

Optimal Traffic-Oblivious Energy-Aware Routing For Multihop Wireless Networks

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Abstract—Energy efficiency is an important issue in multihop wireless networks with energy concerns. Usually it is achieved with accurate knowledge of the traffic pattern and/or the current network information such as the remaining energy level. We investigate the problem of designing a routing scheme to minimize the maximum energy utilization of a multihop wireless network with weak assumption of the traffic pattern and without ongoing collection of network information. We develop polynomial size LP models to design such a routing scheme. We discuss generalizations of the LP models to various radio transmission models. In an interference-limited scenario, we show how to guarantee schedulability of the oblivious routing. We present an extension to consider lossy links. We also discuss implementation issues. The LP models achieve performance close to what an oracle can achieve in the performance study. The results for multihop wireless networks with a single sink are especially good. We make a first stride in designing a traffic-oblivious energy-aware routing framework in multihop wireless networks.

I. INTRODUCTION

Research in multihop wireless networks, such as wireless ad hoc networks, wireless sensor networks, wireless community mesh networks and base-stations connected by wireless links, has drawn much attention recently. Energy efficiency is a paramount issue when the energy source is costly or there are energy constraints. In some wireless systems, it is critical to control heat generation, thus energy efficiency is a key factor. In a multihop wireless network with a single sink, nodes close to the sink tend to be heavily loaded. It is thus more important to balance the load in the network.

Previous work on energy efficiency has made great progress. Singh et al. [21] investigate power-aware routing in wireless ad hoc networks. They propose several routing metrics and study their performance through simulation. The problem of maximizing the lifetime of a wireless ad hoc network with energy constraints is studied in [7], [13], where the lifetime is defined as the length of the time until the first node drains out its energy. It assumes every node is important. Kar et al. [11] investigate how to route the maximal number of messages in wireless ad hoc networks with energy constraints. Sadagopan and Krishnamachari [20] study the problem of maximizing data extraction in wireless sensor networks with energy constraints. If the power supply is renewable, it is desirable that the energy consumption rate is less than the renewal rate. Lin et al. [15] study power-aware routing with renewable energy sources.

Some previous work assumes exact prior knowledge of traffic pattern, e.g. [7], [20]. The traffic pattern may be

known a priori in some applications, such as in a wireless sensor network in which sensors periodically report weather information. With knowledge of the traffic pattern, network flow [2] can be used to model the energy efficiency problem. Chang et al. [7] model the problem of lifetime maximization as a linear program (LP) and give a heuristic solution. Sadagopan and Krishnamachari [20] develop an approximate algorithm and a heuristic algorithm based on a LP formulation of the data extraction problem. In our work we study the optimal routing in the *minimax* sense, i.e., to minimize the maximum energy utilization, in multihop wireless networks. Given the traffic pattern, we can model the problem to minimax energy utilization as a LP optimization problem.

For some classes of applications, e.g. wireless community mesh networks or base stations connected by wireless links, the traffic pattern may not be known a priori. It is difficult to obtain an accurate estimate of the traffic pattern even in the scenario of the Internet [4], [6], [23], where a large amount of measurement data is available. It is likely to be difficult to estimate the traffic pattern accurately in some multihop wireless networks, e.g., in a wireless community mesh network. Recently researchers study traffic characterization in wireless networks (without energy concerns), mainly in a wireless LAN environment such as on a campus or in a corporation, e.g. Meng et al. [16]. Even if an estimate is accurate, it is in a statistical sense, which means there is an error margin with the estimation. Some wireless networks may be designed with the expected traffic pattern in mind. However, in some cases, there are unexpected or unscheduled events. In the sensor network example, where weather information is reported periodically, the sensors may also need to report temperature changes exceeding a certain threshold, which may not be predictable. Furthermore, the actual traffic may deviate from the expectation. Therefore, it is desirable to allow for errors, deviation and uncertainties in traffic prediction when designing a routing scheme. Two approaches may achieve this, namely, adaptive and oblivious.

A routing scheme may be adaptive to the traffic pattern and the network condition such as the remaining energy level in an energy-constrained case. Some adaptive approaches can bound the performance, e.g. [11], [13], [15]. They need to periodically collect information such as the current energy level. The approach in [13] needs a regular traffic pattern to achieve the performance guarantee. The adaptive approaches in [11], [15], which are based on the work of adaptive routing

in a wired network [18], have performance guarantees in the order of logarithmic to network size. It is desirable to design an efficient scheme to collect necessary information for energy efficiency with respect to both computing an energy-efficient route and economizing energy for information collection. We take an alternative approach to investigate feasibility and performance of a routing scheme oblivious to the traffic pattern and network information.

The research on oblivious routing in optimizing link utilization [19], [5], [4] has made great achievements. The oblivious routing problem is to design a routing that achieves close to the optimal performance, with no or only approximate knowledge of the traffic pattern, without considering the current network load. Racke [19] investigates oblivious routing on general symmetric networks. Azar et al. [5] show that an optimal oblivious routing can be computed by an LP with a polynomial number of variables, but infinite number of constraints. Applegate and Cohen [4] design a simple polynomial size LP to obtain traffic-oblivious routing schemes to minimax link utilization that achieve good performance in the scenario of the Internet.

In this paper, we study the problem of traffic-oblivious, energy-aware routing in multihop wireless networks. We focus on wireless networks with stationary topology. Our work is applicable to low mobility. Our goal is to design a routing scheme that achieves minimax energy utilization in a multihop wireless network, with a weak assumption of the traffic pattern, without ongoing collection of network information. We develop polynomial size LP models to design such a routing scheme. The routing is fixed¹, thus it is oblivious to changes and uncertainties of the traffic. It is also oblivious to the current state of the network, such as the current energy level of wireless nodes and the current network load. It does not need to collect network information except for the stationary topology and the initial energy level. The routing achieves minimax energy utilization. Thus it is energy-aware. It achieves energy efficiency nearly optimally as shown in the experimental results. In contrast to the logarithmic performance guarantee of adaptive approaches [11], [15], our LP models give low, constant (close to 1.0) performance guarantee in the studied cases.

The LP models are general enough for several radio transmission models, such as omni-directional and directional antennas and a radio equipped with various possible granularities of transmission power levels. It can also work with a multi-channel and/or multi-radio wireless system.

A wireless network has unique features, such as interference and dynamic channel conditions, in contrast to a wired network. Recently there is increased interest in jointly considering routing and scheduling. Schedulability of a routing is studied in Hajek and Sasaki [8] and Kodialam and Nandagopal [12] for the “free of secondary interference” model, where a node can transmit to or receive from at most one node. Necessary and sufficient conditions are derived. Jain et al. [10] use a conflict

graph to model the interference relationship between links and investigate lower and upper bounds of an achievable network flow. On the other hand, emerging technologies, e.g. the ultra wideband (UWB) system [17], may create an “interference-free” wireless environment which renders schedulability of a routing no longer a (serious) problem. We will discuss schedulability concerns in §III-F.

A wireless channel is usually fluctuating, caused by other ongoing transmissions and the surrounding environment. Quality of service (QoS) is an important issue, especially in a wireless network with interference and time-varying channel conditions. Applications may have various QoS requirements, such as loss rate and end-to-end delay. Our work focuses on energy efficiency. We discuss extensions to consider lossy links in III-G. We take an optimistic treatment of a wireless network by modeling it as a graph with stationary links, in an attempt to gain insight of designing a traffic-independent energy-efficient routing. We discuss an opportunistic implementation of the fixed routing to help address the issue of link fluctuation by locally monitoring link quality in §III-H.

Our major contribution is: we design a routing scheme which is energy-efficient, is independent of the traffic pattern and does not need ongoing network information collection. The experiments show that the oblivious routing can achieve performance close to what an oracle can achieve.

In [14], we have incorporated the schedulability constraints and considered lossy links in formulating LP models. In this paper, we focus on the scenario where the traffic load is relatively low compared with the bandwidth, due to the utilization of ultra-wide band radio technology and/or the traffic itself is sporadic.

The paper is organized as follows. §II presents the network model, notation, and performance metrics. In §III, we develop LP models to compute the optimal oblivious routing. We discuss generalization to various radio transmission models in §III-E. We discuss schedulability concerns in §III-F. An extension to lossy links is presented in §III-G. Implementation issues are discussed in §III-H. We present the experimental results in §IV. Then we draw conclusions.

II. MODEL

Network Model. A multihop wireless network can be abstracted as a digraph $G = (V, E)$, where V is the set of nodes and E is the set of “edges”. There is an edge (u, v) if node u can reach node v . We assume the digraph is strongly connected. Each node u has an initial energy level $pow_0(u)$. We assume stationary channel conditions, e.g. an additive white Gaussian noise (AWGN) channel with constant noise power. We assume a transmitting node uses a fixed modulation scheme. We use $in(u)$ and $out(u)$ to denote the sets of nodes that have edges “into” and “out of” node u respectively, i.e., $in(u) = \{t | (t, u) \in E\}$, and $out(u) = \{v | (u, v) \in E\}$. We use $out(v, -u)$ to denote the set of nodes that have edges out of v , excluding u , i.e., $out(v, -u) = \{w | (v, w) \in E, w \neq u\}$.

