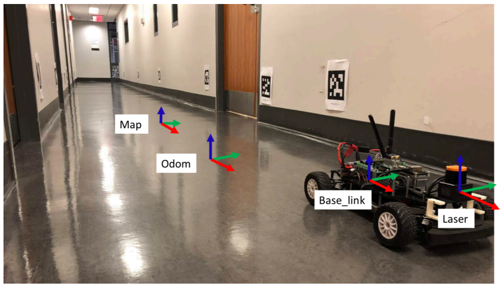
Coordinate frames on/for robots



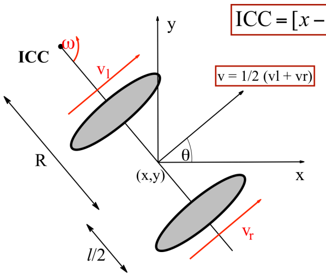
What we will do today ..

- Assignment No. 1, Wednesday (9/18)

$$\operatorname*{argmin}_{\mathcal{X}} \sum_{k=0}^{m} ||h_k(\mathcal{X}_k) - z_k||^2_{\Omega_k} \qquad (4)$$

- Odometry (u_i in Fig. 3 of [Caneda 2016])
  - Wheel odometry
  - LiDAR and LiDAR odometry
  - Inertial measurement unit (IMU) and IMU odometry
- Cameras (RGB and RGB-D)
- On Wed (9/25), we will talk about loop closure (c_i) the course project.

Differential Drive



$$\mathrm{ICC} = [x - R\sin\theta, y + R\cos\theta]$$

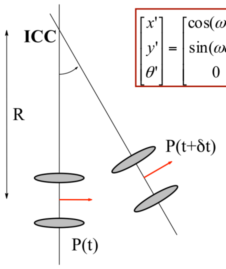
$$\omega(R + l/2) = v_r$$

$$\omega(R - l/2) = v_l$$

$$R = \frac{l}{2}\frac{(v_l + v_r)}{(v_r - v_l)}$$

$$\omega = \frac{v_r - v_l}{l}$$

Differential Drive: Forward Kinematics



$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - \mathrm{ICC}_x \\ y - \mathrm{ICC}_y \\ \theta \end{bmatrix} + \begin{bmatrix} \mathrm{ICC}_x \\ \mathrm{ICC}_y \\ \omega\delta t \end{bmatrix}$$

$$x(t) = \int_0^t v(t') \cos[\theta(t')]dt'$$

$$y(t) = \int_0^t v(t') \sin[\theta(t')]dt'$$

$$\theta(t) = \int_0^t \omega(t')\,dt'$$

Odometric constraint: $u_t\,(z_k)$

From the kinematics of differential drive robot (pp. 4-5), let:

$$x_{t+1} = [x', y', \theta']^T,\ x_t = [x, y, \theta]^T,\ u_t = [v_l,\ v_r]^T$$

the robot kinematic equation is of the form: $x_{t+1} = f(x_t, u_t)$.

If we define $\chi_k = [x_t, x_{t+1}]^T$ and $z_k = u_t$, our SLAM optimization problem needs to be generalized to

$$\mathcal{X}^* = \operatorname{argmin}_{\mathcal{X}} \sum_{k=0}^{m} ||g_k(\chi_k, z_k)||^2_{\Omega_k}$$

where $g_k(\chi_k, z_k) = x_{t+1} - f(x_t, u_t)$.

Odometric constraint: $u_t\,(z_k)$

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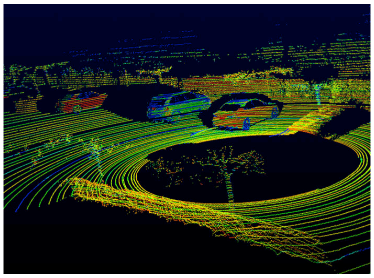
If we define $\chi_k = [x_t, x_{t+1}]^T$ and $z_k = u_t$, our SLAM optimization problem needs to be generalized to

$$\mathcal{X}^* = \operatorname*{argmin}_{x} \sum_{k=0}^{m} ||\ x_{t+1} - f(x_t, u_t)||^2_{\Omega_k}.$$

