Notes on Hex

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Thankyou ...

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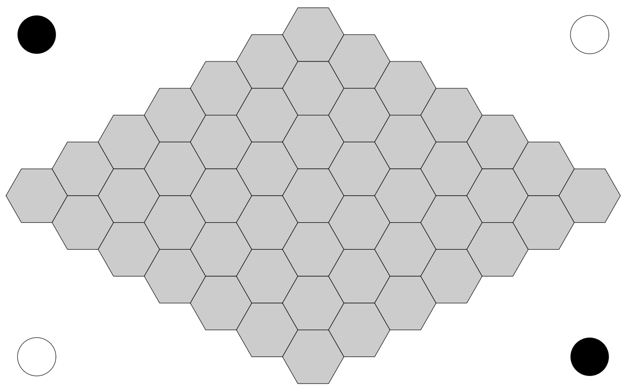
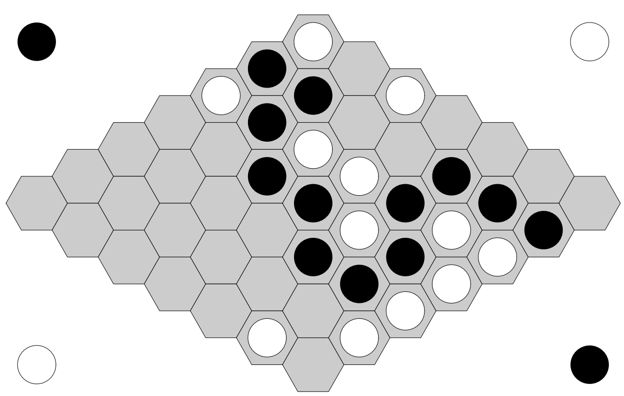
overview

- **history/properties**
- **recent results**
  - **game-playing programs**
  - **solving**
  - **dead cell analysis**

1942 Hein

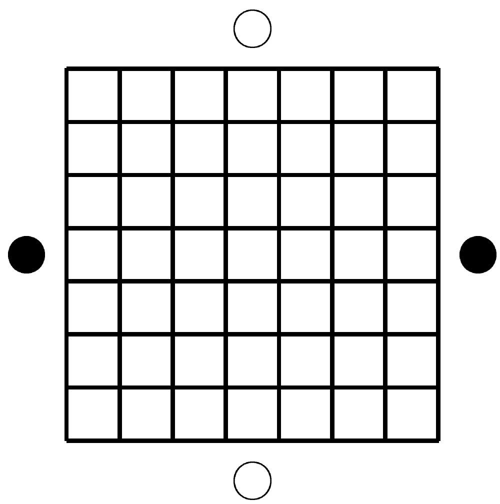
“math & games, games & math”

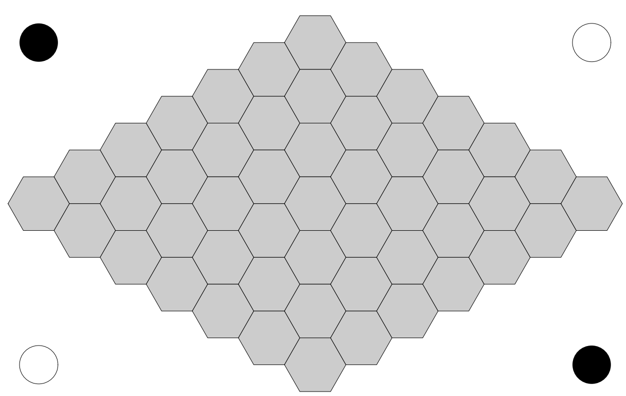




1948 Nash (& Gale)

“connecting topology and game theory”





Theorem (Hein-Nash)
exists 1st-player-win strategy for Hex

Lemma: extra stones don't hurt

- **supp. for P, win-strategy S for gamestate G**
- **supp. G' obtained from G by adding P-stone**
- **then for P, S yields win-strategy S' for G'**

Proof of theorem

- **assume exists 2pw strategy**
- **1st player makes 1st move anywhere**
- **1st player uses lemma and 2pw strategy to obtain 1pw strategy**
- **so exists 1pw contradiction**

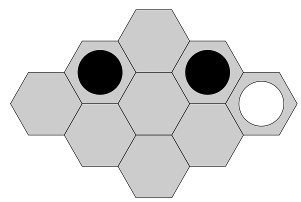
1953 Shannon (& Moore)

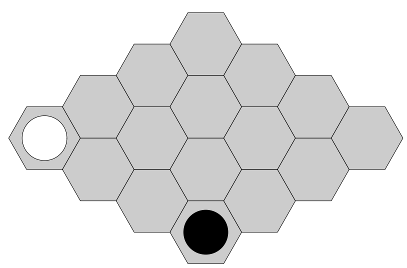
"the problem of designing game-playing machines is fascinating . . .

