

# A BEAUTIFUL GAME FROM THE WAR

## HOW HEX BEGAN

hayward@ualberta.ca + toft

computing.science   university of alberta   supported by NSERC

2019

## THANK YOU

- Hex research team  
Michael Johanson, Yngvi Björnsson, Morgan Kan, Nathan Po, Jack van Rijswijck, Broderick Arneson, Philip Henderson, Jakub Pawlewicz, Aja Huang (AlphaGo), Kenny Young, Noah Weninger, Chao Gao, Martin Müller (Fuego)
- Bjarne Toft
- Liz Greenaway and Audreys Books Edmonton
- UAlberta FacSci events

1942-43 DENMARK: POLYGON  
1949-50 PRINCETON-NYC: HEX  
1957 SCIENTIFIC AMERICAN

# HEX, THE FULL STORY

1942-43 DENMARK: POLYGON  
1949-50 PRINCETON-NYC: HEX  
1957 SCIENTIFIC AMERICAN

# HEX, THE FULL STORY



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1957 SCIENTIFIC AMERICAN

# HEX, THE FULL STORY

<https://webdocs.cs.ualberta.ca/~hayward/hexbook>

# HEX, THE FULL STORY

## sources

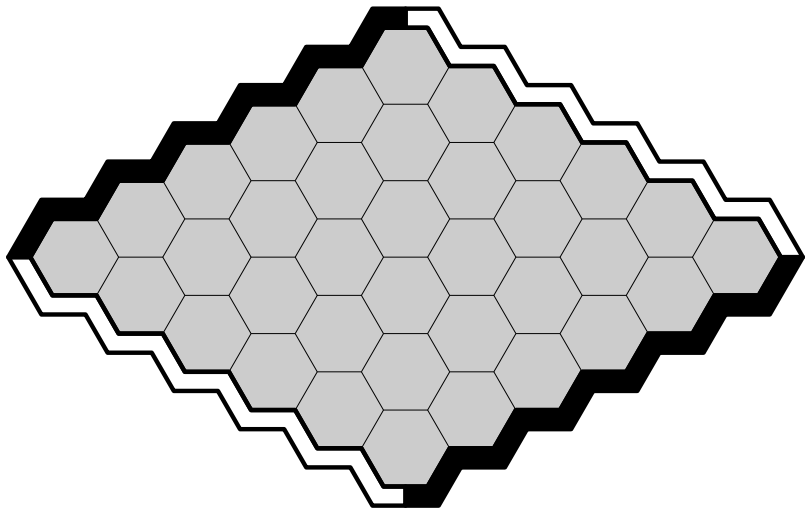
- Berge *L'Art subtil du Hex* 1977
- *Politiken* 1942 – 1943
- Thomas Maarup MSc thesis 2005
- Martin Gardner archive, Stanford U
- Piet Hein archive, collection Hugo Hein
- Jens Lindhard archive, Aarhus University

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# HEX

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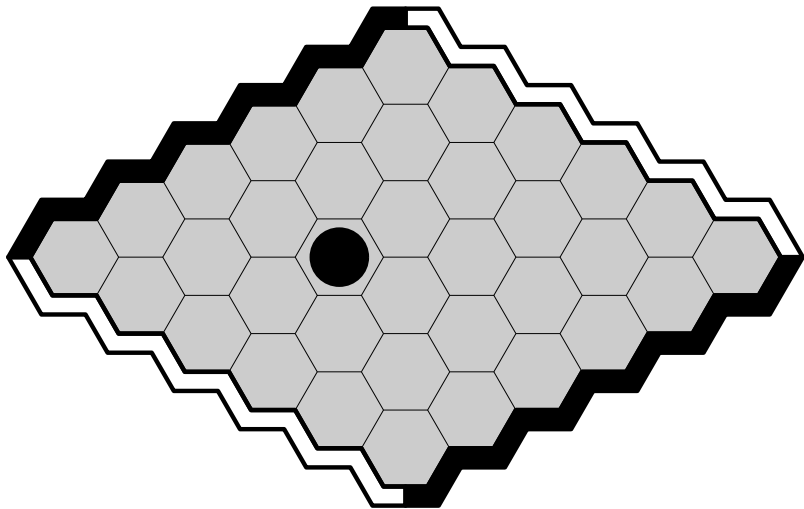
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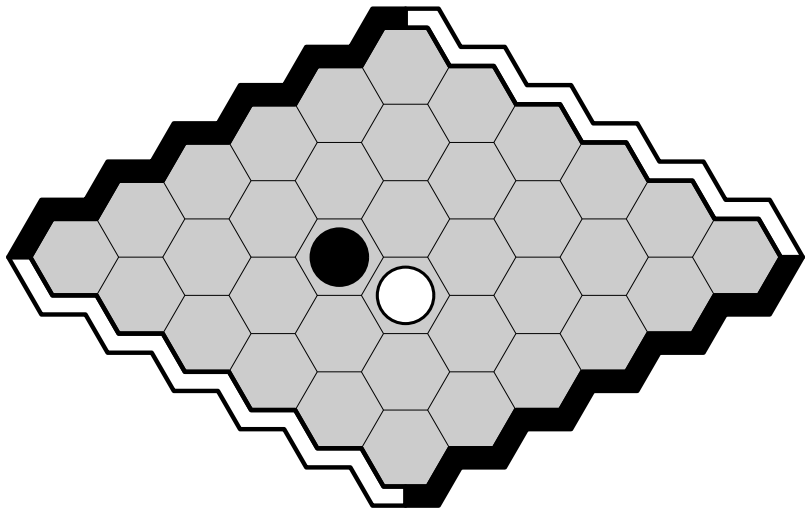
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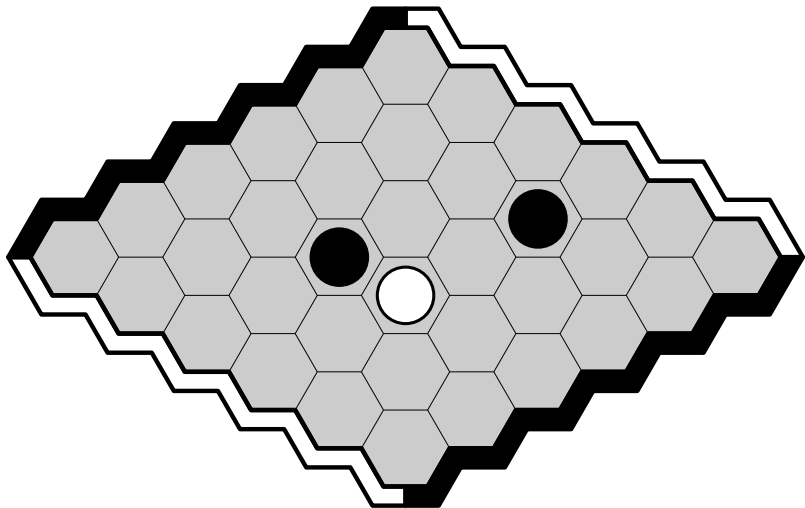
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# HEX