Energy Consumption Model. The energy consumption to transmit a unit amount of data from a node u to another node v

¹The routing is “fixed” in the sense that there is a single output of the LP model. It can be implemented in an opportunistic way as discussed in §III-H.

is $tx(u, v)$. Usually $tx(u, v)$ depends on the distance between u and v . The amount of energy consumption in transmission is proportional to the amount of data to be transmitted. This linear model is used in previous work on energy efficiency, e.g. [7], [9], [11], [13], [20].

We use $r(u)$ and $h(u)$ to model the energy consumption of node u to receive and to overhear a unit of message respectively. Overhearing means receiving a packet by a node not addressed to it. We separate reception and overhearing since they may consume different amounts of energy. For instance, a node may overhear the whole data packet or only the preamble before discarding it. In the former case, overhearing consumes a comparable amount of energy to reception; while in the latter overhearing may consumes much less energy. During the formulation of the LP models, we only need the function forms of these energy consumption models. In the simulation study, we will use specific models. The energy consumption for processing data may be a component of the transmission model and the reception model, thus we do not model it explicitly.

Traffic Matrix. We keep the notation *traffic matrix* (TM) as in the literature of Internet traffic engineering, e.g. [23], [4]. Denoting the number of nodes as n , a traffic matrix is an $n \times n$ nonnegative matrix where the diagonal entries are 0. A traffic matrix provides the amount of traffic between each Origin-Destination (OD) pair over a certain time interval. It characterizes the traffic pattern in an average sense.

Maximum Energy Utilization. We introduce a performance metric, *maximum energy utilization*; and based on it, we model the energy efficiency problem as a LP optimization problem. The definition of this metric is inspired by the derivation of the maximum lifetime, e.g. in [7] and the definition of the maximum link utilization, e.g. in [4]. With this metric, we can handle more problems besides lifetime maximization.

A *routing* \mathbf{f} specifies what fraction of the traffic for each OD pair is routed on each edge. We will give its detailed definition later. For a given routing \mathbf{f} , a given traffic matrix \mathbf{tm} , the maximum energy utilization (MEU) measures the “goodness” of the routing. The lower the maximum energy utilization, the better the routing. We have,

$$\text{MEU}(\mathbf{tm}, \mathbf{f}) = \max_s \frac{\text{energy}_s}{\text{pow}_0(s)}$$

where energy_s denotes the total energy consumption for all the traffic transmitted, received and overheard by node s . We will develop its detailed expression later. Recall $\text{pow}_0(s)$ denotes the initial energy level of s .

Minimax Energy Utilization. For a given traffic matrix \mathbf{tm} , an *optimal routing* minimizes the maximum energy utilization:

$$\text{OPTE}(\mathbf{tm}) = \min_{\mathbf{f}: \mathbf{f} \text{ is a routing}} \text{MEU}(\mathbf{tm}, \mathbf{f})$$

The minimax energy utilization measures the energy consumption rate of a wireless network. It can be regarded as a unification of several studied problems: lifetime maximization with or without energy renewal, maximization of the number

of messages or data extraction, and minimization of power consumption. The lifetime of a wireless network is inversely proportional to the energy consumption rate of the node that consumes energy the fastest. For a given traffic matrix, once we minimax the energy utilization, we effectively maximize the lifetime of the multihop wireless network. The problem of minimizing the renewal rate can be dealt with similarly. To minimax energy utilization is equivalent to maximizing data extraction according to the data rates of the sources, which is a concurrent multi-commodity problem [2]. When wireless nodes have the same initial power reserve, to minimax energy utilization is equivalent to the problem of minimizing power consumption.

Given a traffic matrix, the optimal routing to minimax energy utilization is solvable as a LP multi-commodity flow problem [2]. For now, we focus on routing. We discuss schedulability of a routing in §III-F. The LP to find the optimal routing is:

$$\begin{aligned} & \min p \\ & \mathbf{f} \text{ is a routing} \\ & \forall \text{ nodes } s : \text{energy}_s / \text{pow}_0(s) \leq p \end{aligned} \quad (1)$$

LP (1) minimizes the maximum energy utilization for a given traffic matrix, i.e., LP (1) is equivalent to $\min_{\mathbf{f}} \text{MEU}(\mathbf{tm}, \mathbf{f})$. This LP is similar to that of Chang et al. [7] for maximizing the lifetime of a wireless ad hoc network.

For an application with prior knowledge of traffic pattern, the above LP model is sufficient to compute the optimal routing to minimax energy utilization. For example, it maximizes the lifetime of a wireless sensor network with periodical reports of weather information.

Competitive Ratio. The routing computed by LP (1) does not guarantee performance for other traffic matrices. We will develop LP models to compute the optimal routing that achieves minimax energy utilization with a weak assumption on the traffic pattern. First we introduce the metric of competitive ratio that follows the competitive analysis [18], [4].

For a given routing \mathbf{f} , a given traffic matrix \mathbf{tm} , the *competitive ratio* is defined as the ratio of the maximum energy utilization of the routing \mathbf{f} on the traffic matrix \mathbf{tm} to the maximum energy utilization of the optimal routing. Competitive ratio measures how far the routing \mathbf{f} is from the optimal routing on the traffic matrix \mathbf{tm} . Formally,

$$\text{CR}(\mathbf{f}, \{\mathbf{tm}\}) = \frac{\text{MEU}(\mathbf{tm}, \mathbf{f})}{\text{OPTE}(\mathbf{tm})}$$

The competitive ratio is usually greater than 1. It is equal to 1 only when the routing \mathbf{f} is an optimal routing.

When we are considering a set of traffic matrices \mathbf{TM} , the competitive ratio of a routing \mathbf{f} is defined as

$$\text{CR}(\mathbf{f}, \mathbf{TM}) = \max_{\mathbf{tm} \in \mathbf{TM}} \text{CR}(\mathbf{f}, \mathbf{tm})$$

The competitive ratio with respect to a set of traffic matrices is usually strictly greater than 1, since a single routing can not optimize energy utilization over the set of traffic matrices.

When set \mathbf{TM} includes all possible traffic matrices, $\text{CR}(\mathbf{f}, \mathbf{TM})$ is referred to as the *oblivious competitive ratio* of the routing \mathbf{f} . This is the worst competitive ratio the routing \mathbf{f} achieves with respect to all traffic matrices. An *optimal oblivious routing* is the routing that minimizes the oblivious competitive ratio. Its oblivious ratio is the *optimal oblivious ratio* of the network.

Suppose there is an oracle that knows the instant traffic matrix \mathbf{tm} and computes its optimal routing with energy utilization e . The energy utilization of the optimal oblivious routing for \mathbf{tm} is guaranteed to be within $[e, r * e]$, where r is the oblivious ratio. It may achieve lower energy utilization than $r * e$ for the particular traffic matrix \mathbf{tm} . The oblivious routing guarantees the performance of what an oracle can achieve multiplied by the oblivious ratio for all traffic matrices.

III. TRAFFIC-OBLIVIOUS ENERGY-AWARE ROUTING

In the following, we start with a simpler case of a single sink with all other nodes as sources. A potential application is a wireless sensor network in which every sensor reports to a single node, where there are unknown, unexpected or unscheduled events such that it is difficult to accurately predict the traffic pattern. Then we study the case where communication may happen between every pair of nodes. A potential application is wireless community mesh networks with energy constraints. It is easy to adapt our model to the cases of multiple sources and/or multiple sinks, i.e., communication happens only between certain pair(s) of nodes. The following LPs are developed for the case where energy is a constraint and not renewable. The LP models can be generalized to other problems such as lifetime maximization when the energy is renewable and maximization of data extraction with energy constraints.

The LP models can be generalized to various wireless systems as discussed in §III-E. In an interference-free environment, the routing can work well without considering scheduling. In an interference-limited environment, we can add additional linear constraints to guarantee schedulability of the routing. We discuss schedulability in §III-F. We then discuss an extension to consider lossy links and implementation issues.

A. A Single Sink Case

In the following, we develop LP models to compute the oblivious ratio for a multihop wireless network with a single sink, when we know no or approximate knowledge of the traffic pattern. The sink node is assumed to have infinite energy capacity. We first introduce the detailed definitions of a routing \mathbf{f} and the energy consumption \mathbf{energy}_s .

When there is a single sink in a multihop wireless network, denoted as T , the destination of any OD pair is T . The traffic matrix is reduced to a traffic vector, with each entry d_i denoting the amount of traffic originating from node i . We study what fraction of d_i is routed along each edge.

A routing $f_i(s, t)$ specifies what fraction of d_i is routed along edge (s, t) . The traffic on edge (s, t) for d_i is $d_i f_i(s, t)$.

