Solving 8 $\times$ 8 Hex

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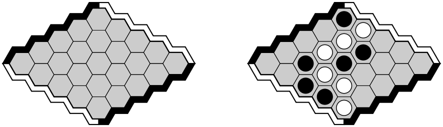
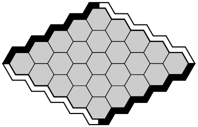
Hex Rules and Properties

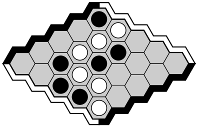


Rules

- Two players alternate turns playing on any empty cell
- Stones are permanent (no moving or capturing)
- Goal is to connect your two sides of the board

Hex Rules and Properties

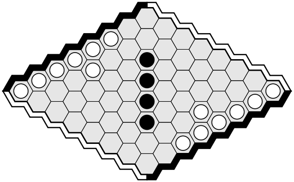




Properties

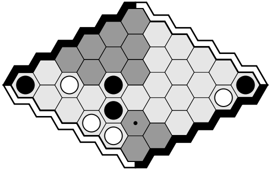
- Extra P-stones never disadvantageous for player P
- Draws are impossible
- First player wins: strategy-stealing argument
- Determining winner is PSPACE-complete

Previously Solved States



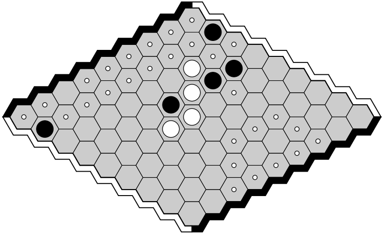
- Last milestone for automated Hex solvers in 2004
- All 7 $\times$ 7 openings solved in two weeks (Hayward et al)
- By hand, humans have solved centre opening on 9 $\times$ 9 (Yang) and a few openings on 8 $\times$ 8 (Mishima et al, Yang)

H-Search



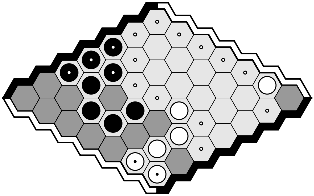
- *H-Search*: algorithm that deduces existing connection strategies in a given Hex position (Anshelevich)
- *Virtual connections* (VC): 2nd-player connection strategy
- *Semi-connections* (SC): 1st-player connection strategy
- *Carrier*: empty cells required for a connection strategy

Mustplay



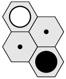
- Identifying a winning VC terminates search
- Identifying winning SCs immediately prunes losing moves
- *Mustplay*: intersection of winning opponent SC carriers

Inferior Cell Analysis



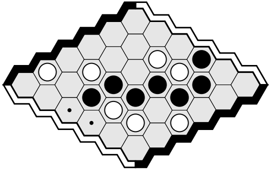
- Graph-theoretic properties and combinatorial game theory
- *Fill-in*: can add stones to the board without changing its win/loss value
- *Reversible* and *dominated* moves: can be pruned from consideration

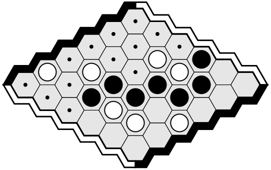
Opposite Color Bridges



- If a P-chain is adjacent to both $\overline{P}$ edges, then splits board into two independent regions
- Easy to detect these decompositions, but very rare
- *Opposite-color bridges*: can treat the two carrier cells as non-adjacent

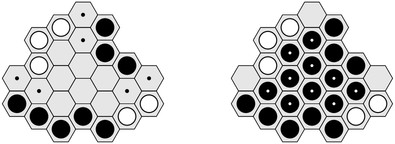
Split Decompositions





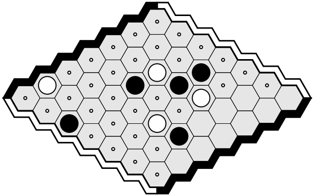
- Two chains *touch* if they are adjacent or form an opposite-color bridge
- *Split decomposition*: when a P-chain touches both $\overline{P}$-edges

Four-Sided Decompositions



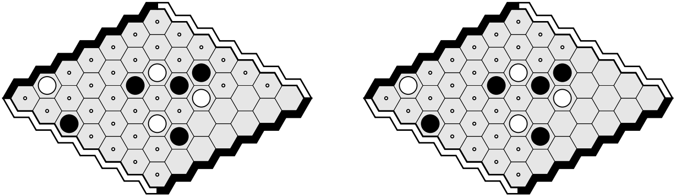
- *Four-sided decomposition*: a 4-cycle of touching Black and White chains
- If player P has a VC connecting the two P-chains of a four-sided decomposition, the region can be filled-in with P-stones

Proof Set Pruning



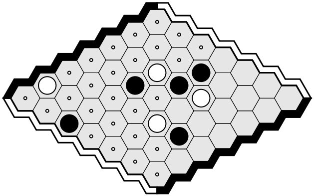
- During search we identify previously-unknown winning SCs
- Can use discovered SCs to further reduce mustplay
- The smaller the SC carrier, the more moves can be pruned

Proof Set Reduction



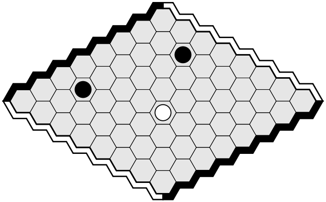
- Given a discovered SC, we try to shrink its carrier
- Cells outside the SC for player P can be assigned to $\overline{P}$
- Inferior cell analysis may identify $\overline{P}$-fill-in
- These cells can be deleted from the SC's carrier

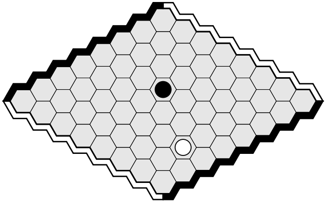
Proof Set Transpositions



- While solving states we track the winning strategy's carrier
- The losing player's stones can be any combination of cells outside of this carrier
- We can store the result for all these combinations as well

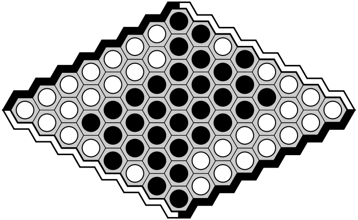
Player Exchange Transpositions





- Want to translate a solved state to equivalent ones with players reversed
- Mirroring stones and reversing their colors is not adequate
- Stone must be added or removed; depends on player to move and who won

Current Results



- 7×7: 10 minutes
- 8×8: 300 hours and 10^8 internal nodes
- 9×9: Cannot solve any opening in two weeks time

Feature Contributions on 7 $\times$ 7

feature f	**only f off**		**only f on**	
	time	**nodes**	**time**	**nodes**
rotation/transposition deduction	2.17	2.22	0.43	0.43
decompositions	1.29	1.51	0.68	0.61
proof set reduction	0.98	1.01	1.03	0.87

Summary

- New: decompositions, proof set reduction, transposition deductions
- Enhanced H-search, inferior cell analysis
- First automated solver for 8×8 Hex openings

Future Work

- 9×9 at least 3 magnitudes more difficult
- Depth-first proof-number search (parallelized)
- Further improve inferior cell analysis, decompositions, etc

Any Questions?

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