In 3D, we need to change the above cost term into a matrix form (see the document "g2o versus Toro: Format and Cost Functions"):

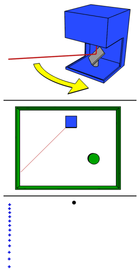
$$\min_{\boldsymbol{R}_i, \boldsymbol{t}_i} \sum_{(i,j)\in\mathcal{E}} \left\|\mathrm{Log}\left(\bar{\boldsymbol{R}}_{ij}^{\mathsf{T}} \boldsymbol{R}_i^{\mathsf{T}} \boldsymbol{R}_j\right)\right\|^2_{\Omega^r_{ij}} + \left\|\bar{\boldsymbol{t}}_{ij} - \boldsymbol{R}_i^{\mathsf{T}}(\boldsymbol{t}_j - \boldsymbol{t}_i)\right\|^2_{\Omega^t_{ij}}$$

LiDAR – Light Detection and Ranging



Source: https://clearpathrobotics.com/blog/2015/04/robots-101-lasers/

Function and Concept of 2D LiDAR



All Lidar units operate using this basic set of steps:

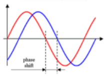
1. Laser light is emitted from the unit (usually infrared).
2. Laser light hits an object and is scattered.
3. Some of the light makes it back to the emitter.
4. The emitter measures the distance (more on how later).
5. Emitter turns, and process begins again.

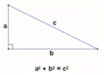
3D LiDAR is built from multiple planes or a pan-tilt mirror system.

Types of LiDAR

How exactly the laser sensor measures the distance to an object depends on how accurate the data needs to be.

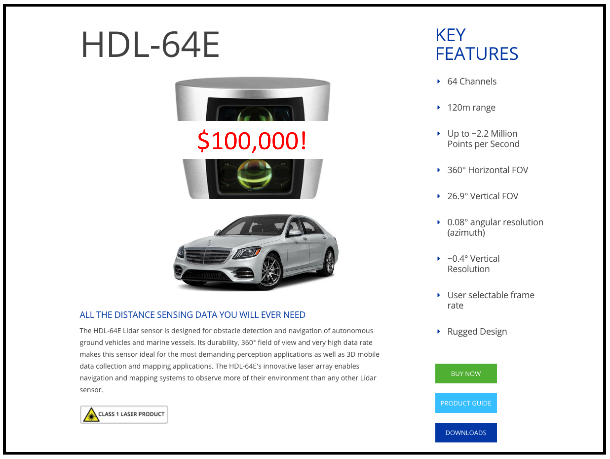
1. Time of Flight:
   distance = (speed of light) x (time).
2. Phase Difference (used by robots):
   modulate laser beam and measure phase shift (e.g. 250 KHz)
3. Angle of incident
   infer distance from the angle of reflected light





LiDAR Selection Criteria

- Range
- Light sensitivity
- Angular resolution
- Field of View
- Refresh rate (a few times/s)
- Accuracy – noise in range measure
- Size
- Cost



HDL-64E

$100,000!

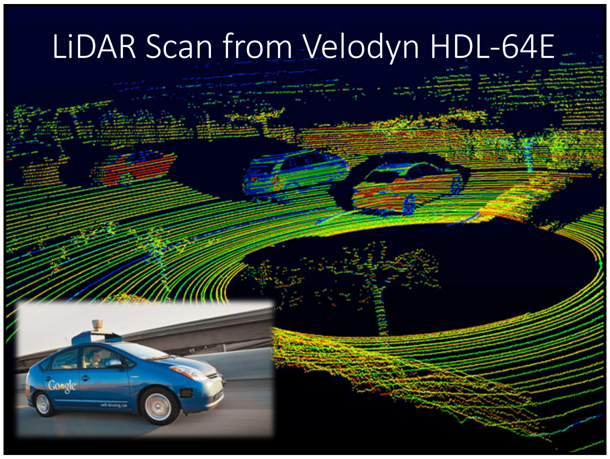
ALL THE DISTANCE SENSING DATA YOU WILL EVER NEED

The HDL-64E Lidar sensor is designed for obstacle detection and navigation of autonomous ground vehicles and marine vessels. Its durability, 360° field of view and very high data rate makes this sensor ideal for the most demanding perception applications as well as 3D mobile data collection and mapping applications. The HDL-64E's innovative laser array enables navigation and mapping systems to observe more of their environment than any other Lidar sensor.

