Playing Reverse Hex

Hayward Henderson Toft

Comp Sci U of A
Edmonton Alberta Canada

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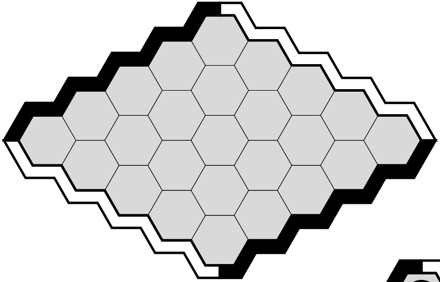
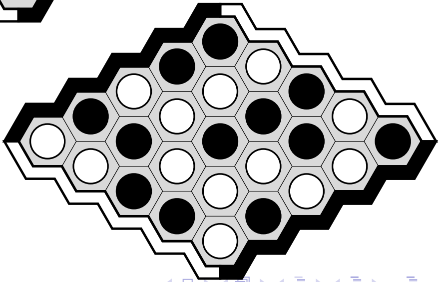
CA USA

IMADA
Odense Denmark

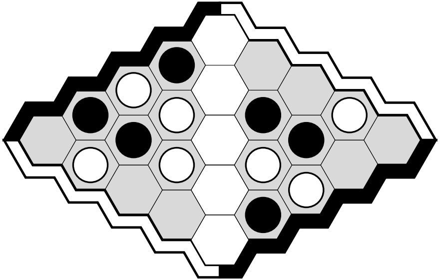
Jan 2011

Rex Reverse Hex Misere Hex

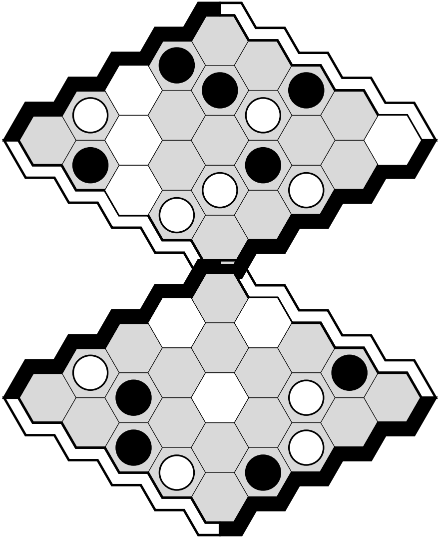




proofs: allow holey boards

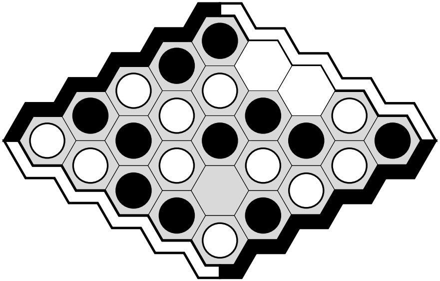


color symmetry



TRex truncated Rex

move only if $\geq$ 2 empty cells
...so can end in draw



Lemma

TRex from psn P with empty cell c
player Z = X or Y:

- X wins (P + Y(c), Z) $\Longrightarrow$ X not-loses (P, Z)
- Y wins (P, Z) $\Longrightarrow$ Y not-loses (P + Y(c), Z)

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Theorem

TRex from color-symmetric psn P with $\geq$ 2 empty cells:

- player to-move not-loses
- player not-to-move not-loses

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TRex from color-symmetric psn P
with $\geq$ 2 empty cells:

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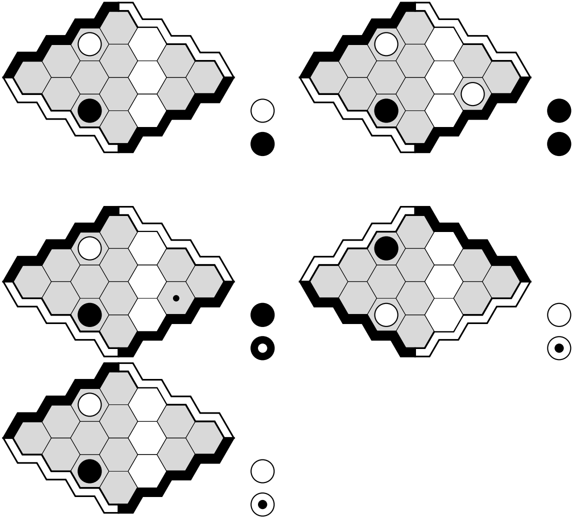
Theorem

TRex from color-symmetric psn P
with ≥ 2 empty cells:

