Solving 8 × 8 Hex

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Hex Rules and Properties

Rules

- Two players alternate turns playing on any empty cell
- Stones are permanent (no moving or capturing)
- Goal is to connect your two sides of the board
Hex Rules and Properties

Properties
- Extra P-stones never disadvantageous for player P
- Draws are impossible
- First player wins: strategy-stealing argument
- Determining winner is PSPACE-complete
Previously Solved States

- Last milestone for automated Hex solvers in 2004
- All $7 \times 7$ openings solved in two weeks (Hayward et al)
- By hand, humans have solved centre opening on $9 \times 9$ (Yang) and a few openings on $8 \times 8$ (Mishima et al, Yang)
H-Search

- **H-Search**: algorithm that deduces existing connection strategies in a given Hex position (Anshelevich)
- **Virtual connections** (VC): 2nd-player connection strategy
- **Semi-connections** (SC): 1st-player connection strategy
- **Carrier**: empty cells required for a connection strategy
Mustplay

- Identifying a winning VC terminates search
- Identifying winning SCs immediately prunes losing moves
- **Mustplay**: intersection of winning opponent SC carriers
Graph-theoretic properties and combinatorial game theory

*Fill-in:* can add stones to the board without changing its win/loss value

*Reversible and dominated* moves: can be pruned from consideration
Opposite Color Bridges

- If a P-chain is adjacent to both $\overline{P}$ edges, then splits board into two independent regions.
- Easy to detect these decompositions, but very rare.
- *Opposite-color bridges*: can treat the two carrier cells as non-adjacent.
Two chains *touch* if they are adjacent or form an opposite-color bridge

*Split decomposition:* when a P-chain touches both \(\overline{P}\)-edges
Four-sided decompositions

- *Four-sided decomposition*: a 4-cycle of touching Black and White chains
- If player P has a VC connecting the two P-chains of a four-sided decomposition, the region can be filled-in with P-stones
During search we identify previously-unknown winning SCs
Can use discovered SCs to further reduce mustplay
The smaller the SC carrier, the more moves can be pruned
Proof Set Reduction

- Given a discovered SC, we try to shrink its carrier
- Cells outside the SC for player $P$ can be assigned to $\overline{P}$
- Inferior cell analysis may identify $\overline{P}$-fill-in
- These cells can be deleted from the SC’s carrier
While solving states we track the winning strategy’s carrier
The losing player’s stones can be any combination of cells outside of this carrier
We can store the result for all these combinations as well
Player Exchange Transpositions

- Want to translate a solved state to equivalent ones with players reversed
- Mirroring stones and reversing their colors is not adequate
- Stone must be added or removed; depends on player to move and who won
Current Results

- 7 × 7: 10 minutes
- 8 × 8: 300 hours and $10^8$ internal nodes
- 9 × 9: Cannot solve any opening in two weeks time
## Feature Contributions on $7 \times 7$

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<th>feature</th>
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<th>only $f$ on</th>
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Summary

- New: decompositions, proof set reduction, transposition deductions
- Enhanced H-search, inferior cell analysis
- First automated solver for $8 \times 8$ Hex openings

Future Work

- $9 \times 9$ at least 3 magnitudes more difficult
- Depth-first proof-number search (parallelized)
- Further improve inferior cell analysis, decompositions, etc
Any Questions?

Thanks to:

- NSERC, iCORE, AIF, Martin Müller, Jonathan Schaeffer, Lorna Stewart for funding support
- University of Alberta GAMES group and referees for helpful comments