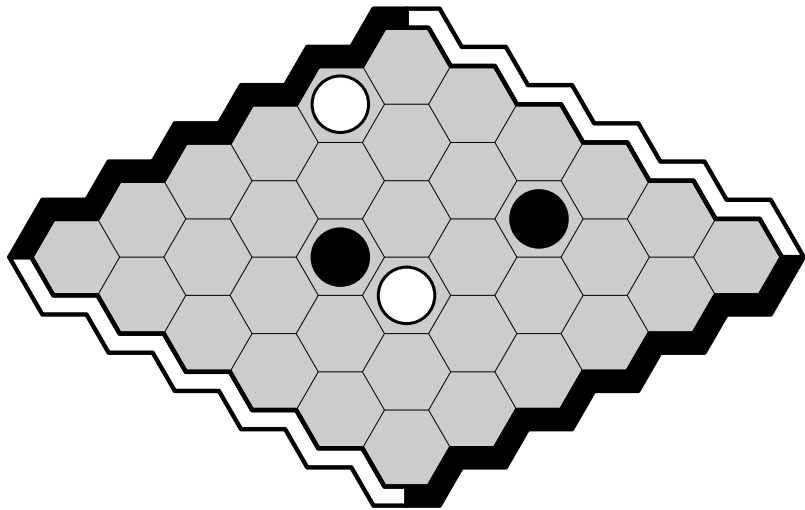
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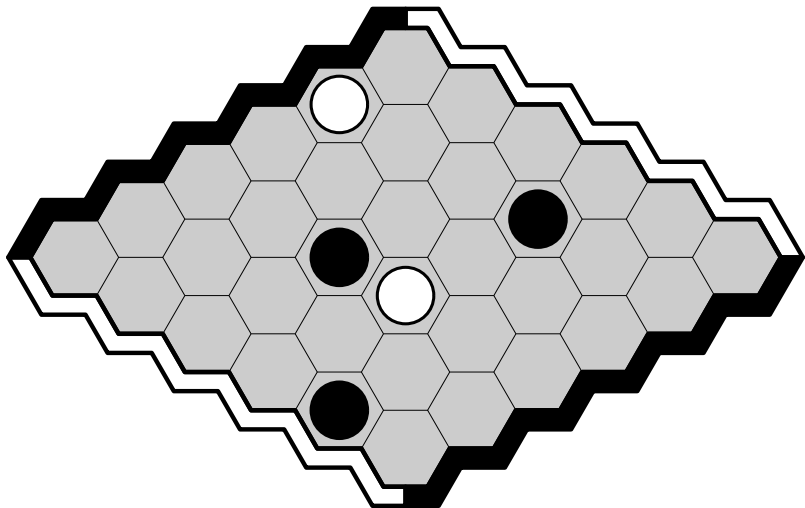


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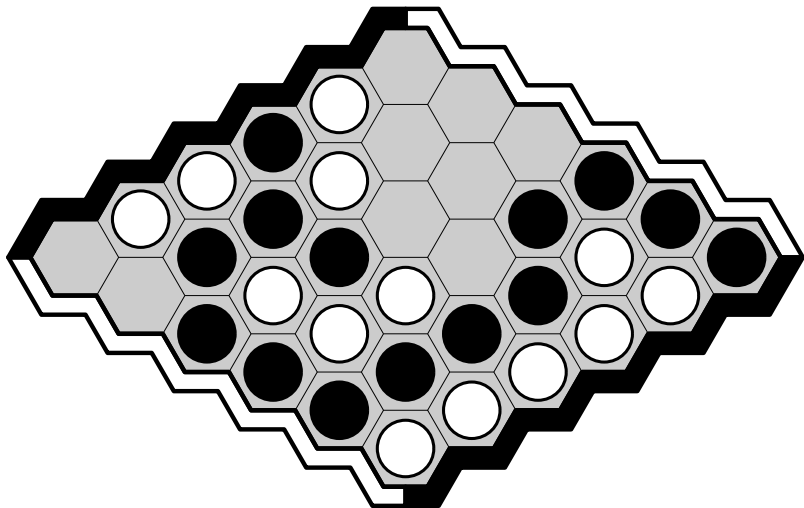


# HEX



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# HEX



1942-43 DENMARK: POLYGON  
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1957 SCIENTIFIC AMERICAN

# HOW HEX BEGAN

## HOW HEX BEGAN

- WWII
- cold war
- post-war

how came the game?

how came the game again?

how spread the game?



## HOW HEX BEGAN

- Hein design 1942
- Lindhard drafts, Thorborg puzzle 1943
  
- Nash theory, Gale board 1949
- Shannon machine 1950
- Parker Brothers game 1950
  
- Gardner column 1957

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# PIET HEIN

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# PIET HEIN





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# WAR

# WAR

- 1940 April 9

Germany invades Denmark, Norway

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1957 SCIENTIFIC AMERICAN

# GROOKS

## GROOKS

### CONSOLATION GROOK

Losing one glove  
is certainly painful,  
but nothing  
    compared to the pain,  
of losing one,  
throwing away the other,  
and finding  
    the first one again.

KUMBEL KUMBELL



# GROOKS

## PROBLEMS

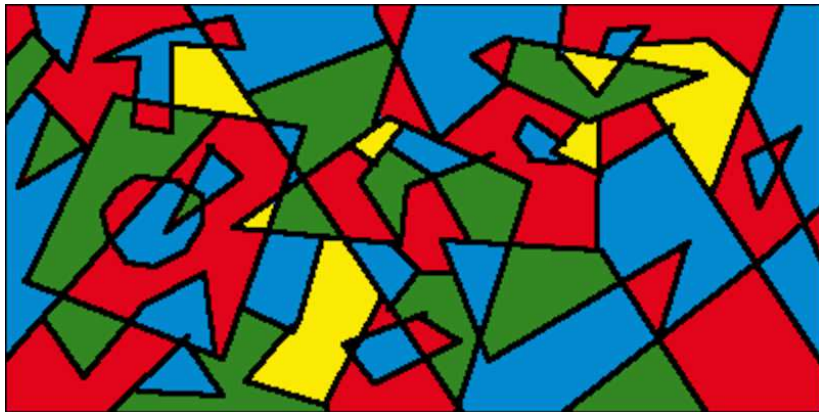
Problems worthy  
of attack  
prove their worth  
by hitting back.