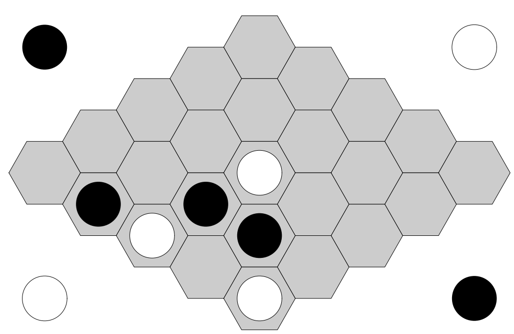
paradoxically, the postional judgment of this machine was good; its chief weakness was in end-game combinatorial play"

1957 Gardner

“played on the tiles of the bathroom floor”

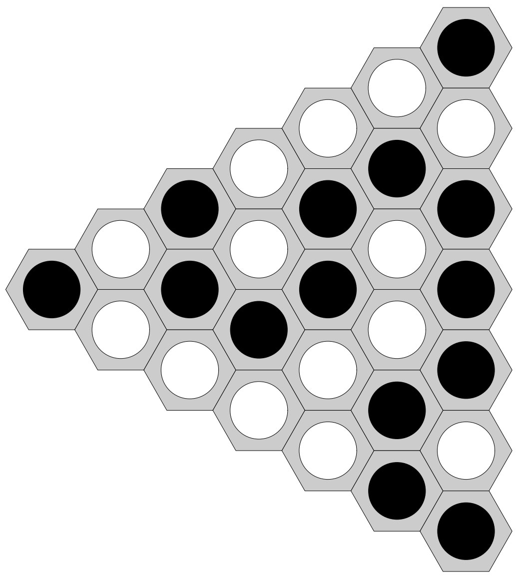




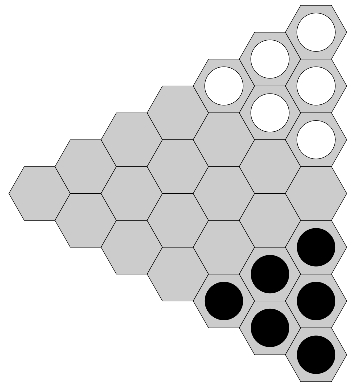


1975 Schensted & Titus

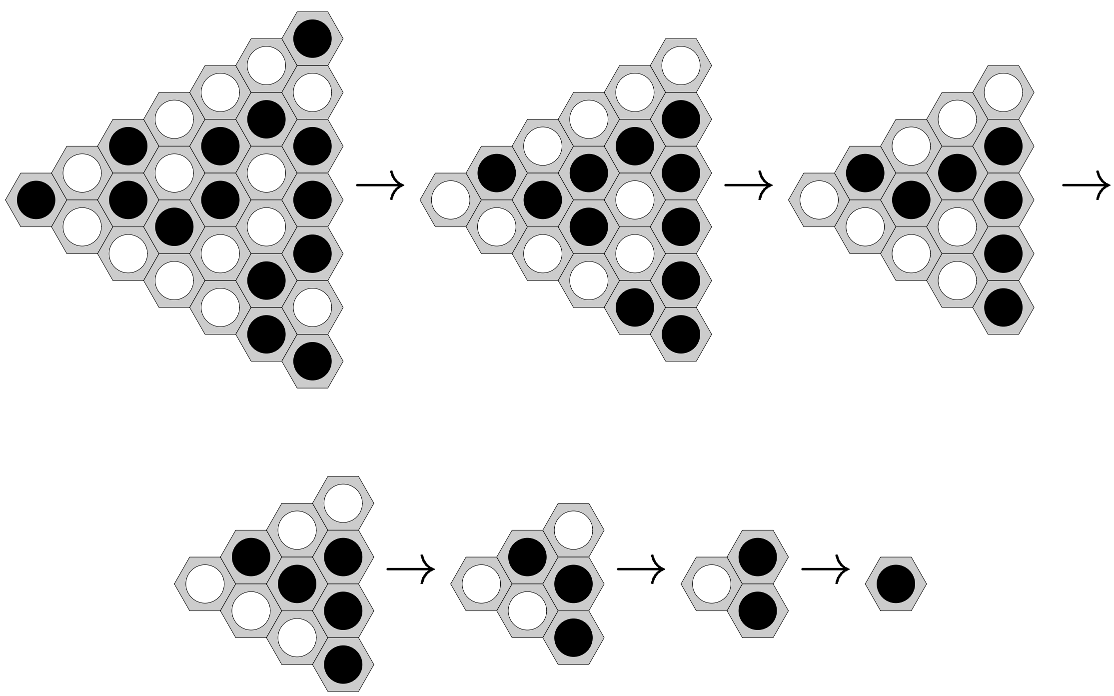
"whenever you feel you must use a car, try playing Y until the feeling passes"