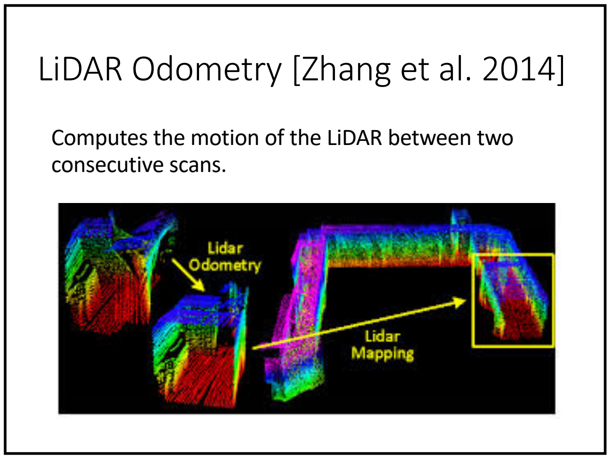
KEY FEATURES

- 64 Channels
- 120m range
- Up to ~2.2 Million Points per Second
- 360° Horizontal FOV
- 26.9° Vertical FOV
- 0.08° angular resolution (azimuth)
- ~0.4° Vertical Resolution
- User selectable frame rate
- Rugged Design

2D LiDAR Scan from a SICK sensor



LiDAR Scan from Velodyn HDL-64E



LiDAR Odometry [Zhang et al. 2014]

Computes the motion of the LiDAR between two consecutive scans.

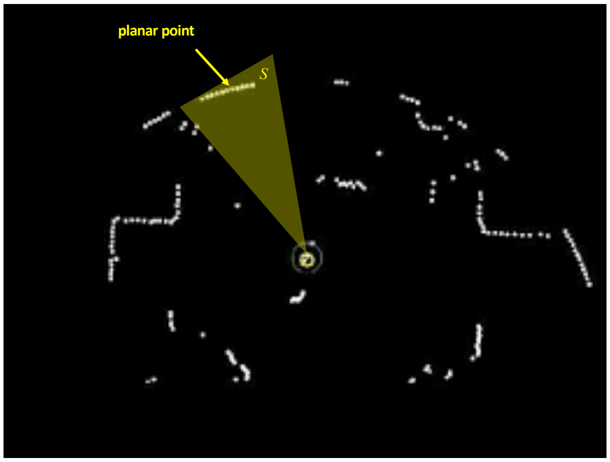
LiDAR Odometry [Zhang et al. 2014]

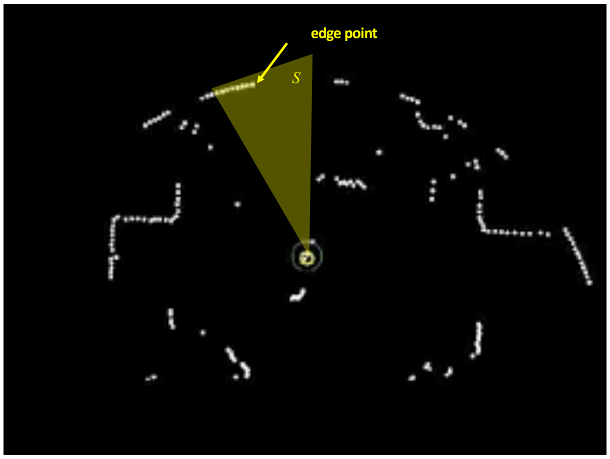
Computes the motion of the LiDAR between two consecutive scans.