KUMBEL

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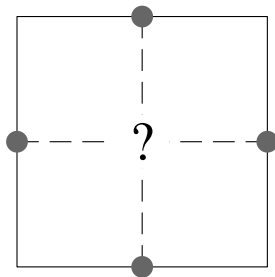
# NEW GAME IDEA

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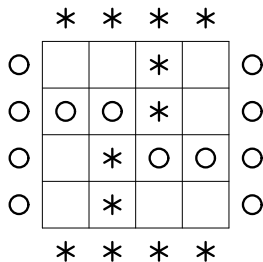
©<https://www.kleemans.ch/four-color-theorem->

# NEW GAME IDEA





## NEW GAME IDEA



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# 1942 POLYGON [HEX]

# 1942 POLYGON [HEX]

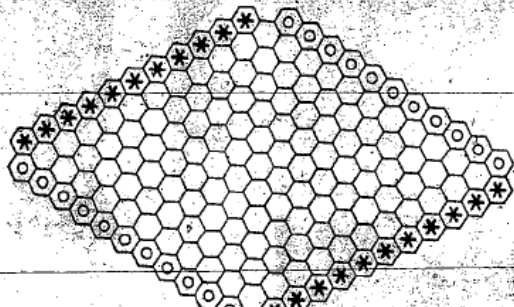
TIKEN

26. December 1942

## Vil De lære Polygon?

Piet Hein har konstrueret et Spil, der med lige stor Glæde kan dyrkes af Skakeksperten og den, der blot kan holde en Blyant

„Politiken“ udskriver i Dag en Præmieopgave, der vil volde Hovedbrud for Begyndere



ten kan afbryde Forbindelsen ved at besætte det mellemste bode. Vist dens Anvendelighed bliver hos Blykammerets Placering i den vidtre Omegn. I det hele taget viser det sig snart nødvendigt at lægge en større Del af Spillebrættet med i Betragtning.

En anden Erfaring, som kommer senere, men som man kan lætte Spillebetyndelse ved at røbe, er, at det betaler sig at begynde i hvert Fald nogenlunde paa Midten. En rimelig, men paa ingen Maade nødvendig Aabning af Spillet er denne:


Paa Spillebrættet i Midten er Hvid begyndt i Midterfeltet. Sort har sat i Kontaktfeltet til det ned imod Midten af Hvids Front og derved gjort to nyttige Felter, som staar i Vinkelstilling til Midterfeltet, usikre. Hvid har ogsaa vaegt et Felt i Kontakt med sin første. Og nu svarer Sort med at besætte et Vinkelfelt, som vilde være meget nyttigt for Hvid. Hvor skal nu Hvid sætte? Der er forskellige gode Muligheder.

Saaen er dette Spil nu begyndt. Nu kan enhver fortsætte. Det er altsaa Hvids Tur. Man skal ikke være udspekuleret fra Begyndelsen. Der er ingen bedre Vej til at lære Spillet end at spille det.

Det er nyttigt at se skitserne offentlig



# 1942 POLYGON [HEX]

  
**POLITIKEN**

TELEGRAM-ADRESSE: POLITIKEN, KØBENHAVN · POSTADRESSE: KØBENHAVN 1190 · TELEFON CENTRAL 8511 · STATSTELEF. 56

AKCIDENSTRYKKERIET      POLITIKENS HUS · KØBENHAVN V

Nr. 1275

**FAKTURA**  
FAKTURA

Den 30. Sept. 1942.

Herr Piet Hein,  
H e r.

N186-2500-7-40

Nr.		Kr.
<del>15xStk</del>	15 Stk. a 50 Kumbel-Spil (Eftertryk)	18.75
13182	10 Blokke a 50 Kumbel-Spil	<u>41.00</u>
		<u>54.75</u>





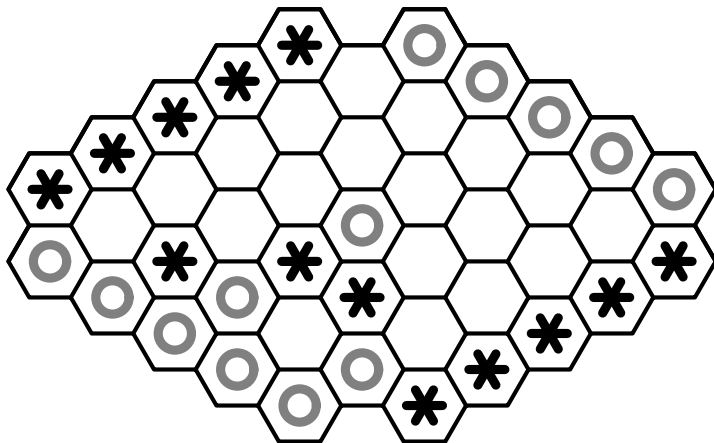




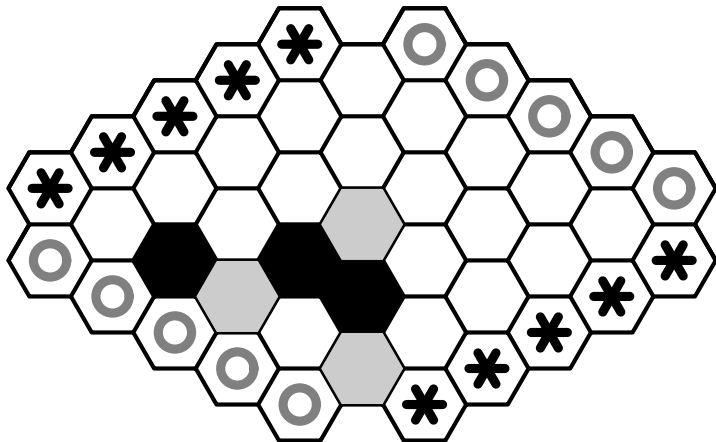
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# PUZZLE 16    WHITE TO PLAY

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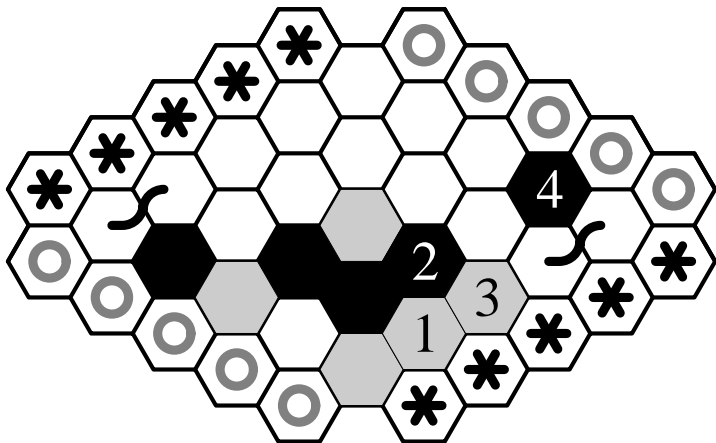