Routing \mathbf{f} is defined as:

$$\begin{cases} \forall \text{ nodes } i \neq T : \sum_{j \in \text{out}(i)} f_i(i, j) = 1 \\ \forall \text{ nodes } i \neq T, \forall k \neq i, T : \\ \quad \sum_{l \in \text{out}(k)} f_i(k, l) - \sum_{j \in \text{in}(k)} f_i(j, k) = 0 \\ \forall i \neq T, \forall \text{ edges } (s, t) : f_i(s, t) \geq 0 \end{cases} \quad (2)$$

From the above, we can derive the routing conservation constraint, $\forall \text{ nodes } i \neq T : \sum_{u \in \text{in}(i)} f_i(u, i) = 1$.

The energy consumption of node s for d_i is, $\mathbf{energy}_s(i) = \sum_{t \in \text{out}(s)} \{d_i f_i(s, t) t x(s, t)\} + \sum_{t \in \text{in}(s)} \{d_i f_i(t, s) r(s)\} + \sum_{t \in \text{in}(s)} \sum_{k \in \text{out}(t, -s)} \{I_{(t, s)}^{(t, k)} d_i f_i(t, k) h(s)\}$ $I_{(t, s)}^{(t, k)}$ is an indicator function defined as,

$$I_{(t, s)}^{(t, k)} = \begin{cases} 1 & \text{if } s \text{ can overhear transmission from } t \text{ to } k; \\ 0 & \text{otherwise.} \end{cases}$$

The first term in $\mathbf{energy}_s(i)$ is the energy consumption for transmission; the second for reception and the third for overhearing. We use $I_{(t, s)}^{(t, k)}$ to indicate that if node s is within the transmission range of the transmission from t to k , s can overhear the transmission and consumes energy for the overhearing. Chang et. al. consider only the energy consumption for transmission [7]. Further investigation consider energy consumption for both transmission and reception [20]. We also consider the energy consumption for overhearing.

The total energy consumption of node s for all d_i 's is,

$$\mathbf{energy}_s = \sum_i \mathbf{energy}_s(i).$$

Since $d_T = 0$, we do not sum over $i \neq T$ for brevity.

B. Routing With No Knowledge of Traffic: A Single Sink Case

Similar to Azar et al. [5], the optimal oblivious routing of a multihop wireless network can be obtained by solving an LP with a polynomial number of variables, but infinitely many constraints. We call this LP ‘‘master LP’’:

$$\begin{aligned} & \min r \\ & \mathbf{f} \text{ is a routing} \\ & \forall \text{ nodes } s \neq T, \forall \text{ TMs } \mathbf{tm} \text{ with } \text{OPTE}(\mathbf{tm}) = 1 : \\ & \quad \sum_i \mathbf{energy}_s(i) / \text{pow}_0(s) \leq r \end{aligned} \quad (3)$$

The oblivious ratio is invariant with the scaling of the traffic matrices or the scaling of the initial energy level. Thus, when calculating the oblivious ratio, it is sufficient to consider traffic matrices with $\text{OPTE}(\mathbf{tm}) = 1$. Another benefit of using traffic matrices with $\text{OPTE}(\mathbf{tm}) = 1$ is that the objective of the LP r , which is the maximum energy utilization of the oblivious routing, is just the oblivious ratio of the network.

Given a routing \mathbf{f} , the constraint of the master LP (3) can be checked by solving the following slave LP for each node

$s \neq T$ to examine whether the objective is $\leq r$ or not.

$$\begin{aligned}
& \max \sum_i \text{energy}_s(i)/\text{pow}_0(s) \\
& g_i(u, v) \text{ is a flow of demand } d_i \\
& \forall \text{ nodes } u \neq T : \\
& \quad \sum_i \sum_{v \in \text{out}(u)} \{g_i(u, v)tx(u, v)\} \\
& \quad + \sum_i \sum_{v \in \text{in}(u)} \{g_i(v, u)r(u)\} \\
& \quad + \sum_i \sum_{v \in \text{in}(u)} \sum_{w \in \text{out}(v, -u)} \{I_{(v, u)}^{(v, w)} g_i(v, w)h(u)\} \\
& \quad \leq \text{pow}_0(u) \\
& \text{node capacity equality constraint} \\
& \forall \text{ nodes } i \neq T : d_i \geq 0
\end{aligned} \tag{4}$$

The constraints of LP (4) guarantee that the traffic can be routed with maximum energy utilization of 1. In the capacity constraint of LP (4), the first term on the left hand side is the energy consumption for transmission; the second for reception and the third for overhearing.

In slave LP (4), flow \mathbf{g} is defined as,

$$\left\{ \begin{array}{l}
\forall \text{ nodes } k \neq T, \forall i \neq k \neq T : \\
\quad \sum_{u \in \text{out}(k)} g_i(k, u) - \sum_{v \in \text{in}(k)} g_i(v, k) = 0 \\
\forall \text{ nodes } i \neq T : \sum_{u \in \text{out}(i)} g_i(i, u) - d_i = 0 \\
\forall \text{ edges } (u, v), u \neq T, \forall i \neq T : g_i(u, v) \geq 0 \\
\forall \text{ nodes } i \neq T : d_i \geq 0
\end{array} \right. \tag{5}$$

From above, we can derive the flow reservation constraint, $\forall \text{ nodes } i \neq T : d_i - \sum_{u \in \text{in}(T)} g_i(u, T) = 0$.

We put a ‘‘node capacity equality constraint’’ in LP (4). This constraint requires that, for at least one node, the node capacity inequality constraint takes the equality form. This constraint prevents the case that, at optimality, all the node capacity constraints take inequality form, which violates the condition $\text{OPTE}(\mathbf{tm}) = 1$. We will discuss how to express it as a linear constraint when we develop the dual of LP (4). LP models in Applegate and Cohen [4] do not express it.

Although the above ‘‘master-slave’’ LPs can solve the optimal oblivious routing problem with polynomial time based on the Ellipsoid algorithm [4], [5], it is not practical for large networks [4]. Inspired by the work of Applegate and Cohen [4], we derive simpler LP models to compute the oblivious ratio.

The formulation can be simplified by collapsing flows g_i on an edge $u \rightarrow v$, i.e., using $g(u, v) = \sum_i g_i(u, v)$. Constraints of LP (4) become:

$$\begin{aligned}
& \forall \text{ nodes } i \neq T : \\
& \quad \sum_{v \in \text{in}(i)} g(v, i) - \sum_{u \in \text{out}(i)} g(i, u) + d_i = 0 \\
& \forall \text{ nodes } u \neq T : \\
& \quad \sum_{v \in \text{out}(u)} \{g(u, v)tx(u, v)\} + \sum_{v \in \text{in}(u)} \{g(v, u)r(u)\} \\
& \quad + \sum_{v \in \text{in}(u)} \sum_{w \in \text{out}(v, -u)} \{I_{(v, u)}^{(v, w)} g(v, w)h(u)\} \leq \text{pow}_0(u) \\
& \text{node capacity equality constraint} \\
& \forall \text{ edges } (u, v), u \neq T : g(u, v) \geq 0 \\
& \forall \text{ nodes } i \neq T : d_i \geq 0
\end{aligned}$$

By relaxing the flow conservation constraint from equality to ≤ 0 , we allow for node i to deliver more flow than demanded, which does not affect the maximum energy utilization

of 1. The slave LP for node $s \neq T$ is thus:

$$\begin{aligned}
& \max \frac{1}{\text{pow}_0(s)} \{ \sum_i \sum_{t \in \text{out}(s)} \{d_i f_i(s, t)tx(s, t)\} \\
& \quad + \sum_i \sum_{t \in \text{in}(s)} \{d_i f_i(t, s)r(s)\} \} \\
& \quad + \sum_i \sum_{t \in \text{in}(s)} \sum_{k \in \text{out}(t, -s)} \{I_{(t, s)}^{(t, k)} d_i f_i(t, k)h(s)\} \\
& \forall \text{ nodes } i \neq T : \\
& \quad \sum_{(v, i) \in \text{in}(i)} g(v, i) - \sum_{(i, u) \in \text{out}(i)} g(i, u) + d_i \leq 0 \\
& \forall \text{ nodes } u \neq T : \\
& \quad \sum_{v \in \text{out}(u)} \{g(u, v)tx(u, v)\} + \sum_{v \in \text{in}(u)} \{g(v, u)r(u)\} \\
& \quad + \sum_{v \in \text{in}(u)} \sum_{w \in \text{out}(v, -u)} \{I_{(v, u)}^{(v, w)} g(v, w)h(u)\} \leq \text{pow}_0(u) \\
& \text{node capacity equality constraint} \\
& \forall \text{ edges } (u, v), u \neq T : g(u, v) \geq 0 \\
& \forall \text{ nodes } i \neq T : d_i \geq 0
\end{aligned} \tag{6}$$

