SOLVING 10x10 HEX
TCGA WORKSHOP KEYNOTE PRESENTATION

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computing UAlberta

2013 June 29
THANK YOU

- invitation Prof Mark Kayll UMontana
- solving $10 \times 10$ Hex joint with Jakub Pawlewicz
- builds on work with B Arneson, P Henderson
- machine Martin Müller
- photo courtesy MIT Museum, MIT, Cambridge MA
- Natural Sciences and Engineering Research Council of Canada
1. HEX
2. KNOWLEDGE
3. SEARCH
4. 10x10
1942 HEX

**RULES**

- 2 players, alternate moves
- win: connect your two sides

Solving 10x10 Hex
1942 HEX

RULES

- 2 players, alternate moves
- win: connect your two sides
**PROOF**

- lemma: extra $X$-cell ok for player $X$
- lemma: no draws
- suppose $P_2$ has win strategy $S_2$
- then $P_1$ can move anywhere, forget move, and follow $S_2$
- thus $P_1$ has win strategy, contradiction $\square$
NO-DRAW

solving 10x10 hex
**n x n+1 Hex: longer-side win**

Hex is a two-player turn-based board game played on a rhombus-shaped grid of hexagonal cells. The players take turns placing stones of their color on the board until no more moves are possible. The objective is to surround as much space as possible with your color.

In a **10x10** Hex game, the longer side wins. This property is a consequence of the game's unique rules and the geometry of the board.

**Properties**
- Shannon machine
- Provably hard
- Humans vs. Computers
N x n+1 Hex: longer-side win
1951 Shannon machine
1951 Shannon Machine

- play on any graph
- two marked vertices
- black move: ‘short’ any vertex (make nbrs clique)
- white move: ‘cut’ any vertex (delete)
- black wins iff two marked vertices are shorted (connected)

- generalizes Hex
1951 Shannon Machine
1951 Shannon machine
1951 Shannon machine
PROVABLY HARD

- 1975 Even & Tarjan  
  Shannon v-switching: PS-c
- 1981 Stefan Reisch  
  Hex: PS-c
- 2000 Clay Math Inst  
  P vs NP: $1 000 000
## HUMANS

### SOLVED OPENINGS

<table>
<thead>
<tr>
<th>Year</th>
<th>Player</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>Yang</td>
<td>17/49 7x7</td>
</tr>
<tr>
<td>2002</td>
<td>Yang</td>
<td>8x8</td>
</tr>
<tr>
<td>2003</td>
<td>Yang</td>
<td>9x9</td>
</tr>
<tr>
<td>2004</td>
<td>Noshita</td>
<td>7x7</td>
</tr>
<tr>
<td>2005</td>
<td>Noshita</td>
<td>8x8</td>
</tr>
<tr>
<td>2006</td>
<td>Mishima</td>
<td>8x8</td>
</tr>
</tbody>
</table>
# Computers

## Solved Openings

<table>
<thead>
<tr>
<th>Year</th>
<th>Player(s)</th>
<th>Date</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>Enderton</td>
<td>6x6</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>van Rijswijck</td>
<td>6x6</td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>H Bjö Joh Kan Po vRij</td>
<td>5d</td>
<td>7x7</td>
</tr>
<tr>
<td>2007</td>
<td>Rasmussen et al.</td>
<td>7x7</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>Arneson H Henderson</td>
<td>4d</td>
<td>8x8</td>
</tr>
<tr>
<td>2010</td>
<td>A H H</td>
<td>25d</td>
<td>some 9x9</td>
</tr>
<tr>
<td>2012</td>
<td>Pawlewicz H</td>
<td>110d x 24 thread</td>
<td>9x9</td>
</tr>
<tr>
<td>2013</td>
<td>Pawlewicz H</td>
<td>63d x 24 thread</td>
<td>centre 10x10</td>
</tr>
</tbody>
</table>
COMPUTERS
COMPUTERS

SOLVING 10x10 HEX
COMPUTERS

solving 10x10 hex
COMPUTERS
COMPUTERS

Solving 10x10 Hex
COMPUTERS

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Solving 10x10 Hex
COMPUTERS

SOLVING 10x10 HEX
COMPUTERS

Solving 10x10 Hex

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virtual connections: combining rules, mustplay
inferior cells: dead, captured, etc.
A VIRTUAL CONNECTION