# PUZZLE 16 WHITE TO PLAY

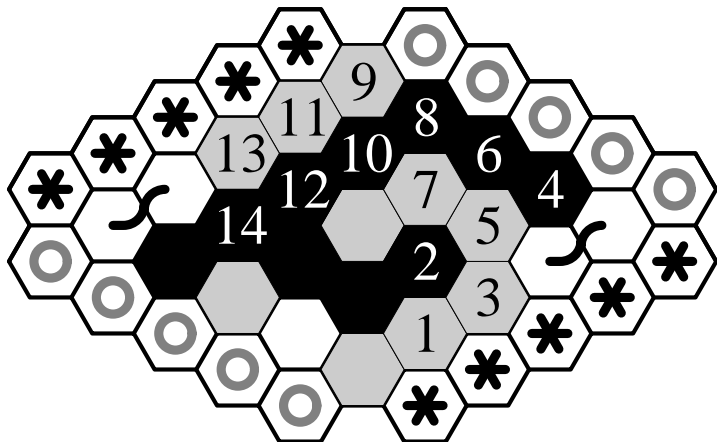




# PUZZLE 16 WHITE TO PLAY



# PUZZLE 16 WHITE TO PLAY



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# WHO POSED THE PUZZLES?

## WHO POSED THE PUZZLES?

- *I am by no means an excellent Polygon player and I do not feel obliged to be so merely because I happen to be the inventor of the game.*

Piet Hein

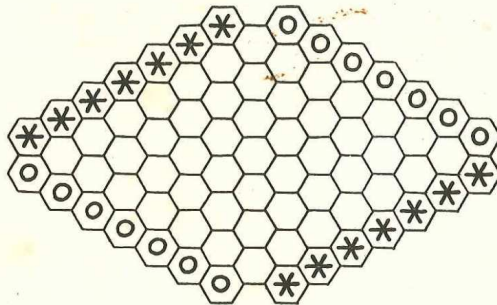
- puzzle drafts ?

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## WHO POSED THE PUZZLES?

# POLYGON — SPILLEBRÆT

7 × 7



# WHO POSED THE PUZZLES?

## SPILLETS ENESTE REGEL

POLYGON spilles af to Spillere: Hvid og Sort, som har Mærkerne henholdsvis en Cirkel og en Stjerne. Hver Spiller har to modstaaende Fronter, afmærkede i Felterne. — Man skiftes til at sætte sit Mærke i et hvilket som helst tomt Felt — og fortsætter indtil den ene har vundet. — Det gælder for hver Spiller om at besætte Felter paa saadan en Maade, at de danner en sammenhængende omend nok saa snoet Forbindelse mellem vedkommende Spillers to Fronter.

## EFTERLYSNING


For at POLYGON-Spillet kan lanceres for en videre Kreds — hvilket er en Forudsætning for, at der fortsat kan fremstilles Spillediagrammer — behøves et ret stort Opgave-Materiale.

Jeg opfordrer derfor alle, som har Interesse for Spillet, til at ha Opmærksomheden henvendt paa Ideer til POLYGON-Opgaver. Saadan Ideer opstaar ofte under Spillet. Der er Brug for Opgaver paa Spillebrætter af alle Størrelserne fra  $5 \times 5$  Felter indtil den normale Spillestørrelse  $11 \times 11$  — især paa Brætter af de smaa Antal Felter. POLYGON-Opgaver med Løsninger bedes indsendt til mig. POLYGON-Spillebrætter tilsendes Opgaveindsendere gratis, saa vidt Oplaget rækker.

Blandt de bedste af de hver Uge indtil videre indkomne Opgaver udtrækkes én, som belønnes med en *Præmie* — i Ugen indtil Lørdag d. 12. December 1942: en Julegans.

En POLYGON-Opgave bestaar i et POLYGON-Spillebræt, paa hvilket nogen af Felterne er besat med Hvids og Sorts Mærker. Denne Afmærkning behøver ikke at kunne være fremkommet under et normalt Spil, specielt kan der udmærket være mange flere af den ene Spillers Mærker end af den andens. Til en Opgave hører 3 Oplysninger: 1. *hvilken Spiller, der skal begynde*, og 2. *hvilken* (ikke nødvendigvis altid den samme) *Spiller, der kan vinde*, og 3. (helst) *i hvor mange Træk*, hvis begge spiller stærkest muligt. I Overensstemmelse med den Maade, hvorpaa en Løsning angives (se nedenfor), regnes Antallet af Træk efter begge Spilleres samlede Antal.

En Opgave maa være nogenlunde éntydig, dvs. naar begge spiller bedst muligt, skal der praktisk talt kun være én Fremgangsmaade, især skal det første Træk ligge fast. En Opgave bør ikke indeholde overflødige Mærker.

En Opgaves *Løsning* noteres saaledes paa Spillebrættet: de Mærker, som hører med til den stillede Opgave, sættes paa sædvanlig Maade; de Mærker, som hører til Løsningen, erstattes i kronologisk Orden med Tallene 1, 2, 3 osv., saaledes at den som trækker først altsaa har de ulige Tal, den anden de lige. For Overskueligheden kan Sorts Tal gøres særlig fede. Naar Udfyldningen af de sidste Felter i en Løsning er rent rutinemæssig, er det en Fordel, at den ikke udføres. Naar (hvad der ofte sker) to Nabo-Felter, som er blevet besat af henholdsvis Hvid og Sort, ligesaa godt kunde være blevet besat at dem omvendt, kan det angives ved, at de to Felter forbindes med dette Tegn: .

Venlig Hilsen.

## WHO POSED THE PUZZLES?

### SPILLET'S ENESTE

POLYGON spilles af to Spillere: Hvid og Sort, som har  
Hver Spiller har to modstaaende Fronter, afmærkede i  
i et hvilket som helst tomt Felt — og fortsætter indtil  
Spiller om at besætte Felter paa saadan en Maade, at  
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### EFTERLYSNING

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fortsat kan fremstilles Spillediagrammer — behøves et

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Ideer til POLYGON-Opgaver. Saadan Ideer opstaar ofte  
Spillebrætter af alle Størrelserne fra  $5 \times 5$  Felter indtil  
Brætter af de smaa Antal Felter. POLYGON-Opgaver med  
Spillebrætter tilsendes Opgaveindsendere gratis, saa vid

Blandt de bedste af de hver Uge indtil videre indkom  
en *Præmie* — i Ugen indtil Lørdag d. 12. December 19

## WHO POSED THE PUZZLES?

THE GAME'S ONLY RULE. Polygon is played by two players: White and Black, whose symbols are a circle and a star. Each player has two opposing fronts, marked with their symbols. Each player in turn marks any empty field with their symbol, The game continues until one player wins, A player wins by joining their two sides with a path, which can twist and turn.



