SOME QUESTIONS ON HEX

U MONTANA TALK

hayward@ualberta.ca

computing UAlberta

2014 April 14
THANK YOU

- invitation Prof Mark Kayll UMontana
- solving $10 \times 10$ Hex joint with Jakub Pawlewicz
- builds on work with B Arneson, P Henderson
- machine Martin Müller
- photo courtesy MIT Museum, MIT, Cambridge MA
- Natural Sciences and Engineering Research Council of Canada
1 SOME QUESTIONS

2 HEX

3 KNOWLEDGE

4 SOME ANSWERS
is hex fair?
when will computers solve $11 \times 11$ hex?
$11 \times 11$ hex, 10-1 odds, 1st 2 stones: wager?
write hex player in 8 hours: algorithms?
1942 HEX

**RULES**

- 2 players, alternate moves
- win: connect your two sides
1942 HEX

RULERS

- 2 players, alternate moves
- win: connect your two sides
PROOF

- lemma: extra X-cell ok for player X
- lemma: no draws
- suppose P2 has win strategy S2
- then P1 can move anywhere, forget move, and follow S2
- thus P1 has win strategy, contradiction $\square$
NO-DRAW

SOME QUESTIONS
HEX
KNOWLEDGE
SOME ANSWERS

PROPERTIES
SHANNON MACHINE
PROVABLY HARD
HUMANS
COMPUTERS

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SOME QUESTIONS ON HEX
\textbf{N x N+1 Hex: longer-side win}
**N x N+1 Hex: longer-side win**

A diagram of an N x N+1 Hex board is shown, highlighting the winning condition where the longer side wins. The diagram includes labels for each cell, with letters A to U arranged in a hexagonal pattern.
1951 Shannon machine
Some questions on hex

1951 Shannon machine

- play on any graph
- two marked vertices
- black move: ‘short’ any vertex (make nbrs clique)
- white move: ‘cut’ any vertex (delete)
- black wins iff two marked vertices are shorted (connected)
- generalizes Hex
1951 Shannon Machine
1951 Shannon machine
1951 **Shannon Machine**

- Some Questions
- Hex
- Knowledge
- Some Answers

Shannon Machine

Provably Hard

Humans

Computers

T

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Some questions on hex
PROVABLY HARD

- 1975 Even & Tarjan  
  Shannon v-switching: PS-c
- 1981 Stefan Reisch  
  Hex: PS-c
- 2000 Clay Math Inst  
  P vs NP: $1,000,000
SOLVED OPENINGS

- 2001 Yang 17/49 7x7
- 2002 Yang 8x8
- 2003 Yang 9x9
- 2004 Noshita 7x7
- 2005 Noshita 8x8
- 2006 Mishima 8x8
<table>
<thead>
<tr>
<th>Year</th>
<th>Authors</th>
<th>Size</th>
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<tbody>
<tr>
<td>1995</td>
<td>Enderton</td>
<td>6x6</td>
</tr>
<tr>
<td>2000</td>
<td>van Rijswijck</td>
<td>6x6</td>
</tr>
<tr>
<td>2003</td>
<td>H Bjö Joh Kan Po vRij</td>
<td>5d</td>
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<tr>
<td>2007</td>
<td>Rasmussen et al.</td>
<td>7x7</td>
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<tr>
<td>2009</td>
<td>Arneson H Henderson</td>
<td>4d</td>
</tr>
<tr>
<td>2010</td>
<td>A H H</td>
<td>25d</td>
</tr>
<tr>
<td>2012</td>
<td>Pawlewicz H</td>
<td>110d x 24 thread</td>
</tr>
<tr>
<td>2013</td>
<td>Pawlewicz H</td>
<td>63d x 24 thread</td>
</tr>
</tbody>
</table>