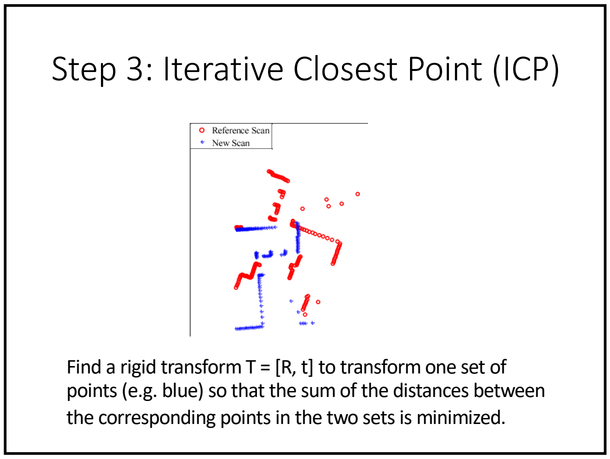
1. Select feature points that are on either sharp edges or planar surface patches

$$c = \frac{1}{|\mathcal{S}| \cdot ||\boldsymbol{X}^L_{(k,i)}||} || \sum_{j\in\mathcal{S}, j\neq i} (\boldsymbol{X}^L_{(k,i)} - \boldsymbol{X}^L_{(k,j)})||.$$

2. Match edge points and planar points in consecutive scans.
3. Estimate LiDAR motion from matched points.



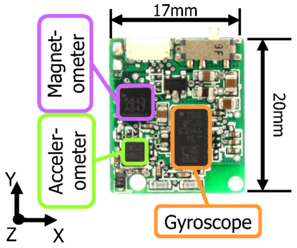




Step 3: Iterative Closest Point (ICP)

Find a rigid transform T = [R, t] to transform one set of points (e.g. blue) so that the sum of the distances between the corresponding points in the two sets is minimized.

IMU – Inertial Measurement Unit

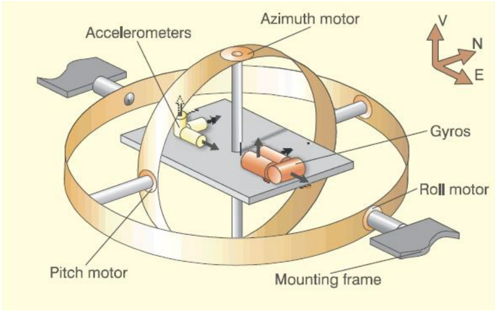


What is in an IMU?

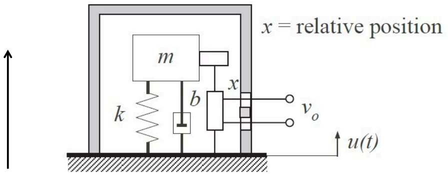
- Gyroscopes -> Angular Velocity (radians/s)
- Accelerometer -> Linear Acceleration (m/s^2 or g)
- Magnetometer -> Magnetic field strength (micro-Tesla or Gauss)

In addition, sometimes, barometric or GPS measurements are integrated in an IMU as well.

IMU – Inertial Measurement Unit



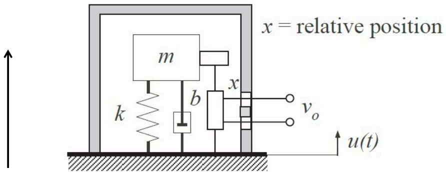
Accelerometer



When m accelerates, it will experience a force, which will cause spring K to move. The displacement of k can be measured through sensor x, and the force F can be sensed as a result. Given F and m, the acceleration $\ddot{x}$ can be measured based the Newton's 2nd law.

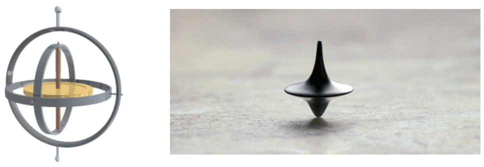
$$\ddot{x} \quad = F/m$$

Accelerometer



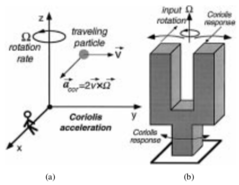
Three (3) accelerometers along the x, y, and z axes provide three linear accelerations whose double-integration results in linear displacement measurements (odometry)

Gyroscope or Gyro



Newton's first law of motion states: a body in motion will remain in motion unless it is acted upon by an external force.