## WHO POSED THE PUZZLES?

MISSING PERSON'S REPORT. In order to bring Polygon to a larger audience — which we all want — we need a large supply of Polygon puzzles. So I urge anyone with an interest in the game to construct such puzzles. ... Send POLYGON puzzles, with commentary, to me. Among the best of those puzzles submitted this week, one will be picked at random on December 12, 1942 to receive a special prize: a Christmas goose.

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# IF NOT HEIN ... ?

## IF NOT HEIN ... ?

*Politiken Jan 26 1943* Mon Feb 1 8pm: Polygon salon. In response to the spreading Polygon-mania, the game's creator Piet Hein, with expert player Jens Lindhard, will demonstrate the game's strategy and finer points. They will use a giant game board to play members of the audience and answer questions. Tickets at "Polygon" ticket office, Town Hall Square, today 9am to 6pm. Free subscriber admission with Coupon 1 from subscription receipt. Cloakroom payment 25 cents.

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... MAYBE THE SALON EXPERT ?

## ... MAYBE THE SALON EXPERT ?

- at salon, played simultaneous Polygon games
- chess expert, studied with Aage Bohr under Niels Bohr, physics prof Aarhus U (while Toft student)
- never married, post-retirement to office each day
- any papers left?
- 2017 Nov Toft goes to Aarhus

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# JENS LINDHARD, POLYGON PUZZLIST

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# JENS LINDHARD, POLYGON PUZZLIST



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# JENS LINDHARD, POLYGON PUZZLIST



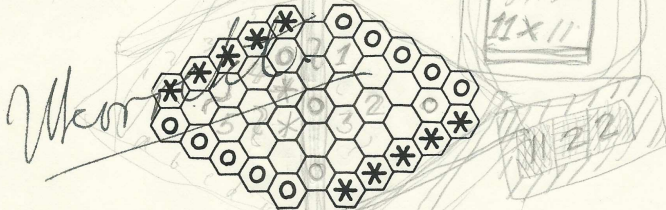


# JENS LINDHARD, POLYGON PUZZLIST

## POLYGON

5 × 5

*Sort trækker og vinder. Temmelig let.*



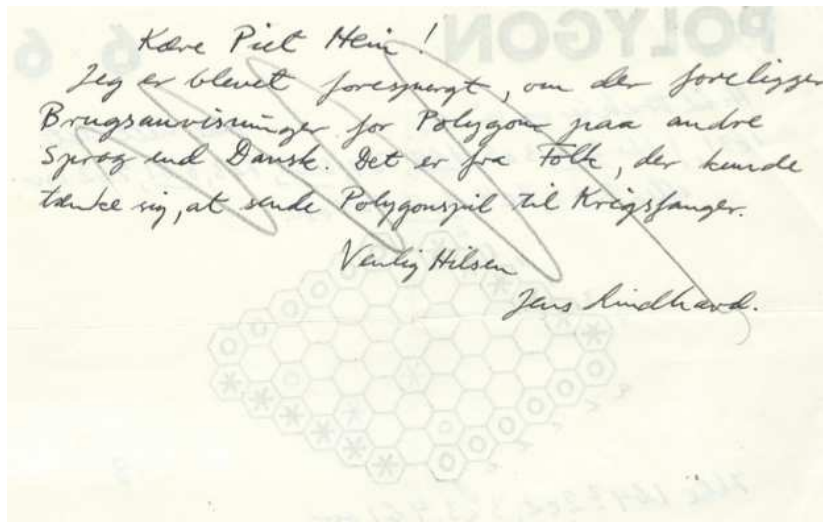
*Paa 2c4 følger 3d4, 4c5, 5d2, 6e2, 7d3 osv.*

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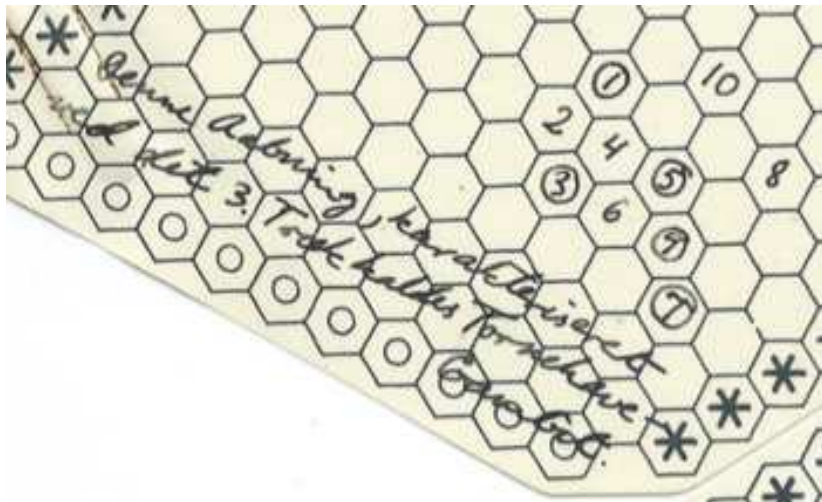
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## JENS LINDHARD, POLYGON PUZZLIST