2003 H Bjö Joh Kan Po vRij 5d 7x7
2007 Rasmussen et al. 7x7
2009 Arneson H Henderson 4d 8x8
2010 A H H 25d some 9x9
2012 Pawlewicz H 110d x 24 thread 9x9
2013 Pawlewicz H 63d x 24 thread centre 10x10
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SOME QUESTIONS ON HEX
virtual connections: combining rules, mustplay
inferior cells: dead, captured, etc.
A VIRTUAL CONNECTION
A VIRTUAL CONNECTION
COMBINING RULE: AND (FULL)
COMBINING RULE: AND (FULL)
COMBINING RULE: AND (FULL)
COMBINING RULE: AND (FULL)
COMBINING RULE: AND (FULL)
COMBINING RULE: AND (FULL)
COMBINING RULE: AND (SEM I)
COMBINING RULE: AND (SEMI)
COMBINING RULE: AND (SEMI)
COMBINING RULE: OR

some questions on hex
COMBINING RULE: OR
COMBINING RULE: OR
COMBINING RULE: OR
WHERE MUST WHITE PLAY?
WHERE MUST WHITE PLAY?
WHERE MUST WHITE PLAY?
WHERE MUST WHITE PLAY?
DEAD

SOME QUESTIONS
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VIRTUAL CONNECTIONS
INFERIOR CELLS
BLACK-DOMINATED (DOT SUPERIOR)
BLACK-CAPTURED
BLACK-DOMINATED (DOT SUPERIOR)
BLACK-CAPTURE-REVERSIBLE (TO WHITE DOT)
BLACK FILL DECOMPOSITION
STAR DECOMPOSITION
BLACK STAR DECOMP DOMINATION
modify H-search

- and/or combining rules + capture
FAIR ?
Some questions on Hex

Knowledge

Some answers

FAIR?

- $n \times n$, $n \geq 2$, most win positions have losing moves
- $n \times n$, random play, $n$ even: $\text{Prob}(1\text{pw}) = 0.5$
- $n \times n$, random play, $n$ odd: $\text{Prob}(1\text{pw}) \to 0.5$ (?)
HOW LONG UNTIL 11x11?
### HOW LONG UNTIL 11x11?

<table>
<thead>
<tr>
<th>yr</th>
<th>size</th>
<th>states (approx)</th>
<th>center cell: solver fn calls</th>
</tr>
</thead>
<tbody>
<tr>
<td>42</td>
<td>2x2</td>
<td>9.0 e 0</td>
<td>0</td>
</tr>
<tr>
<td>42</td>
<td>3x3</td>
<td>5.5 e 1</td>
<td>0</td>
</tr>
<tr>
<td>42</td>
<td>4x4</td>
<td>7.6 e 5</td>
<td>0</td>
</tr>
<tr>
<td>42</td>
<td>5x5</td>
<td>4.0 e 9</td>
<td>0</td>
</tr>
<tr>
<td>42-95</td>
<td>6x6</td>
<td>4.0 e 14</td>
<td>2</td>
</tr>
<tr>
<td>01-03</td>
<td>7x7</td>
<td>1.5 e 20</td>
<td>68</td>
</tr>
<tr>
<td>02-09</td>
<td>8x8</td>
<td>1.0 e 27</td>
<td>19 554</td>
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<tr>
<td>03-12</td>
<td>9x9</td>
<td>2.7 e 34</td>
<td>912 352</td>
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<tr>
<td>13-</td>
<td>10x10</td>
<td>1.2 e 43</td>
<td>5 821 097 789</td>
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<tr>
<td>11x11</td>
<td>2.2 e 52</td>
<td>??? ??? ??? ??? ??? ????</td>
<td></td>
</tr>
</tbody>
</table>
11x11 HANDICAP: WAGER?
11x11 HANDICAP: WAGER?

- exists simple strategy that wins
  - up to $5 \times 5$ with 1 stone
  - up to $11 \times 11$ with 2 stones
  - up to $17 \times 17$ with 3 stones
  - ...
8 HOURS TO CODE PLAYER: ALGORITHMS?
8 hours to code player: algorithms?

Basics

- search? flat monte-carlo (random simulations, keep stats)
- detect wins? union-find
- stats: all-moves-as-first (each winning stone gets bonus)
- move selection: highest AMAF score

Improvements

- in simulations, save bridges
- monte carlo tree search
- code sample: https://github.com/ryanbhayward/miowy

webdocs.cs.ualberta.ca/~hayward/670gga/jem/gga.html
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