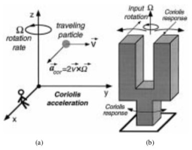
Gyroscope's Principle of Operation

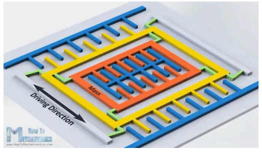


"Imagine a particle traveling in space with a velocity vector $\vec{V}$. An observer sitting on the x-axis in Fig. (a) is watching this particle. If the coordinate system along with the observer starts to rotate around the z-axis with an angular velocity $\vec{\Omega}$, the observer thinks that the particle is changing its trajectory toward the x-axis an an acceleration $2\times\vec{V}\times\vec{\Omega}$" or **Coriolis** force $F=m\times2\times\vec{V}\times\vec{\Omega}$.

(Navid Yazdi et al., "Micromachined Inertial Sensors", *Proc. IEEE*, 1998).

Gyroscope: tuning fork design

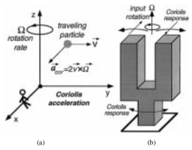


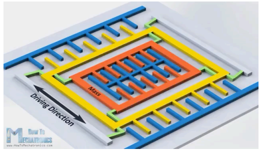


Tuning fork design uses two tines that resonate ($\vec{V}$). Coriolis force is detected either as differential bending of the tines or as a torsional vibration of the tuning-fork stem, along direction that is orthogonal to the main vibration.

From the Coriolis force, angular velocity ($\vec{\Omega}$) can be calculated.

Gyroscope or Gyro





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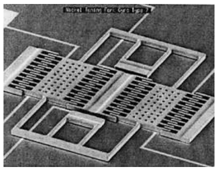
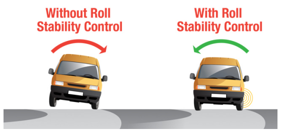
Gyroscope or Gyro



Three gyroscopes along three principal axes provide three angular velocities whose integration results in angular change (odometry). Modern gyros are built with MEMS (right).

Gyroscope applications



Gyros are responsible for UAV control, ESC (electronic stability control) for cars to prevent rollovers, Wii by Nintendo, video games on smart phones, etc.

Summary

- IMU uses gyroscopes to measure angular velocities and accelerometers to measure accelerations, about the three axes.
- Integration of angular velocity over a short time interval provides angular change.
- Double integration of acceleration over a short time interval provides linear change.
- IMU is inexpensive but requires calibration and is subject to ambient noise in environments.

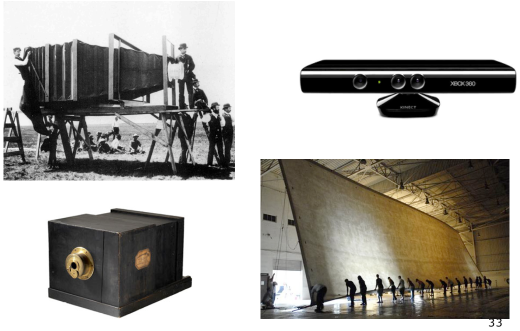
Cameras and Images

- Why camera?
- Images and Pixels
- Image formation
- Coordinate frames: image, camera, and world (map)
- Camera calibration: intrinsics and extrinsics

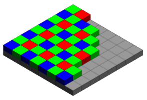
Why camera?