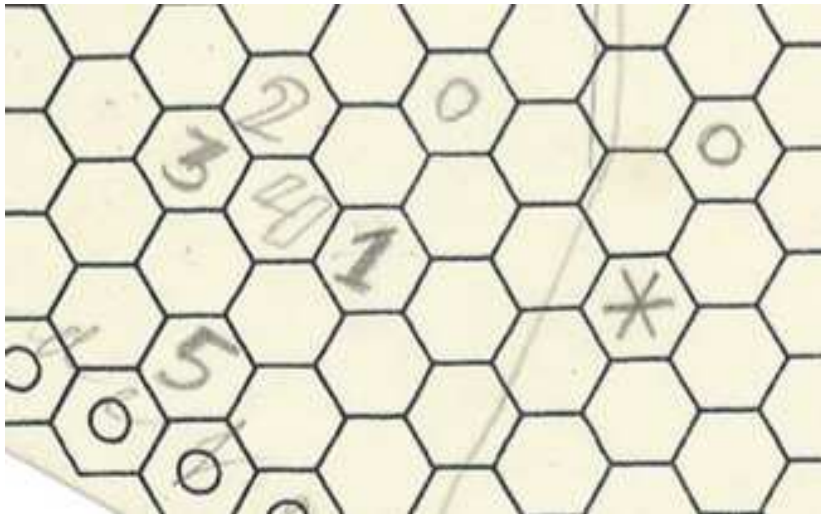


# JENS LINDHARD, POLYGON PUZZLIST



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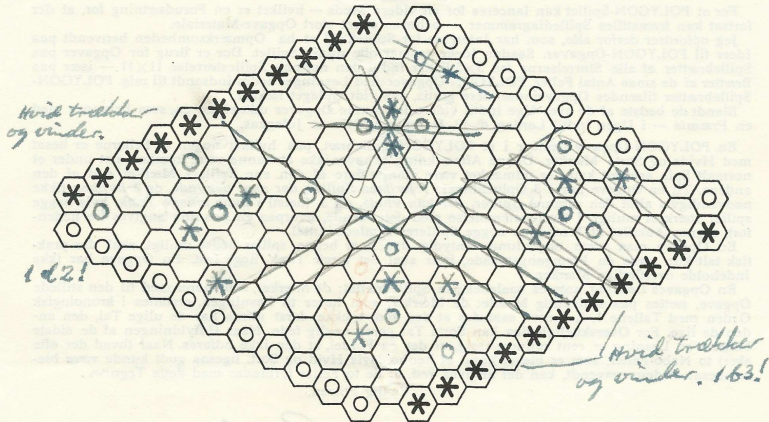


# JENS LINDHARD, POLYGON PUZZLIST

## POLYGON — SPILLEBRÆT

11 × 11

*Svart trækker og vinder: 165, 2/2, 3-05.*

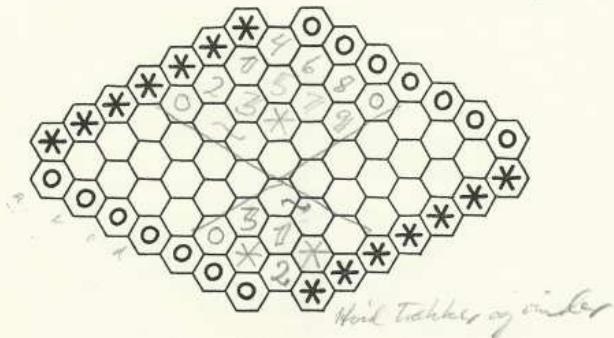


## JENS LINDHARD, POLYGON PUZZLIST

# POLYGON — SPILLEBRÆT

**7** × **7**

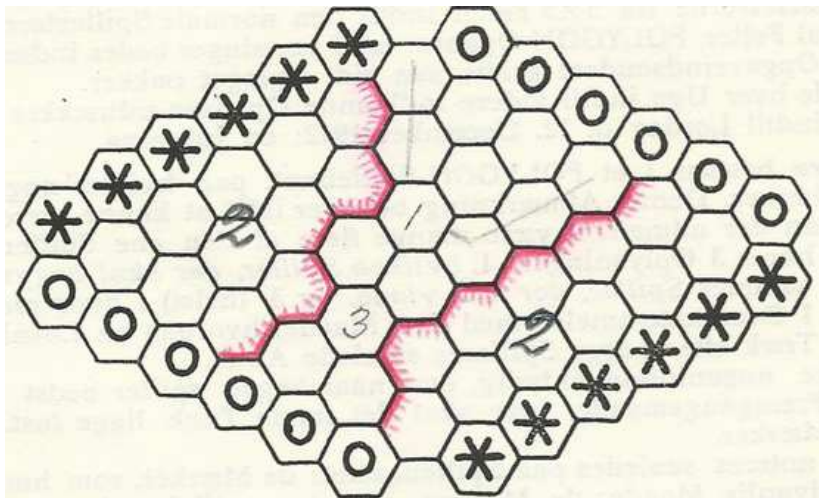
*Heid Takker: Kan Heidi vinde? Ja!*



*Heid Takker, og vinde*

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1957 SCIENTIFIC AMERICAN

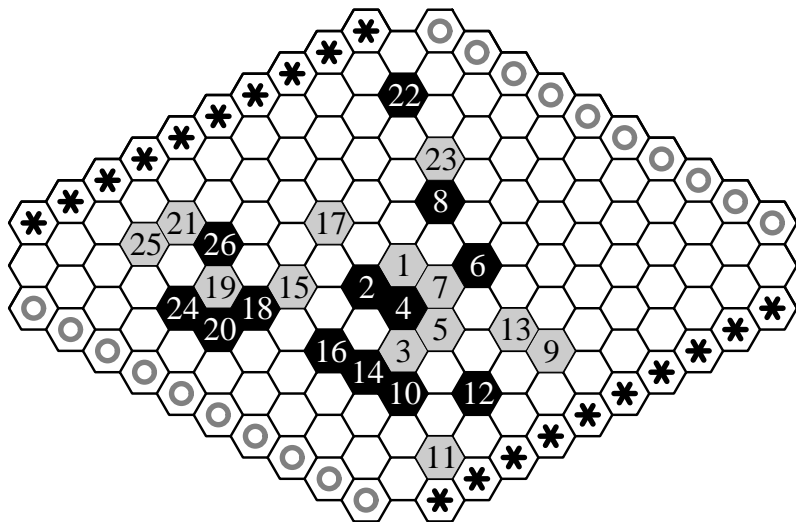
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# KAREN MARGRETHE THORBORG

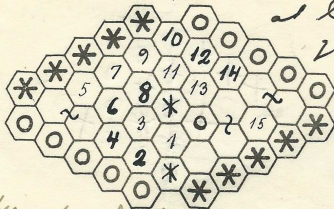
# KAREN MARGRETHE THORBORG

## POLYGON — SPILLEBRÆT

5 × 5

22. 1. 43.

Koen J.L.! Se det meget vord,  
saa bevis det. Det er  
jo ellers meget nok  
at lave en til  
genneanalyse  
Opgave!



F

af denne Opgave begynder lidt-og-omg, men først efter  
15 Træk og under skadige Træk fra poske Lide.  
Jeg har prøvet adskillige (men ikke alle!) andre Mø-  
ligheder for hvids første Træk, men har kun fået  
fundt et Modstrik fra poske Lide.

## KAREN MARGRETHE THORBORG

White starts and wins, but only after 15 moves, and under constant threat from Black. I tried very many (but not all!) other possibilities for White's first move, but in each case I found a winning Black reply.

