points
  - obtain camera matrix by minimizing error
  - obtain intrinsic parameters from camera matrix
- Error minimization:
  - Linear least squares
    - easy problem numerically
    - solution can be rather bad
  - Minimize image distance
    - more difficult numerical problem
    - solution usually rather good, but can be hard to find
      - start with linear least squares
  - Numerical scaling is an issue

Geometric properties of projection

- Points go to points
- Lines go to lines
- Planes go to whole image
- Polygons go to polygons
- Degenerate cases
  - line through focal point to point
  - plane through focal point to line



Polyhedra project to polygons

- (because lines project to lines)