Hex and Mathematics

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Natural Sciences and Engineering
Research Council of Canada

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Games Research
• history
• properties
• recent progress
  – game-playing programs
  – solving
  * dead cell analysis

Hex and Mathematics
H, van Rijswijck
to appear in *Discrete Mathematics*

Solving 7x7 Hex
H, Björnsson, Johanson, Kan, Po, vR
to appear in *Theoretical Computer Science*

Dead Cell Analysis in Hex and the Shannon Game
H, B, J, vR
submitted to *Proc. Graph Theory 2004*
1942 Hein

“Matematik betragtet som ’Spil
– Spillenes Matematik”
1948 Nash (& Gale)

“connecting topology and game theory”
1950 Parker Brothers

1952 Nash

Some Games and Machines for Playing Them

• no draws
• extra stones don’t hurt
• exists 1st-player-win strategy
1953 Shannon (& Moore)

Computers and Automata

“the problem of designing game-playing machines is fascinating . . .

“paradoxically, the positional judgment of this machine was good; its chief weakness was in end-game combinatorial play”
1957 Gardner

“played on the tiles of the bathroom floor”
1975 Schensted & Titus

Mudrack Y and Poly-Y

“whenever you feel you must use a car, try playing Y until the feeling passes”
no draws in $Y$  (or Hex)

proof: Schensted’s $Y$-reduction
reversing Y-reduction
1977 Berge

“l’art subtil du Hex”
virtual connections, mustplay
1981 Berge

“...to solve some Hex problem by using nontrivial theorems about combinatorial properties of sets ...”
1976  Even & Tarjan

generalization of Hex  PSACE-complete

1981  Reisch

Hex  PSPACE-complete

1984  Berge

“computers will never beat humans at Hex”
Hex computer programs

H, Yngvi Björnsson, Mike Johanson, Maryia Kazekевич, Morgan Kan, Nathan Po, Jack van Rijswijck

- resistance network
- virtual connections
- dead cell analysis
- mustplay

Game 4. Mongoose (black) defeats Six.
dead cell analysis (H-vR-B-J)
dead cells and induced paths

- node \( v \) is *dead* if,
  - for every completion of \( G - v \),
  - colour of \( v \) does not change winner
- *live* iff not dead
- Theorem: live iff on terminal-terminal induced path of reduced graph
death has consequences

• nodes \( P\)-captured: \( P \) has 2p-kill strategy

adding \( P\)-stones doesn’t change game

• nodes \( P\)-dominated: \( P \) has 1p-kill strategy

\( P \) need only consider first move

algorithmic considerations

• induced path closure NP-complete (Fellows)

• dead nodes often simplicial
dead cell analysis
solving Hex: opening winning moves