HEX
KNOWLEDGE
SEARCH
10x10
VIRTUAL CONNECTIONS
INFERIOR CELLS

solving 10x10 hex

PAWEWICZ + HAYWARD@UALBERTA.CA
A VIRTUAL CONNECTION

Hex Knowledge Search 10x10 Virtual Connections Inferior Cells

Pawlewicz + Hayward@ualberta.ca Solving 10x10 Hex
COMBINING RULE: AND (FULL)
COMBINING RULE: AND (FULL)
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COMBINING RULE: AND (FULL)
COMBINING RULE: AND (FULL)
<table>
<thead>
<tr>
<th>HEX</th>
<th>KNOWLEDGE</th>
<th>SEARCH</th>
<th>10x10</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIRTUAL CONNECTIONS</td>
<td>INFERIOR CELLS</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**COMBINING RULE:** AND (SEMI)
COMBINING RULE: AND (SEMI)
COMBINING RULE: AND (SEMI)
COMBINING RULE: OR
COMBINING RULE: OR
COMBINING RULE: OR
COMBINING RULE: OR
WHERE MUST WHITE PLAY?
WHERE MUST WHITE PLAY?
WHERE MUST WHITE PLAY?
WHERE MUST WHITE PLAY?
BLACK-DOMINATED (DOT SUPERIOR)
BLACK-CAPTURED
BLACK-DOMINATED (DOT SUPERIOR)
BLACK-CAPTURE-REVERSIBLE (TO WHITE DOT)
BLACK FILL DECOMPOSITION

```
HEX
KNOWLEDGE
SEARCH
10x10
VIRTUAL CONNECTIONS
INFERIOR CELLS
```

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STAR DECOMPOSITION
BLACK STAR DECOMP DOMINATION
modify H-search

- and/or combining rules + capture
proof number search

negamax p,d values

1,4

2,3

3,5

1,3 1,1 1,1 1,1 2,2

1,3 1,1 1,1 1,1 1,1 1,1 1,1 1,1

1,2 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1

1,2 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1

1,2 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1

1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1

1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1

1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1

1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1

1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1

1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1

1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1

1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1

1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1
proof number search

```
  2,5
 /    /
1,4   2,2
|      |
1,3 1,1 1,1 1,1 1,1 2
|      |    |
3,1   1,3 1,1 1,1 1,1 1,1 1,2
|      |    |    |    |    |
1,3 1,1 1,1 1,1 1,1 1,1 0,∞ 1,1
|      |    |    |    |    |    |    |    |
1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1
    |    |    |    |    |    |    |    |    |
1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1
    |
1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1
```

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proof number search

3,4

1,4

1,3

2,4

1,3 1,1 1,1 1,1 1,2 1,1

0,2 1,1 1,1 1,1 1,4 1,1 1,1 1,1 1,3 1,2

1,1 1,1 1,1 1,1 ∞,0 1,1 1,2 1,1 1,1 1,1 1,1

1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1
F-DFPNS

- PNS  Allis et al
F-DFPNS

- PNS  Allis et al
- DFPNS  Nagai
F-DFPNS

- PNS  Allis et al
- DFPNS  Nagai
- DFPNS in Hex?
F-DFPNS

- PNS       Allis et al
- DFPNS     Nagai
- DFPNS in Hex ?
- ... requires non-incremental H-search :(
F-DFPNS

- PNS    Allis et al
- DFPNS   Nagai
- DFPNS in Hex ?

...requires non-incremental H-search :(
...uniform branching factor :(
F-DFPNS

- PNS  Allis et al
- DFPNS  Nagai
- DFPNS in Hex ?

... requires non-incremental H-search :(
... uniform branching factor :(
idea: move ordering + DFPNS = F-DFPNS
F-DFPNS (1)

- expand node
- consider first $b + \lceil f \times 6 \rceil = 4$ (of 6) live children
F-DFPNS (2)

- discover move 3 loses
- consider first $b + \lceil f \times 5 \rceil = 4$ (of 5) live children
F-DFPNS (3)

- discover move 5 loses
- consider first $b + \lceil f \times 4 \rceil = 3$ (of 4) live children
F-DFPNS (4)

- discover move 2 wins, so . . .
- . . . root solved without exploring 6th move

Σ

\[ \begin{array}{c}
\hline
\text{F-DFPNS (4)} \\
\hline
\end{array} \]
SOLVING 10x10

- stronger VC computations
- scalable parallel DFPN S
Pawlewicz: stronger VC computations

- faster and/or-rule VC computation
- limit form of new VCs, so never redundant
- find fewer VCs, but solve 2 to 10 times faster
EXAMPLE: VCS TO SIDE
EXAMPLE: SEMIS
EXAMPEL: SEMIS
EXAMPLE: SEMIS

solving 10x10 hex
EXAMPLE: SEMIS
GREEDY UNION SEMIS TO GET FULL
BLOCK CELL TO GET ANOTHER VC
BLOCK CELL TO GET ANOTHER VC
BLOCK CELL TO GET ANOTHER VC
BLOCK CELL TO GET ANOTHER VC
Pawlewicz: scalable parallel DFPNS