- Vision provides rich information about the world in which a robot operates (humans derive 80-90% of information from vision).
- Types of information computed with vision in robotics:
  - Geometry of a scene (SfM, single-image depth, surface normal)
  - Robot motion (visual odometry)
  - Object detection (pedestrians, cars, doors, windows, etc.)
  - Object classification (scene semantics)

Cameras

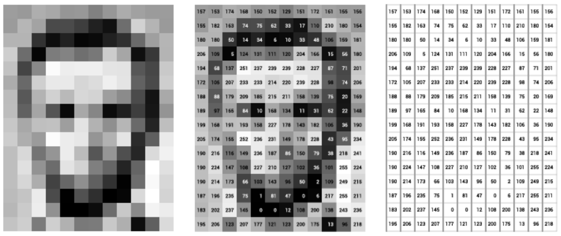


Images and pixels



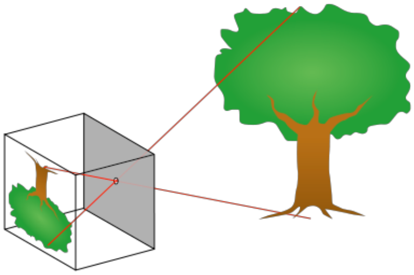
- A camera captures either color or grayscale images
- Each grayscale image is a matrix of N columns and M rows (e.g., 640x480)
- A color image consists of three separate images, one for each color component or channel (e.g., RGB).
- A color image can be captured with one array of lighting sensing elements covered with a Bayer filter mosaic or three separate arrays.
- Each element of an image array is called a picture element or pixel, indexed by its column and row number.
- The intensity (grayscale or a color channel) value of a pixel is encoded typically in 8-bits (0-255).
- Therefore, a grayscale image is just an array of 8-bit numbers, which a computer vision algorithm processes to extract info.

Images and pixels

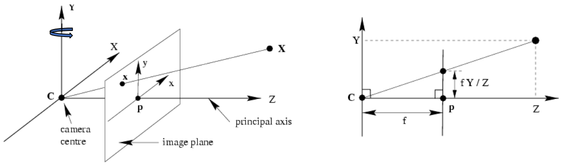


157	153	174	168	150	152	129	151	172	161	155	156
155	182	163	74	75	62	33	17	110	210	180	154
180	180	50	14	34	6	10	33	48	106	159	181
206	109	5	124	131	111	120	204	166	15	56	180
194	68	137	251	237	239	239	228	227	87	71	201
172	105	207	233	233	214	220	239	228	98	74	206
188	88	179	209	185	215	211	158	139	75	20	169
189	97	165	84	10	168	134	11	31	62	22	148
199	168	191	193	158	227	178	143	182	106	36	190
205	174	155	252	236	231	149	178	228	43	95	234
190	216	116	149	236	187	86	150	79	38	218	241
190	224	147	108	227	210	127	102	36	101	255	224
190	214	173	66	103	143	96	50	2	109	249	215
187	196	235	75	1	81	47	0	6	217	255	211
183	202	237	145	0	0	12	108	200	138	243	236
195	206	123	207	177	121	123	200	175	13	96	218

Image formation: pinhole model



Pinhole camera

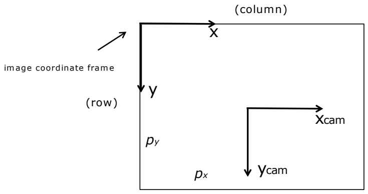


space point image point

$$(X,Y,Z)^T \mapsto \mathbf{x} = (x,y)^T = (fX/Z, fY/Z)^T$$

Image coordinate vs camera cameraframe

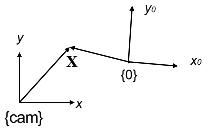
- *Image coordinate frame* is related to the *camera coordinate frame* by two translations p_x and p_y



World coordinate frame

- If the world coordinate frame {0} is defined with respect to {cam}, the camera coordinate frame, by *R* and *t*,

$$\mathbf{X}_c = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \mathbf{X}_0$$



- [*R, t*]$^{-1}$ are called the extrinsic parameters of the camera and *K* the intrinsic parameters.

Camera parameters

- Intrinsics: *K*
  - Focal lengths (2)
  - Image center (2)
  - Skew coefficient (1)
  - Distortion coefficients (3)
- Extrinsics: R, t
  - Rotation (3), *R*
  - Translation (3), *t*
- Intrinsics are calibrated once whereas extrinsics change whenever the camera (robot) moves.

Coordinate frames on/for robots