Kind regards, univ. stud. Karen Thorborg  
Horserødvej 20 Copenhagen 0

# KAREN MARGRETHE THORBORG

22.1.43. Dear JL! If worthwhile, use this puzzle.  
But of course it is easy enough to construct an  
unanalyzed puzzle!

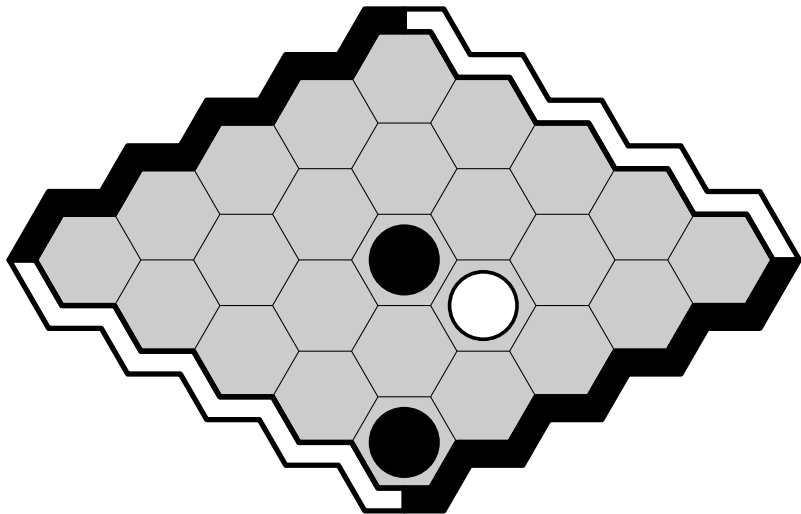
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# THORBORG'S PUZZLE

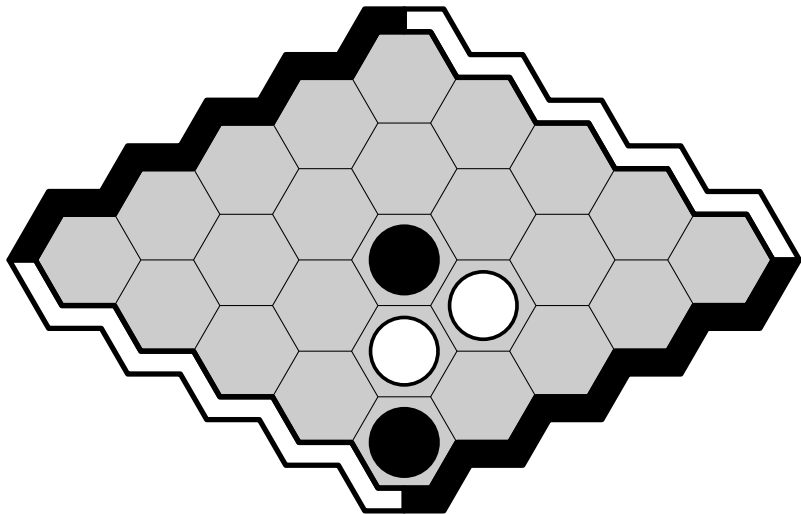
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# THORBORG'S PUZZLE



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# THORBORG'S PUZZLE: DOES WHITE WIN?





# WAR

- 29 August 1943

Germany declares martial law in Denmark

# GAME THEORY

- chess: can 1st player always win or draw?

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# JOHN F NASH    DAVID GALE

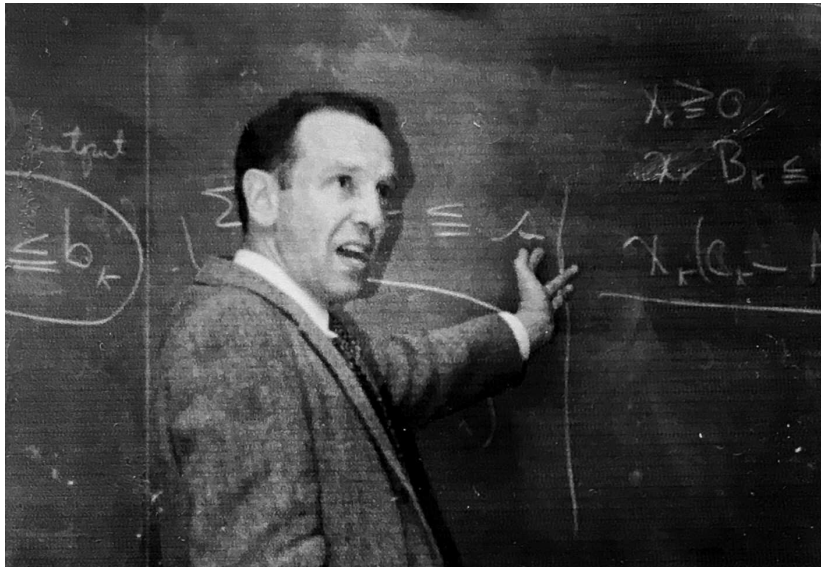
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1957 SCIENTIFIC AMERICAN

# JOHN F NASH    DAVID GALE



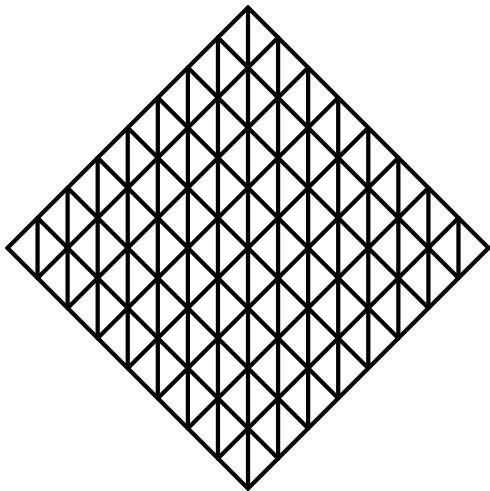
1942-43 DENMARK: POLYGON  
1949-50 PRINCETON-NYC: HEX  
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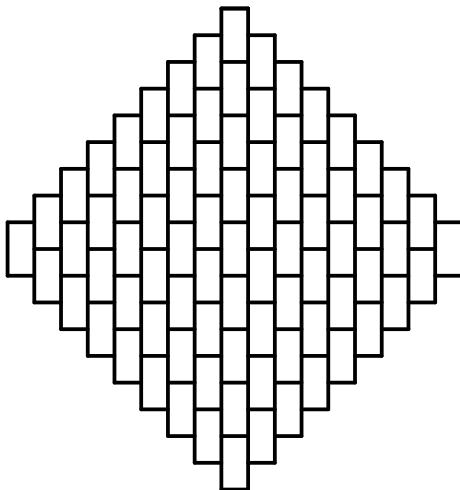
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# FIRST PLAYER WINS