- parallel PNS: keep tree in memory? e.g. I-Chen Wu connect6
- Hex: leaf computations fast, so tree too big
- how to assign jobs to processors?
  - jobs too long: computation redundant
  - jobs too short: too much client/server traffic
- solution: MaxWorkPerJob
SP DFPN S FEATURES

- $1 + \varepsilon$ variant of DFPN
- advanced TT resolution: upon collision, search next $k$ (say 4) cells for empty location; if none found, overwrite location with smallest work job
- once node computation assigned to leaf, use virtual win/loss so new threads go elsewhere
- ...so compute virtual (dis)proof numbers
- shared TT: many-read / 1-write locks
- tune MaxWorkPerJob
- for Hex: use Focussed DFPN S
SP DFPNS PERFORMANCE

- speedup test: 8 hardest 8x8 openings, 8 11x11 positions
- speedup performance: 11.8 on 16 threads (.74)
- solved all 9x9 openings
- solved centre 10x10 opening
SP DFPNS PERFORMANCE

Solving 10x10 hex
SP DFPN S PERFORMANCE

Solving 10x10 hex

Pawlewicz: stronger VC engine
Pawlewicz: SPDFPNS

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## SP DFPN S Performance

<table>
<thead>
<tr>
<th>opening</th>
<th>#threads</th>
<th>time</th>
<th>winner</th>
</tr>
</thead>
<tbody>
<tr>
<td>a2</td>
<td>8/24</td>
<td>68d09:40:18</td>
<td>black</td>
</tr>
<tr>
<td>a3</td>
<td>8</td>
<td>80d08:37:34</td>
<td>white</td>
</tr>
<tr>
<td>a4</td>
<td>8</td>
<td>33d14:06:03</td>
<td>black</td>
</tr>
<tr>
<td>a5</td>
<td>8</td>
<td>65d04:14:52</td>
<td>black</td>
</tr>
<tr>
<td>a6</td>
<td>24</td>
<td>110d14:35:06</td>
<td>black</td>
</tr>
<tr>
<td>a7</td>
<td>24</td>
<td>4d08:56:03</td>
<td>white</td>
</tr>
<tr>
<td>a8</td>
<td>24</td>
<td>6d14:21:30</td>
<td>black</td>
</tr>
</tbody>
</table>
# SP DFPNS Performance

<table>
<thead>
<tr>
<th>opening</th>
<th>#threads</th>
<th>time</th>
<th>winner</th>
</tr>
</thead>
<tbody>
<tr>
<td>b2</td>
<td>8</td>
<td>53d15:18:21</td>
<td>black</td>
</tr>
<tr>
<td>b4</td>
<td>8</td>
<td>29d23:53:14</td>
<td>black</td>
</tr>
<tr>
<td>b6</td>
<td>8</td>
<td>1d21:52:28</td>
<td>black</td>
</tr>
<tr>
<td>b7</td>
<td>8</td>
<td>4d17:19:13</td>
<td>black</td>
</tr>
<tr>
<td>c2</td>
<td>24</td>
<td>1d08:42:57</td>
<td>black</td>
</tr>
<tr>
<td>i1</td>
<td>24</td>
<td>6d00:51:25</td>
<td>black</td>
</tr>
<tr>
<td>10x10:f5</td>
<td>24</td>
<td>63d20:44:30</td>
<td>black</td>
</tr>
</tbody>
</table>
### How Long Until 11x11?

<table>
<thead>
<tr>
<th>Size</th>
<th>States (approx)</th>
<th>Center Cell: Solver Fn Calls</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x2</td>
<td>9.0 e 0</td>
<td>0</td>
</tr>
<tr>
<td>3x3</td>
<td>5.5 e 1</td>
<td>0</td>
</tr>
<tr>
<td>4x4</td>
<td>7.6 e 5</td>
<td>0</td>
</tr>
<tr>
<td>5x5</td>
<td>4.0 e 9</td>
<td>0</td>
</tr>
<tr>
<td>6x6</td>
<td>4.0 e 14</td>
<td>2</td>
</tr>
<tr>
<td>7x7</td>
<td>1.5 e 20</td>
<td>68</td>
</tr>
<tr>
<td>8x8</td>
<td>1.0 e 27</td>
<td>19 554</td>
</tr>
<tr>
<td>9x9</td>
<td>2.7 e 34</td>
<td>912 352</td>
</tr>
<tr>
<td>10x10</td>
<td>1.2 e 43</td>
<td>5 821 097 789</td>
</tr>
<tr>
<td>11x11</td>
<td>2.2 e 52</td>
<td>??? ??? ??? ???</td>
</tr>
</tbody>
</table>
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