The dual of the simplified slave LP (6) (for node $s \neq T$) is:

$$\begin{aligned}
& \min \sum_{u \neq T} \pi_s(u) \text{pow}_0(u) \\
& \forall \text{ nodes } i \neq T : \\
& \quad p_s(i) \geq \frac{1}{\text{pow}_0(s)} \{ \sum_{t \in \text{out}(s)} \{f_i(s, t)tx(s, t)\} \\
& \quad + \sum_{t \in \text{in}(s)} \{f_i(t, s)r(s)\} \\
& \quad + \sum_{t \in \text{in}(s)} \sum_{k \in \text{out}(t, -s)} \{I_{(t, s)}^{(t, k)} f_i(t, k)h(s)\} \} \\
& \forall \text{ edges } (u, v), u \neq T : \\
& \quad tx(u, v)\pi_s(u) + r(v)\pi_s(v) \\
& \quad + \sum_{k \in \text{out}(u, -v)} \{I_{(u, k)}^{(u, v)} h(k)\pi_s(k)\} - p_s(u) + p_s(v) \geq 0 \\
& \sum_{u \neq T} \pi_s(u) > 0 \\
& \forall \text{ nodes } i \neq T : \pi_s(i), p_s(i) \geq 0 \\
& p_s(T) = 0, \pi_s(T) = 0
\end{aligned} \tag{7}$$

The dual variable $p_s(i)$ corresponds to the flow conservation constraint for the demand d_i . Since there is no conservation constraint for the demand d_T , we introduce $p_s(T) = 0$ for convenience. The dual variable $\pi_s(u)$ corresponds to the capacity constraint for node u . Since there is no capacity constraint for node T , we introduce $\pi_s(T) = 0$.

We introduce the constraint $\sum_{u \neq T} \pi_s(u) > 0$ in the dual LP (7) to express the ‘‘node capacity equality constraint’’ in the primal LP (6), i.e. to guarantee that the node capacity inequality constraint takes the equality form for at least one node. According to the complementary slackness theorem [2], we know that if the dual variable corresponding to the capacity constraint in the primal LP is non-zero, the primal constraint is tight. That is, if the dual variable $\pi_s(u)$ is non-zero, the primal constraint for $\text{pow}_0(u)$ is tight (the maximum energy utilization is 1). Thus in dual LP (7), we express the condition $\text{OPTE}(\mathbf{tm}) = 1$ in master LP (3).

According to the LP duality theory [2], the primal LP and its dual LP have the same optimal values if they exist. That is, LP (6) and LP (7) are equivalent. Replacing the constraint in the master LP (3) with LP (7), we obtain a polynomial size LP to compute the optimal oblivious ratio when there is a single sink. It has $O(n^2 + nm)$ variables and $O(n^2 + nm)$ constraints, where n and m are the numbers of nodes and edges.

min r

\mathbf{f} is a routing

\forall nodes $s \neq T$:

$$\sum_{u \neq T} \pi_s(u) pow_0(u) \leq r$$

\forall nodes $i \neq T$:

$$p_s(i) \geq \frac{1}{pow_0(s)} \left\{ \sum_{t \in out(s)} \{f_i(s, t) tx(s, t)\} + \sum_{t \in in(s)} \{f_i(t, s) r(s)\} + \sum_{t \in in(s)} \sum_{k \in out(t, -s)} \{I_{(t, s)}^{(t, k)} f_i(t, k) h(s)\} \right\}$$

\forall edges $(u, v), u \neq T$:

$$tx(u, v) \pi_s(u) + r(v) \pi_s(v) + \sum_{k \in out(u, -v)} \{I_{(u, k)}^{(u, v)} h(k) \pi_s(k)\} - p_s(u) + p_s(v) \geq 0$$

$$\sum_{u \neq T} \pi_s(u) > 0$$

\forall nodes $i \neq T$: $\pi_s(i), p_s(i) \geq 0$

$$p_s(T) = 0, \pi_s(T) = 0$$

(8)

C. Routing with Approximate Knowledge of Traffic: A Single Sink Case

In this section, we derive an LP model to compute the optimal oblivious routing² for a multihop wireless network with a single sink when approximate knowledge of traffic pattern is known, in the form that the traffic demand d_i is within the range of $[a_i, b_i]$ ($0 \leq a_i \leq b_i$).

Without the restriction $\text{OPTE}(\mathbf{tm}) = 1$, LP (3) becomes:

min r

\mathbf{f} is a routing

$\forall \alpha > 0, \forall$ nodes $s \neq T$:

\forall TMs \mathbf{tm} with $\text{OPTE}(\mathbf{tm}) = \alpha$, and $\forall i \neq T$ $a_i \leq d_i \leq b_i$:

$$\sum_i \text{energy}_s(i) / pow_0(s) \leq r \alpha$$

(9)

Since the oblivious ratio r is invariant with respect to the scaling of traffic demands, we can consider a scaled TM $\mathbf{tm}' = \alpha \cdot \mathbf{tm}$. With $\alpha = 1/\text{OPTE}(\mathbf{tm})$, we have $\text{OPTE}(\mathbf{tm}') = 1$. Under these conditions, the master LP (9) becomes:

min r

\mathbf{f} is a routing

\forall nodes $s \neq T, \forall$ TMs \mathbf{tm} with $\lambda > 0$ such that (10)

$\text{OPTE}(\mathbf{tm}) = 1$ and $\forall i \neq T$ $\lambda a_i \leq d_i \leq \lambda b_i$:

$$\sum_i \text{energy}_s(i) / pow_0(s) \leq r$$

As in §III-B, we can derive the constraints that traffic matrices can be routed with maximum energy utilization of 1. We need to add the constraint $\lambda a_i \leq d_i \leq \lambda b_i$, when we know the range restriction of the traffic matrix. We also use $g(u, v) = \sum_i g_i(u, v)$ to simplify the formulation as before.

²With a slight misuse of terms, we also call the routing that minimizes the competitive ratio over the range restriction on the traffic matrix the ‘‘optimal oblivious routing’’, and its competitive ratio the ‘‘optimal oblivious ratio’’.

\forall nodes $i \neq T$:

$$\sum_{v \in in(i)} g(v, i) - \sum_{u \in out(i)} g(i, u) + d_i \leq 0$$

\forall nodes $u \neq T$:

$$\sum_{v \in out(u)} \{g(u, v) tx(u, v)\} + \sum_{v \in in(u)} \{g(v, u) r(u)\} + \sum_{v \in in(u)} \sum_{w \in out(v, -u)} \{I_{(v, u)}^{(v, w)} g(v, w) h(u)\} \leq pow_0(u)$$

node capacity equality constraint

\forall nodes $i \neq T$: $0 \leq \lambda a_i \leq d_i \leq \lambda b_i$

\forall nodes $i \neq T$: $d_i \geq 0$

$\lambda > 0$

The slave LP for node $s \neq T$ with range restriction is thus:

$$\begin{aligned} \max \quad & \frac{1}{pow_0(s)} \left\{ \sum_i \sum_{t \in out(s)} \{d_i f_i(s, t) tx(s, t)\} + \sum_i \sum_{t \in in(s)} \{d_i f_i(t, s) r(s)\} + \sum_i \sum_{t \in in(s)} \sum_{k \in out(t, -s)} \{I_{(t, s)}^{(t, k)} d_i f_i(t, k) h(s)\} \right\} \\ \forall \text{ nodes } i \neq T : & \Leftrightarrow p_s(i) \\ & \sum_{(v, i) \in in(i)} g(v, i) - \sum_{u \in out(i)} g(i, u) + d_i \leq 0 \\ \forall \text{ nodes } u \neq T : & \Leftrightarrow \pi_s(u) \\ & \sum_{v \in out(u)} \{g(u, v) tx(u, v)\} + \sum_{v \in in(u)} \{g(v, u) r(u)\} + \sum_{v \in in(u)} \sum_{w \in out(v, -u)} \{I_{(v, u)}^{(v, w)} g(v, w) h(u)\} \\ & \leq pow_0(u) \end{aligned}$$

node capacity equality constraint

\forall nodes $i \neq T$: $d_i - \lambda b_i \leq 0 \Leftrightarrow w_s^+(i)$

\forall nodes $i \neq T$: $-d_i + \lambda a_i \leq 0 \Leftrightarrow w_s^-(i)$

\forall edges (u, v) : $g(u, v) \geq 0$

\forall nodes $i \neq T$: $d_i \geq 0$

$\lambda \geq 0$

(11)