## FIRST PLAYER WINS

- ① ~~When the board is~~ ~~field~~ filled one or the other of the players will have connected but not both.
- ② ~~One~~ Either the first player or the second will have a winning strategy.
- ③ Suppose the second player could force a win.
- ④ Consider a defensive strategy

## FIRST PLAYER WINS

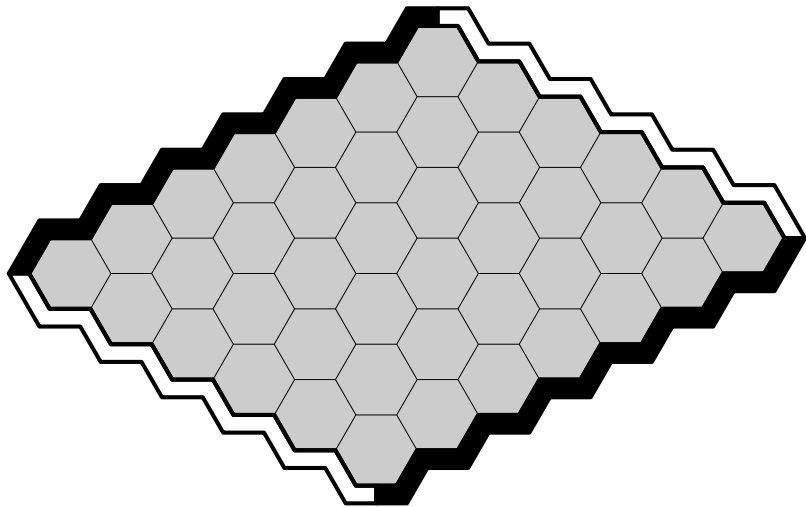
the board is always an asset,  
never a handicap in connecting,  
at the end of the game  
first player will be <sup>assumed</sup> better off  
using the adapted second player  
strategy than he would have  
been if simply playing as  
second player. So he will  
win.

⑥ Since this contradicts the  
hypothesis ③ that second  
player can win it follows that

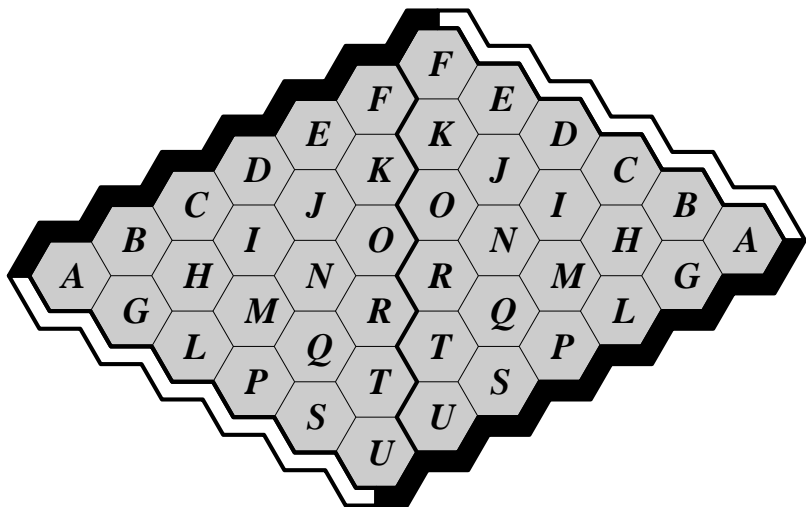
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## SHORTER DISTANCE WINS

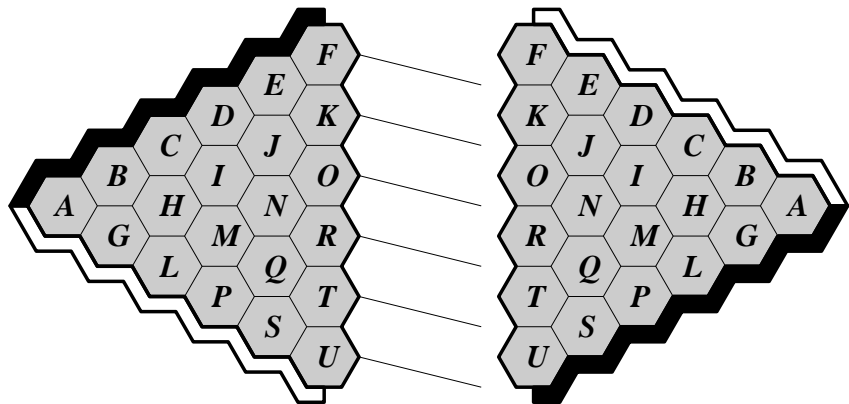
## SHORTER DISTANCE WINS



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# CLAUDE SHANNON



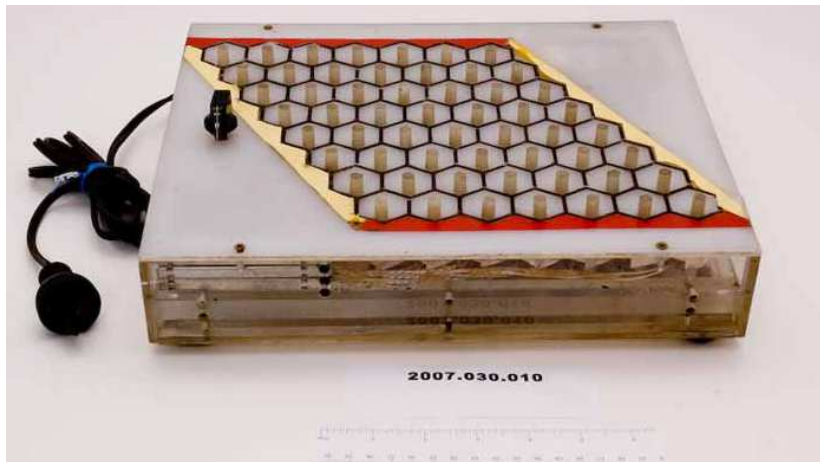
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# CLAUDE SHANNON



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# CLAUDE SHANNON



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# MARTIN GARDNER

1942-43 DENMARK: POLYGON  
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1957 SCIENTIFIC AMERICAN

# MARTIN GARDNER



# MARTIN GARDNER

- Scientific American
- 1956 Dec hexaflexagons
- 1957 monthly Mathematical Games
- 1957 July *Concerning the game of Hex, which may be played on the tiles of the bathroom floor*
  
- find 1pw ?      find win/loss for each 1st move ?
- 4x4 easy                      5x5 harder                      6x6 ?

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THANK YOU



HAYWARD@UALBERTA.CA + TOFT

A BEAUTIFUL GAME FROM THE WAR