- player to-move not-loses
- player not-to-move not-loses

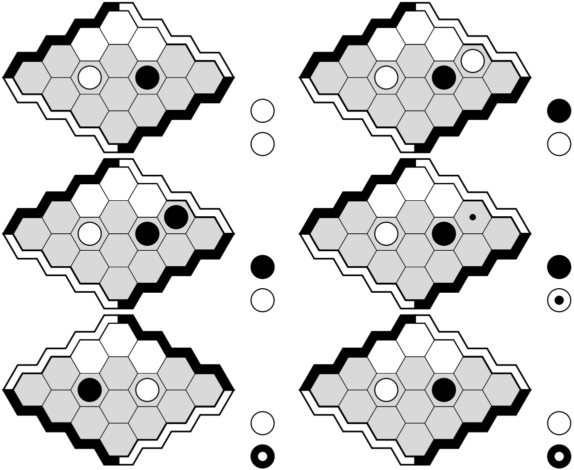
to-move not-loses

pf by cont'n



not-to-move not-loses

pf by cont'n



Theorem

Rex color-symmetric P k empty cells:

- $k \geq 2 \implies$ exists non-losing move
- k even $\implies$ 1st player not-loses$*$
- k odd $\implies$ 2nd player non-loses$*$

$*$ wins if P has no holes

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- $k \geq 2$ even:
  X wins (P, Y) $\Longrightarrow$ $\forall$ empty c, X wins (P + X(c), Y)

- $k \geq 3$ odd:
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Theorem

Rex n×n P $2t$ empty cells
long diagonal color-symmetry
c in corner empty wedge and X-peripheral:

- X not-loses (P + X(c), Y)

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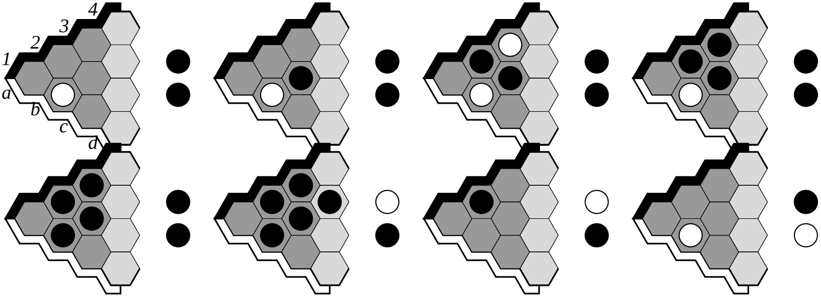
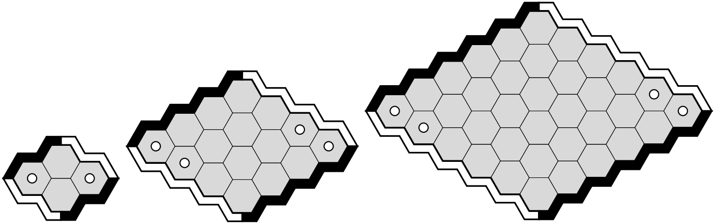
X not-loses (P+X(c),Y) pf by cont'n



some winning $2t \times 2t$ openings



Theorem

Rex n×n P $2t + 1$ empty cells
P color-symmetric P' = P+X(c):

• c-reflection d $\neq$ c $\Longrightarrow$
Y not-loses P'+Y(d)

• long diag'l symmetry, d Y-peripheral
in empty corner wedge of P' $\Longrightarrow$
Y not-loses P'+Y(d)

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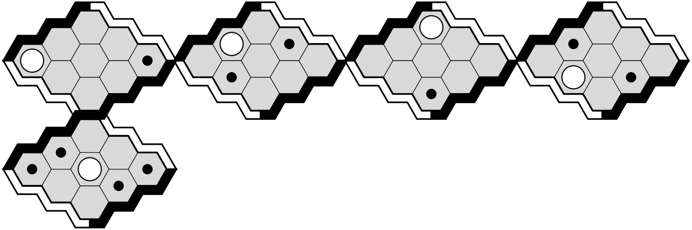
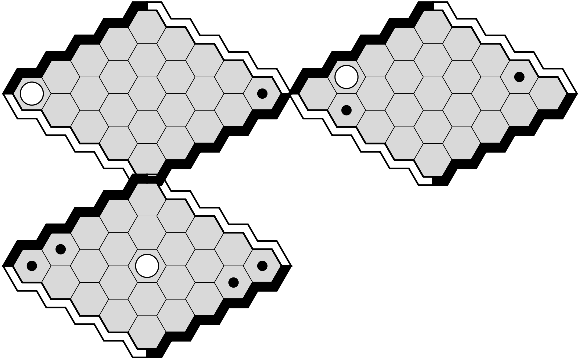
some winning 3×3 replies

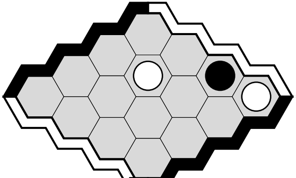


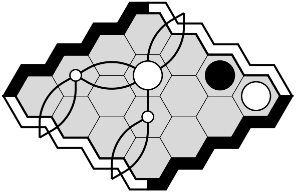
some winning 5×5 replies

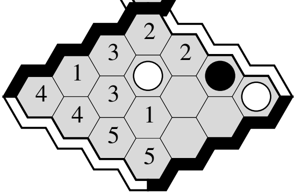


a 4×4 example

black-to-play wins from here so . . .