The dual of slave LP (11) for node s is:

$$\begin{aligned} \min \quad & \sum_{u \neq T} \pi_s(u) pow_0(u) \\ \forall \text{ nodes } i \neq T : & \Leftrightarrow d_i \\ & p_s(i) + w_s^+(i) - w_s^-(i) \geq \\ & \frac{1}{pow_0(s)} \left\{ \sum_{t \in out(s)} \{f_i(s, t) tx(s, t)\} + \sum_{t \in in(s)} \{f_i(t, s) r(s)\} + \sum_{t \in in(s)} \sum_{k \in out(t, -s)} \{I_{(t, s)}^{(t, k)} f_i(t, k) h(s)\} \right\} \\ \forall \text{ edges } (u, v), u \neq T : & \Leftrightarrow g(u, v) \\ & tx(u, v) \pi_s(u) + r(v) \pi_s(v) \\ & + \sum_{k \in out(u, -v)} \{I_{(u, k)}^{(u, v)} h(k) \pi_s(k)\} - p_s(u) + p_s(v) \geq 0 \\ \sum_{u \neq T} \pi_s(u) & > 0 \\ \forall \text{ nodes } i \neq T : & \sum_i \{w_s^-(i) a_i - w_s^+(i) b_i\} \geq 0 \Leftrightarrow \lambda \\ \forall \text{ nodes } i \neq T : & \pi_s(i), p_s(i), w_s^+(i), w_s^-(i) \geq 0 \\ p_s(T) = 0, \pi_s(T) & = 0 \end{aligned}$$

(12)

The dual variable $p_s(i)$ corresponds to the flow conservation constraint for the node i . The dual variable $\pi_s(u)$ corresponds to the capacity constraint for node u . The dual variables $w_s^+(i)$ and $w_s^-(i)$ are associated with the range constraints $d_i - \lambda b_i \leq 0$ and $-d_i + \lambda a_i \leq 0$ respectively. To help make the derivation of the dual LP (12) clearer, we use leftarrow ‘‘ \Leftarrow ’’ to indicate dual variables corresponding with primal constraints in LP (11). In dual LP (12), we indicate primal variables corresponding to dual constraints.

Therefore, when we have approximate knowledge of the traffic pattern in the form that the traffic demand d_i is within the range $[a_i, b_i]$, we can compute the optimal oblivious

ratio of a multihop wireless network with a single sink by a polynomial size LP. It has $O(n^2 + nm)$ variables and $O(n^2 + nm)$ constraints. The LP follows:

$$\begin{aligned}
& \min r \\
& \mathbf{f} \text{ is a routing} \\
& \forall \text{ nodes } s \neq T : \\
& \quad \sum_{u \neq T} \pi_s(u) \text{pow}_0(u) \leq r \\
& \quad \forall \text{ nodes } i \neq T : \\
& \quad \quad p_s(i) + w_s^+(i) - w_s^-(i) \geq \\
& \quad \quad \frac{1}{\text{pow}_0(s)} \left\{ \sum_{t \in \text{out}(s)} \{f_i(s, t) tx(s, t)\} \right. \\
& \quad \quad \left. + \sum_{t \in \text{in}(s)} \{f_i(t, s) r(s)\} \right. \\
& \quad \quad \left. + \sum_{t \in \text{in}(s)} \sum_{k \in \text{out}(t, -s)} \{I_{(t, s)}^{(t, k)} f_i(t, k) h(s)\} \right\} \\
& \quad \forall \text{ edges } (u, v), u \neq T : \\
& \quad \quad tx(u, v) \pi_s(u) + r(v) \pi_s(v) \\
& \quad \quad + \sum_{k \in \text{out}(u, -v)} I_{(u, k)}^{(u, v)} h(k) \pi_s(k) - p_s(u) + p_s(v) \geq 0 \\
& \quad \sum_{u \neq T} \pi_s(u) > 0 \\
& \quad \forall \text{ nodes } i \neq T : \sum_i \{w_s^-(i) a_i - w_s^+(i) b_i\} \geq 0 \\
& \quad \forall \text{ nodes } i \neq T : \pi_s(i), p_s(i), w_s^+(i), w_s^-(i) \geq 0 \\
& \quad p_s(T) = 0, \pi_s(T) = 0
\end{aligned} \tag{13}$$

D. All Pair Case

The multihop wireless network with a single sink is a special case of communication over multihop wireless networks, where there may be traffic between all pairs of nodes. When all pairs of nodes may have traffic, an entry d_{ij} in a traffic matrix denotes the amount of traffic of OD pair $i \rightarrow j$. Usually no node is assumed to have infinite energy capacity.

Similar to a single sink case, a routing $f_{ij}(s, t)$ specifies the fraction of traffic demand d_{ij} on edge (s, t) . The traffic on edge (s, t) for d_{ij} is $d_{ij} f_{ij}(s, t)$. Routing \mathbf{f} is defined as:

$$\left\{ \begin{array}{l} \forall \text{ pairs } i \rightarrow j : \sum_{u \in \text{out}(i)} f_{ij}(i, u) = 1 \\ \forall \text{ pairs } i \rightarrow j, \forall \text{ nodes } k \neq i, j : \\ \quad \sum_{u \in \text{out}(k)} f_{ij}(k, u) - \sum_{v \in \text{in}(k)} f_{ij}(v, k) = 0 \\ \forall \text{ pairs } i \rightarrow j, \forall \text{ edges } (s, t) : f_{ij}(s, t) \geq 0 \end{array} \right. \tag{14}$$

From the above, we can derive the conservation constraint, $\forall \text{ pairs } i \rightarrow j : \sum_{u \in \text{in}(j)} f_{ij}(u, j) = 1$.

$$\begin{aligned}
& \text{The energy consumption of node } s \text{ for } d_{ij} \text{ is,} \\
& \text{energy}_s(i, j) = \sum_{t \in \text{out}(s)} \{d_{ij} f_{ij}(s, t) tx(s, t)\} \\
& \quad + \sum_{t \in \text{in}(s)} \{d_{ij} f_{ij}(t, s) r(s)\} \\
& \quad + \sum_{t \in \text{in}(s)} \sum_{k \in \text{out}(t, -s)} \{I_{(t, s)}^{(t, k)} d_{ij} f_{ij}(t, k) h(s)\}
\end{aligned}$$

The total energy consumption for node s is,

$$\text{energy}_s = \sum_{i, j} \text{energy}_s(i, j).$$

Flow \mathbf{g} is defined as,

$$\left\{ \begin{array}{l} \forall \text{ pairs } i \rightarrow j, k \neq i, j : \\ \quad \sum_{u \in \text{out}(k)} g_{ij}(k, u) - \sum_{v \in \text{in}(k)} g_{ij}(v, k) = 0 \\ \forall \text{ pairs } i \rightarrow j : \sum_{t \in \text{in}(j)} g_{ij}(t, j) - d_{ij} = 0 \\ \forall \text{ pairs } i \rightarrow j, \forall \text{ edges } (u, v) : g_{ij}(u, v) \geq 0 \\ \forall \text{ pairs } i \rightarrow j : d_{ij} \geq 0 \end{array} \right.$$

From above, we can derive the flow reservation constraint,

$$\forall \text{ pairs } i \rightarrow j : d_{ij} = \sum_{u \in \text{out}(i)} g_{ij}(i, u).$$

Similar techniques in §III-B and III-C for the formulation of LP models can be used here, e.g. we can collapse flows g_{ij} on an edge $u \rightarrow v$ with the same origin by $g_i(u, v) = \sum_j g_{ij}(u, v)$ to simplify the LP formulation. In LP (15), we directly give the LP model to compute the optimal oblivious ratio for a multihop wireless network, when we know approximate knowledge of the traffic pattern that d_{ij} is within the range of $[a_{ij}, b_{ij}]$. When we have no knowledge of the traffic pattern, i.e., the range is $[0, +\infty]$, the LP to compute the oblivious ratio can be obtained by removing the constraints, $\forall \text{ nodes } s, \forall \text{ nodes } i, j \neq i : \sum_i \{w_s^-(i, j) a_{i, j} - w_s^+(i, j) b_{i, j}\} \geq 0$, and the variables $w_s^+(i, j)$ and $w_s^-(i, j)$ for the range restrictions.

$$\begin{aligned}
& \min r \\
& \mathbf{f} \text{ is a routing} \\
& \forall \text{ nodes } s : \\
& \quad \sum_u \pi_s(u) \text{pow}_0(u) \leq r \\
& \quad \forall \text{ nodes } i, j \neq i : \\
& \quad \quad p_s(i, j) + w_s^+(i, j) - w_s^-(i, j) \geq \\
& \quad \quad \frac{1}{\text{pow}_0(s)} \left\{ \sum_{t \in \text{out}(s)} \{f_{ij}(s, t) tx(s, t)\} \right. \\
& \quad \quad \left. + \sum_{t \in \text{in}(s)} \{f_{ij}(t, s) r(s)\} \right. \\
& \quad \quad \left. + \sum_{t \in \text{in}(s)} \sum_{u \in \text{out}(t, -s)} \{I_{(t, s)}^{(t, u)} f_{ij}(t, u) h(s)\} \right\} \\
& \quad \forall \text{ nodes } i, \forall \text{ edges } (u, v) : \\
& \quad \quad tx(u, v) \pi_s(u) + r(v) \pi_s(v) \\
& \quad \quad + \sum_{k \in \text{out}(u, -v)} \{I_{(u, k)}^{(u, v)} h(k) \pi_s(k)\} \\
& \quad \quad - p_s(u, i) + p_s(v, i) \geq 0 \\
& \quad \sum_u \pi_s(u) > 0 \\
& \quad \forall \text{ nodes } i, j \neq i : \sum_i \{w_s^-(i, j) a_{ij} - w_s^+(i, j) b_{ij}\} \geq 0 \\
& \quad \forall \text{ nodes } u : \pi_s(u) \geq 0 \\
& \quad \forall \text{ nodes } i, j \neq i : p_s(i, j), w_s^+(i, j), w_s^-(i, j) \geq 0 \\
& \quad \forall \text{ nodes } i : p_s(i, i) = 0
\end{aligned} \tag{15}$$

Theorem 1. *The optimal oblivious ratio and the optimal oblivious routing of a multihop wireless network can be computed by a polynomial size linear program with $O(n^3 + n^2m)$ variables and $O(n^3 + n^2m)$ constraints, where n and m are the numbers of nodes and edges.*

We can prove Theorem 1 when we develop the LP model, by counting the number of variables and constraints. A multihop wireless network with a single sink is a special case. Its optimal oblivious ratio and optimal oblivious routing can be computed by a polynomial size LP with $O(n^2 + nm)$ variables and $O(n^2 + nm)$ constraints by LP (8) or LP (13). The n -fold reduction in complexity is due to the n -fold reduction in the number of OD pairs, since there is a single destination.

E. Generalization

Energy Consumption Model. The energy consumption model energy_s (whose detailed definition is developed in §III-A or §III-D) is general enough to take into account several issues in radio transmission. By properly defining the indicator function, we can handle the case in which a node can vary its transmission range with arbitrary precision, at several discrete levels, or with a fixed transmission range. Note a node may

transmit on different links with different power; but the power on a link is constant.

For example, assume a disk model for radio transmission, i.e., the maximum transmission range is the same in different directions. Also assume omni-directional transmission. When a node can vary its transmission range with arbitrary precision, the indicator function is defined as,

$$I_{(t,s)}^{(t,k)} = \begin{cases} 1 & \text{if } dist(t,k) \geq dist(t,s); \\ 0 & \text{otherwise.} \end{cases}$$

When there are n_t transmission ranges for a wireless node t , r_1, r_2, \dots, r_{n_t} , with $r_1 < r_2 < \dots < r_{n_t}$, we need to replace $dist(t,k)$ with r_i , where $r_i \geq dist(t,k)$, and if $i \neq 1$, $r_{i-1} < dist(t,k)$. That is, we replace $dist(t,k)$ with the smallest transmission range which is $\geq dist(t,k)$. We have,

$$I_{(t,s)}^{(t,k)} = \begin{cases} 1 & \text{if } r_i \geq dist(t,s); \\ 0 & \text{otherwise.} \end{cases}$$

In the case where the transmission range for node t is fixed, denoted as $TxR(t)$, we can obtain the proper indicator function by replacing r_i with $TxR(t)$.

As well, we can handle the radio irregularity problem studied recently, e.g. in [22], that a radio has different maximum transmission ranges in different directions. This affects the neighborhood relationship. The energy model can be used for the wireless communication using either omni-directional antenna or smart antenna. For smart antenna with directional communication, the model for transmission $tx(s,t)$ and reception $r(s)$ need to include a component to account for the energy consumption for managing the antenna. With smart antennae, the indicator function will be affected, which will affect the energy consumption for overhearing.

Multi-channel and multi-radio. In a wireless network with multi-channel and/or multi-radio, there will be multiple edges between a pair of nodes in a graph representation. The LP models still work on this multigraph. The channels operated by a radio usually interfere with each other. But there is no interference among channels operated by multiple radios orthogonal with each other. The work to handle interference in Kodialam and Nandagopal [12] and Jain et al. [10] are applicable here. Interference issues will be further discussed in §III-F. The LP models is thus extensible to a multi-input multi-output (MIMO) system.

F. Schedulability

In the following, we discuss schedulability of a routing in two scenarios, “interference-free” and interference-limited.

Interference-free. When the traffic load is relatively low compared with the bandwidth, the interference is less severe or negligible. With the emerging ultra-wideband (UWB) radio technology, a wireless node has a wide bandwidth, which may result in a large number of orthogonal channels between a pair of nodes, e.g. by MIMO; at the same time, the power is limited, thus the traffic load is low compared to the large bandwidth. Consequently, concurrent transmissions may be arranged in a manner such that there is negligible interference.

This “interference-free” feature of UWB is exploited, e.g. in [17]. In some applications, such as some sensor networks, traffic may be sporadic. Interference will not be a major concern in such cases. Note that in a network with low traffic load, certain load/energy balancing technique like ours is still needed for energy efficiency. In interference-free scenarios, our LP model can work well without considering scheduling.

Interference-limited. In an interference-limited wireless network, it is necessary to consider schedulability of a routing. To optimize a schedulable routing is essentially to solve an optimization problem at the intersection of the feasible flow space and the feasible scheduling space. The problem of scheduling with interference is NP-hard itself. Thus researchers seek necessary and sufficient condition for a flow to be schedulable. In the problem to maximize network flow (throughput), researchers look for lower and upper bounds for the schedulable flow. We discuss how to guarantee schedulability of an oblivious routing.

The free of secondary interference model receives considerable attention. Hajek and Sasaki [8] investigate the schedulability of a routing in polynomial time. Kodialam and Nandagopal [12] give necessary and sufficient conditions implicitly in [8]. These conditions are expressed as linear constraints over the flows and data rate on neighboring edges of a node. For the case of multihop wireless networks with a single sink, in terms of our notation, the necessary and sufficient conditions can be expressed as follows for each node s when β takes the values of 1 and $\frac{2}{3}$ respectively:

$$\sum_{t \in out(s)} \frac{g(s,t)}{c(s,t)} + \sum_{t \in in(s)} \frac{g(t,s)}{c(t,s)} \leq \beta. \quad (16)$$

Here $c(s,t)$ denotes the data rate the edge (s,t) can support. These conditions for the case of multihop wireless network can be expressed similarly. To guarantee schedulability, we use the sufficient condition, i.e., by setting $\beta = \frac{2}{3}$.

Our traffic-oblivious energy-aware routing scheme enjoys the feature that it can incorporate more linear constraints, e.g., to make the routing schedulable. For the case of multihop wireless network with a single sink, we can add constraints (16) to a slave LP, e.g., LP (6) to guarantee the flow schedulability. As well, we can add linear constraints based on Jain et al. [10] or exploit new progress to guarantee schedulability of the flow. See [14] for more details.

G. Lossy Links

When developing the LP models, we implicitly assume the wireless links are lossless. This assumption is made (implicitly) in most previous work that are based on a LP model, e.g., [7], [13], [20]. A wireless link is usually lossy and some applications need reliable transmission. As a consequence, a packet may take several transmissions. Thus modifications need to be made to the usual flow conservation constraints, by considering some link loss factor, which measures the average number of transmissions to successfully transmit a packet on the link. If there is a *constant* link loss factor $\gamma_{ij} \geq 1$ for each edge (i,j) , we have linear flow conservation constraints.

Taking a single sink case as an example, the definition of flow \mathbf{g} in (5) becomes:

$$\left\{ \begin{array}{l} \forall \text{ nodes } k \neq T, \forall i \neq k \neq T : \\ \quad \sum_{u \in \text{out}(k)} \frac{g_i(k,u)}{\gamma_{ku}} - \sum_{v \in \text{in}(k)} \frac{g_i(v,k)}{\gamma_{vk}} = 0 \\ \forall \text{ nodes } i \neq T : \sum_{u \in \text{out}(i)} \frac{g_i(i,u)}{\gamma_{iu}} - d_i = 0 \\ \forall \text{ edges } (u,v), u \neq T, \forall i \neq T : g_i(u,v) \geq 0 \\ \forall \text{ nodes } i \neq T : d_i \geq 0 \end{array} \right. \quad (17)$$

In (17), $g_i(u,v)$ denotes the actual flow originating from node i on edge (u,v) (due to retransmissions); while $\frac{g_i(u,v)}{\gamma_{uv}}$ denotes the effective flow. The routing definition (2) remains the same. The energy consumption model needs to change as well. We need to multiply γ_{st} with the term $d_i f_i(s,t)tx(s,t)$ for energy consumption for transmission to reflect multiple transmissions. A term for energy consumption for reception remains the same, since there is only one successful reception. The case for overhearing may be more complex. The number of transmissions overheard may be related to the two γ 's of the transmission link and the overhearing link.

Our LP models in §III-A through §III-D can be regarded as an optimistic treatment of a lossy environment, by taking all γ 's as 1. LP formulations in [14] take lossy links into account.

H. Implementation Issues

As the performance study will show in §IV, the traffic-oblivious energy-aware routing has excellent theoretical results, i.e., the oblivious ratios are close to 1.0. In the following, we discuss several issues in a potential implementation of the routing scheme.