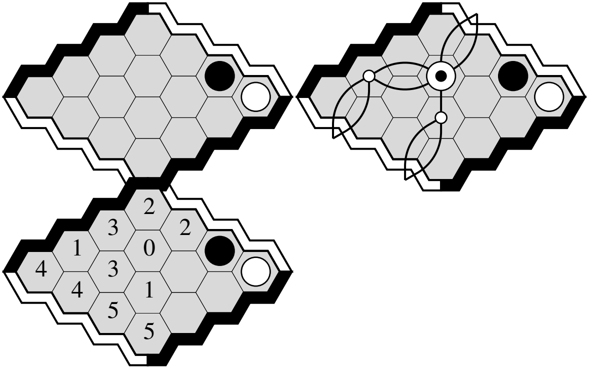






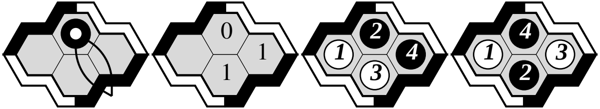
a 4×4 example

...black-to-play wins from here

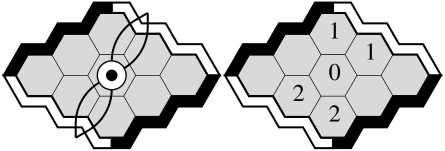


2×2 1st-player win

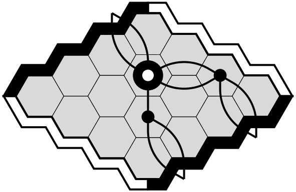
e.g. force 2nd-player this way

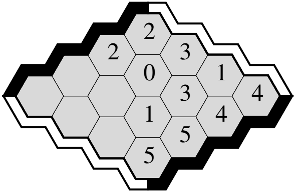


3×3 2nd-player win

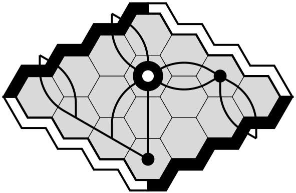


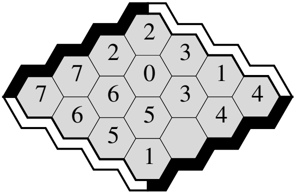
4×4 1st-player win



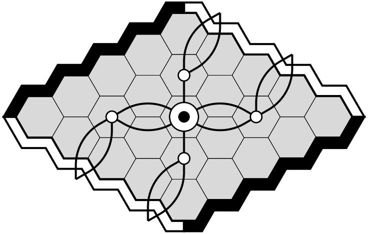


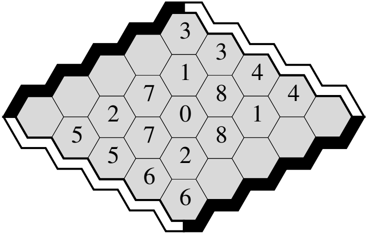
4×4 another way to win



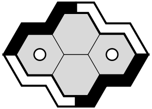


5×5 2nd-player win

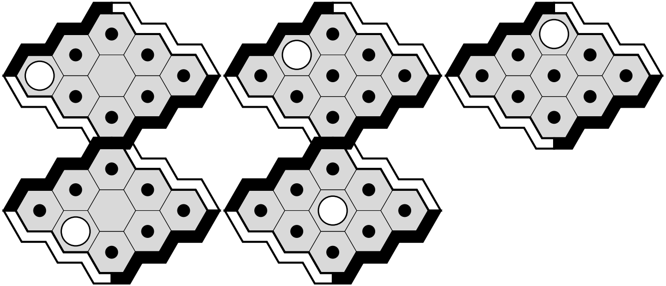




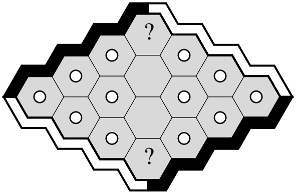
2×2 all winning openings



3×3 all winning replies



4×4 all (but 2?) winning openings



thanks to

- NSERC   Alberta Ingenuity
- UofA GAMES   UofA Hex
- M Mueller   J Schaeffer