For a stationary network, our tools only need to collect information of topology and initial energy level once. We need information about node positions and their connectivity. It is possible to construct the graph of the network with good links, using the techniques in Woo et al. [22] to estimate link quality. The criteria for the goodness of link quality is that its average quality is good and relatively stable. After that, we take a centralized way to compute the optimal oblivious routing. We need a round of message exchanges to implement the routing, so that each node knows, for each OD pair, what fraction of traffic to transmit to which neighbor. Once the routing is implemented, it does not need to collect global network information any more. With only two rounds of message exchanges, our approach has a low message complexity. Once the routing is implemented, it is fully distributed. In contrast, the distributed algorithms in [7], [11], [20] and the hierarchical algorithm in [13] need ongoing collection of network information such as the remaining energy level.

The routing computed by our LP model can be implemented in an opportunistic manner, i.e., each node transmits data packets opportunistically to its neighbors according to the fraction specified by the routing. Such an implementation has the potential to combat the fluctuating channel condition in practice. This is achieved by monitoring the outgoing links and choosing the one with good quality at the time of transmission. Recently there are experimental results on

link quality estimation, e.g. Woo et al. [22], and interference detection, e.g. Zhou et al. [24]. We can exploit their techniques to estimate the variation of link quality caused by temporary link failures and interference. The estimate of link quality and the routing fraction determine which link to transmit a packet. This is amenable to a distributed implementation, in which each node only needs to monitor the quality of the neighboring links. Once the fraction of traffic load on each link is satisfied, the energy efficiency is accomplished. In this way, our “single fixed” routing makes “rerouting” transparent. It is desirable that the routing fractions are satisfied in a relatively short time interval, so there will be a tradeoff between using links with good quality and satisfying the routing fractions. The opportunistic implementations of the oblivious routing could help alleviate the impact of channel quality fluctuation.

In an energy-constrained multihop wireless network, when one or several nodes have used up energy, they are disconnected from the network. The network may still be working for a while. Reoptimization of the routing may be needed. A similar problem, how to optimize the “oblivious restoration” when one or several nodes fail in the scenario of the Internet, is studied in [3]. We may use similar techniques to obtain an optimal oblivious restoration. However, collecting remaining energy capacities consumes energy. Thus, further investigation is needed to justify the benefit of routing reoptimization. A simple approach is, for a node s having flow to the failed node, to bypass the failed node by assigning additional fraction of flow to the other downstream nodes of node s . The adjustment of routing fractions for downstream nodes is determined by the original routing fractions, in an attempt to balance the load. For example, suppose in the single sink case, for origin i , node s has routing fractions $f_i(s,u)$, $f_i(s,v)$ and $f_i(s,w)$ to nodes u , v and w , respectively. When node u fails, node s adjusts routing fractions to v and w as $\alpha(v)f_i(s,v)$ and $\alpha(w)f_i(s,w)$, where $\alpha(x) = 1 + f_i(s,x)/\{f_i(s,v) + f_i(s,w)\}$. This simple approach of detouring around the failed node is amenable to a distributed implementation. It is worth further investigating how to handle failure scenarios.

IV. PERFORMANCE STUDY

We study the performance of the LP models developed in §III-A through §III-D on multihop wireless networks where energy is a constraint and non-renewable. We also give results when it is interference-limited and links are lossy in §IV-C.

We use random topologies. We put nodes on a $k \times k$ grid, each cell of which represents a $10m \times 10m$ area. In each cell of the grid, we put a node at a random position. The initial energy level of each node is set randomly, uniformly within $[20J, 30J]$ (note the oblivious ratio is invariant with the scaling of the initial energy level). For brevity, we use a disk model for radio transmission. That is, suppose the maximum transmission range of node u is R_{max} , there is an edge (u,v) if $R_{max} \geq \text{dist}(u,v)$, where $\text{dist}(u,v)$ denotes the distance between u and v . In the simulations, every node has the same maximum transmission range. We conduct experiments on networks of various sizes. For each size of the network, we

study two maximum transmission ranges, 15m and 20m. We use CPLEX [1] to solve the LP programs.

We use the energy model in [9], i.e., we set $tx(u, v) = E_{elec} + \epsilon_{amp} \times dist^2(u, v)$ and $r(u) = E_{elec}$, where E_{elec} represents the energy consumption for running the transmitter or the receiver circuitry, ϵ_{amp} represents the energy consumption for running the transmitter amplifier to achieve an acceptable signal-noise ratio. We set $h(u) = r(u)$, i.e., we assume that the overhearing consumes the same amount of energy per unit of message as the reception. As in [9], we set $E_{elec} = 50nJ/bit$ and $\epsilon_{amp} = 100pJ/bit/m^2$.

A. A Single Sink Case

We first study a single sink case. We conduct experiments on networks of sizes 25, 36, 49, 81, 100 and 121. We choose the node either in the center or in the corner cell as the sink.

The oblivious ratio of a multihop wireless network is computed by LP (8). The oblivious ratios of the studied networks are shown in the last column in Table I under ∞ (which means we have no knowledge of the traffic pattern, as will be clear later in this section). We have encouraging results, considering the oblivious ratios are achieved without any knowledge of the traffic pattern, and only an oracle can achieve the ratio of 1.

It is expected that with some knowledge of traffic demands, we can achieve lower competitive ratios. In the following we study the performance of LP (13) if we know the degree of accuracy of the traffic estimation, for a topology, a traffic matrix \mathbf{tm} and an “error margin” $\epsilon > 1$. We will study the oblivious ratio of a network, given the knowledge of traffic demand in the range of $[d_i/\epsilon, \epsilon d_i]$, with respect to the base traffic matrix d_i 's. First, we need to decide the d_i 's.

We may have some rough estimation of the traffic pattern in a multihop wireless network. We use four traffic models to determine the base traffic matrix \mathbf{tm} : Gravity, Bimodal, Random and Uniform, to attempt to capture some broad classes of traffic patterns in multihop wireless networks. They are denoted as G, B, R and U respectively in the tables. In the Gravity model, the amount of traffic originating from node i , d_i , is proportional to $pow_0(i)$, the initial energy level of node i . In the Bimodal, a small portion of nodes have a large amount of traffic, while a large number of nodes have small amount of traffic. In our study, 80% of the nodes have traffic demands determined by a normal distribution $N(1.0, 0.1)$; while traffic demands of 20% of the nodes are determined by $N(10.0, 1.0)$. $N(\mu, \sigma^2)$ denotes a normal distribution with mean μ and variance σ^2 . Random model is self-explanatory. In our study, we use a uniform distribution on the range $[1, 100]$. In a Uniform model, all the nodes have the same amount of traffic. The Gravity and the Bimodal traffic models are inspired by the study on the Internet traffic estimation in [23] and [6] respectively, which may reflect the technical expectation to and the social phenomenon of a network (the Internet). In a wireless network, we may imagine that a node with high energy capacity may tend to transmit more data. It is possible that, in some applications, some “hotspot” area may have much more data to transmit.

N	R_{max}	TM	1.5	2.0	3.0	∞
49	15m	U	1.2264	1.3892	1.5808	1.8239
		G	1.2256	1.3879	1.5810	
		B	1.2612	1.4430	1.5831	
		R	1.2177	1.3773	1.5815	
	20m	U	1.2852	1.4884	1.7138	1.9651
		G	1.2846	1.4875	1.7163	
		B	1.3018	1.5169	1.6943	
		R	1.2954	1.4955	1.6772	
81	15m	U	1.1441	1.2725	1.5154	1.9964
		G	1.1439	1.2727	1.5169	
		B	1.1406	1.2824	1.4966	
		R	1.1377	1.2681	1.5208	
	20m	U	1.1605	1.3086	1.5650	2.0242
		G	1.1600	1.3080	1.5663	
		B	1.1883	1.3493	1.5819	
		R	1.1576	1.3037	1.5544	
100	15m	U	1.0673	1.1442	1.3221	1.9065
		G	1.0669	1.1429	1.3195	
		B	1.0887	1.1851	1.3901	
		R	1.0556	1.1273	1.3037	
	20m	U	1.0920	1.1834	1.3761	1.9752
		G	1.0918	1.1825	1.3743	
		B	1.1219	1.2360	1.4676	
		R	1.0995	1.1923	1.3872	
121	15m	U	1.0604	1.1268	1.3012	1.9071
		G	1.0609	1.1268	1.3007	
		B	1.0780	1.1529	1.3373	
		R	1.0573	1.1305	1.3166	
	20m	U	1.0883	1.1886	1.4129	2.0751
		G	1.0875	1.1874	1.4113	
		B	1.0784	1.2016	1.4803	
		R	1.0960	1.2007	1.4263	

TABLE I
OBLIVIOUS RATIOS: SINGLE SINK IN THE CENTER

Tables I shows the results for the sink in the center with the error margin ϵ of 1.5, 2.0 and 3.0, for the four base traffic models respectively. For each network size N , each maximum transmission range R_{max} , the oblivious ratios for the four base traffic models are on the same topology. We can see that LP (13) can achieve fairly low oblivious ratios with large error margins. Note that, with 50% error in traffic estimation, the performance is close to the optimal (the oblivious ratio is close to 1.0). To save space, we do not present results for networks of sizes 25 and 36. They have similarly low oblivious ratios.

N	R_{max}		1.5	2.0	3.0	∞	
49	15m	min	1.000+	1.000+	1.000+	1.000+	
		max	1.2329	1.3946	1.6091	1.9340	
	20m	min	1.0100	1.0203	1.0438	1.1800	
		max	1.3771	1.5880	1.7750	1.9767	
	81	15m	min	1.000+	1.000+	1.000+	1.0353
			max	1.0945	1.2125	1.4442	1.9652
20m		min	1.0044	1.0090	1.0224	1.1872	
		max	1.1706	1.3402	1.6354	2.0770	

TABLE II
MIN AND MAX OBLIVIOUS RATIOS OVER 9 RUNS
A SINGLE SINK IN THE CORNER (UNIFORM BASE TM)

When the sink is in the corner, the oblivious ratio can be much lower. We conduct experiments with 9 seeds for the random number generator, which may change the locations of

N	R_{max}	TM	1.5	2.0	3.0	∞
49	75m	G	1.2985	1.4891	1.7232	2.1356
		R	1.3110	1.5294	1.7328	
	100m	G	1.3128	1.5465	1.8378	2.2395
		R	1.3405	1.5877	1.8332	
81	75m	G	1.1102	1.2111	1.4423	2.1934
		R	1.0987	1.1957	1.4251	
	100m	G	1.1580	1.3206	1.6335	2.2678
		R	1.1615	1.3331	1.6350	
100	75m	G	1.0419	1.0886	1.1973	1.9786
		R	1.0427	1.0840	1.2036	
	100m	G	1.0720	1.1539	1.3308	2.1054
		R	1.0659	1.1404	1.3253	
121	75m	G	1.0372	1.0713	1.1648	1.9499
		R	1.0317	1.0605	1.1899	
	100m	G	1.0764	1.1545	1.3463	2.3008
		R	1.0572	1.1310	1.3324	

TABLE III

OBVIOUS RATIOS: A SINGLE SINK IN THE CENTER, TRANSMISSION DOMINATES ENERGY CONSUMPTION

the nodes (thus the graph), the initial energy level and the base traffic matrix (for Bimodal and Random model). In Table II, we show the min and max of the oblivious ratios over 9 seeds for Uniform model for the case the sink is in the corner for 49 nodes and 81 nodes. The results of 1.000+ represent those slightly greater than 1.0.

The energy consumption for reception and overhearing may be insignificant in some cases such as long-range transmission. We attempt to study how our LP models perform under such circumstances. We still use a $k \times k$ grid. However, each cell of the grid represents a $50m \times 50m$ area. we study two maximum transmission ranges, 75m and 100m. Recall we set $tx(u, v) = E_{elec} + \epsilon_{amp} \times dist^2(u, v)$ and $r(u) = h(u) = E_{elec}$. Thus the distance plays an important role in energy consumption. It seems that our LP models perform similarly over the four base traffic models. In this set of experiments, we use the Gravity model and Random model to determine the base traffic matrix when we have approximate knowledge of the traffic pattern. Experimental results in Table III shows that when the energy consumption for reception and overhearing is less significant, our LP models can still achieve low oblivious ratios, especially when we have some weak knowledge of the traffic pattern.

We also conduct experiments using the energy model in [13], where $tx(u, v) = 0.0001 \times dist^3(u, v)$. Since the reception and overhearing are not considered in [13], we set $h(u) = r(u) = 0$. We obtain similarly low oblivious ratios.

B. All Pair Case

Next we study the all pair case. We study topologies of sizes 25 and 36. The oblivious ratios of the studied networks are shown in the last column in Table IV (denoted by the error margin ∞). These results are encouraging, since they are achieved without any knowledge of the traffic pattern and without the ongoing network information collection.

Similar to §IV-A, we use four traffic models to determine the base TM **tm**, when we have approximate knowledge of the traffic pattern. In the Gravity model, the amount of traffic

of the OD pair d_{ij} is proportional to $pow_0(i) \times pow_0(j)$. In the Bimodal, 20% of the pairs have traffic demands determined by $N(10.0, 1.0)$, while 80% of the pairs determined by $N(1.0, 0.1)$. We use a uniform distribution on $[1, 100]$ for the Random model. In the Uniform model, all OD pairs have the same amount of traffic.

With rough knowledge of the traffic pattern, the competitive ratios can be much lower than that without any knowledge of traffic. With error margin $\epsilon = 1.5$, we can achieve an oblivious routing that is at most 33.5%–46.5% worse than the oracle optimal routing. We do not intend to claim that for all the topologies we can achieve a competitive ratio within this range. It may be lower or higher. The competitive ratio depends on the topology and the relative energy capacity levels. If the oblivious ratio is acceptably low, the oblivious routing is a competitive option for optimizing energy efficiency.

N	R_{max}	TM	1.5	2.0	3.0	∞
25	15m	U	1.3351	1.4928	1.6532	2.1671
		G	1.3346	1.4924	1.6532	
		B	1.3357	1.4925	1.6516	
		R	1.3318	1.4882	1.6448	
	20m	U	1.3732	1.5451	1.7113	2.2237
		G	1.3730	1.5449	1.7110	
36	15m	U	1.4287	1.6151	1.8094	2.4054
		G	1.4288	1.6151	1.8097	
		B	1.4277	1.6143	1.8019	
		R	1.4237	1.6134	1.8085	
	20m	U	1.4642	1.6826	1.8866	2.4397
		G	1.4646	1.6830	1.8870	
		B	1.4575	1.6819	1.8827	
		R	1.4648	1.6831	1.8866	

TABLE IV

OBVIOUS RATIOS: ALL PAIR CASE

We also conduct experiments in the case that each cell of the grid represents a $50m \times 50m$ area to attempt to study how the LP models perform in multihop wireless networks when energy consumption for reception and overhearing is less significant. We use two maximum transmission ranges, 75m and 100m. We use the Gravity model and Random model when we know the range restriction on the base traffic matrix. Table V shows that our LP models can achieve low oblivious ratios (close to 1.0, the oracle optimal performance).

N	R_{max}	TM	1.5	2.0	3.0	∞
25	75m	G	1.4048	1.6377	1.8718	2.4018
		R	1.4054	1.6297	1.8647	
	100m	G	1.4868	1.7438	1.9655	2.4098
		R	1.4813	1.7342	1.9547	
36	75m	G	1.4253	1.6596	1.9346	2.6166
		R	1.4309	1.6652	1.9386	
	100m	G	1.5215	1.8107	2.1031	2.6479
		R	1.5230	1.8113	2.1039	

TABLE V

OBVIOUS RATIOS: ALL PAIR CASE, TRANSMISSION DOMINATES ENERGY CONSUMPTION

C. Interference-limited lossy-links case

We conduct experiments for the case where it is interference-limited and links are lossy. Loss ratio of each edge is uniformly set within $[0\%, 50\%]$. We set $\beta = \frac{2}{3}$ for schedulability constraint (16). Table VI shows the results for nine topologies of 81 nodes with a single sink in the center cell and $R_{max} = 15m$. We have low oblivious ratios. See [14] for the LP formulation and more results.

TM		1.5	2.0	3.0	∞	
U	min	1.1831	1.3320	1.6252	2.4053 (min)	
	max	1.5835	1.6356	1.9240		
G	min	1.1838	1.3330	1.6279		
	max	1.5848	1.6382	1.9204		
B	min	1.2378	1.4380	1.6249		2.7418 (max)
	max	1.5506	1.7279	2.0580		
R	min	1.2684	1.3895	1.5625		
	max	1.6096	1.6688	1.9251		

TABLE VI
MIN AND MAX OBLIVIOUS RATIOS OVER 9 RUNS
81 NODES WITH $R_{max} = 15m$, A SINGLE SINK IN THE CENTER,
INTERFERENCE-LIMITED LOSSY-LINKS

V. CONCLUSIONS

Energy efficiency is an important issue in multihop wireless networks with energy concerns. We investigate the problem of designing optimal traffic-oblivious energy-aware routing to minimax energy utilization in multihop wireless networks. We design LP models of polynomial sizes in both the number of variables and the number of constraints with a fairly weak assumption of the traffic pattern. With no or approximate knowledge of the traffic pattern, our LP models can achieve the performance close to what an oracle can achieve (with oblivious ratios close to 1.0). The performance is particularly good when there is a single sink.

Our LP model is general enough to model various wireless systems, such as MIMO. The routing scheme in this paper can work well in an interference-free wireless network. We discuss several implementation issues. In [14], we have incorporated the schedulability constraints and considered lossy links in formulating LP models. It is interesting to compare our work with an adaptive approach, e.g. [11], [15]. With several issues to further study and implementation details to fulfill, we have made a first stride in designing a traffic-oblivious energy-aware routing framework in multihop wireless networks.

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