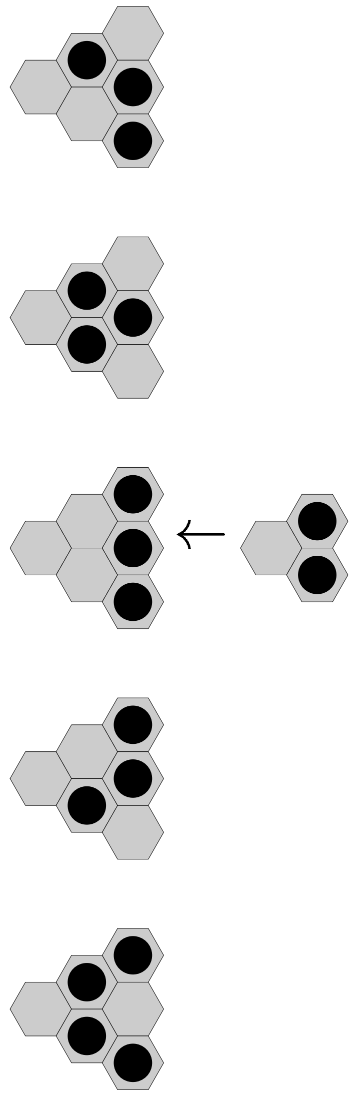
Hex special case of Y



Schensted's Y-reduction



reversing Y-reduction



Theorem (Schensted)
If Y-board completely covered with B/W stones, then B XOR W has win-set.

Proof (induction)

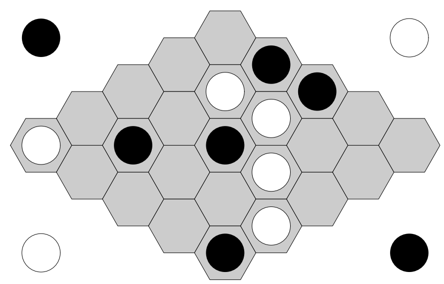
- **start with covered Y_n board**
- **Y-reduce to obtain covered Y_{n-1} board**
- **w.l.o.g. Y_{n-1} board has B-win-set**
- **corresp. B-cells on Y_n board form B-win-set**

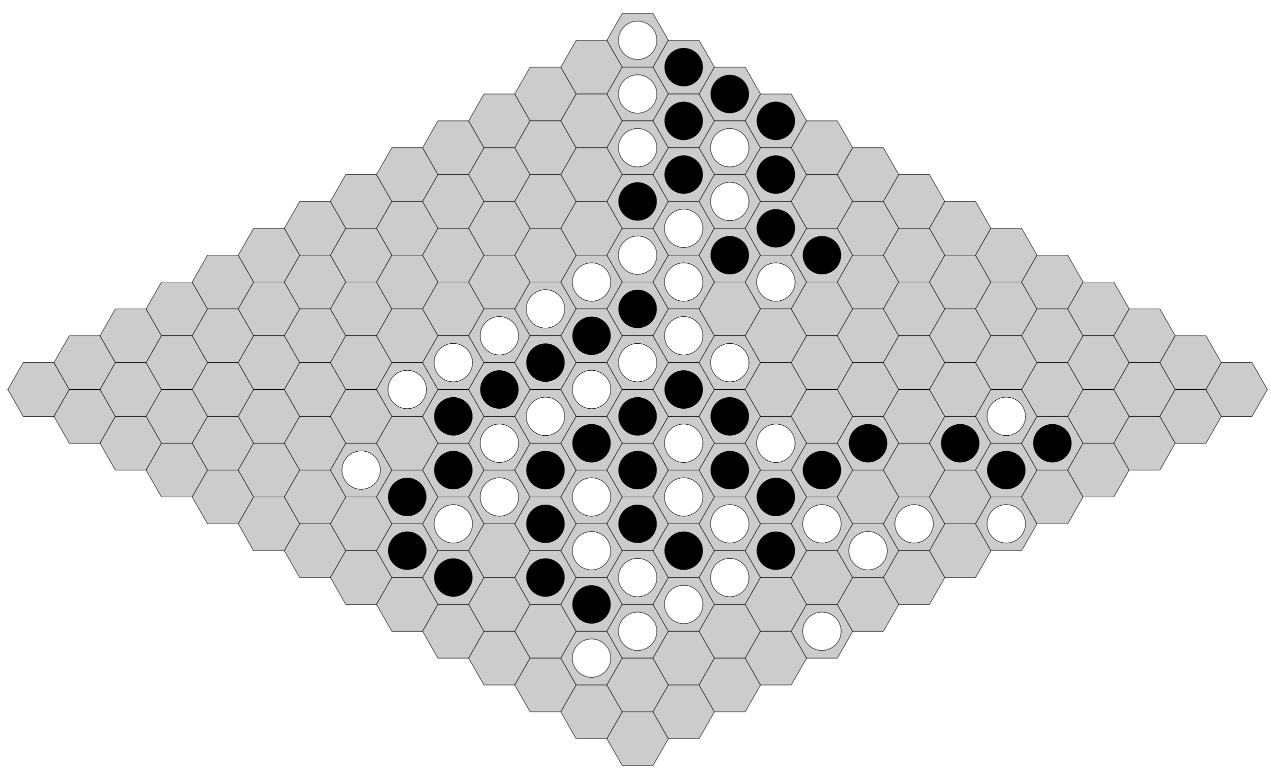
Corollary no draws in Y

Corollary no draws in Hex

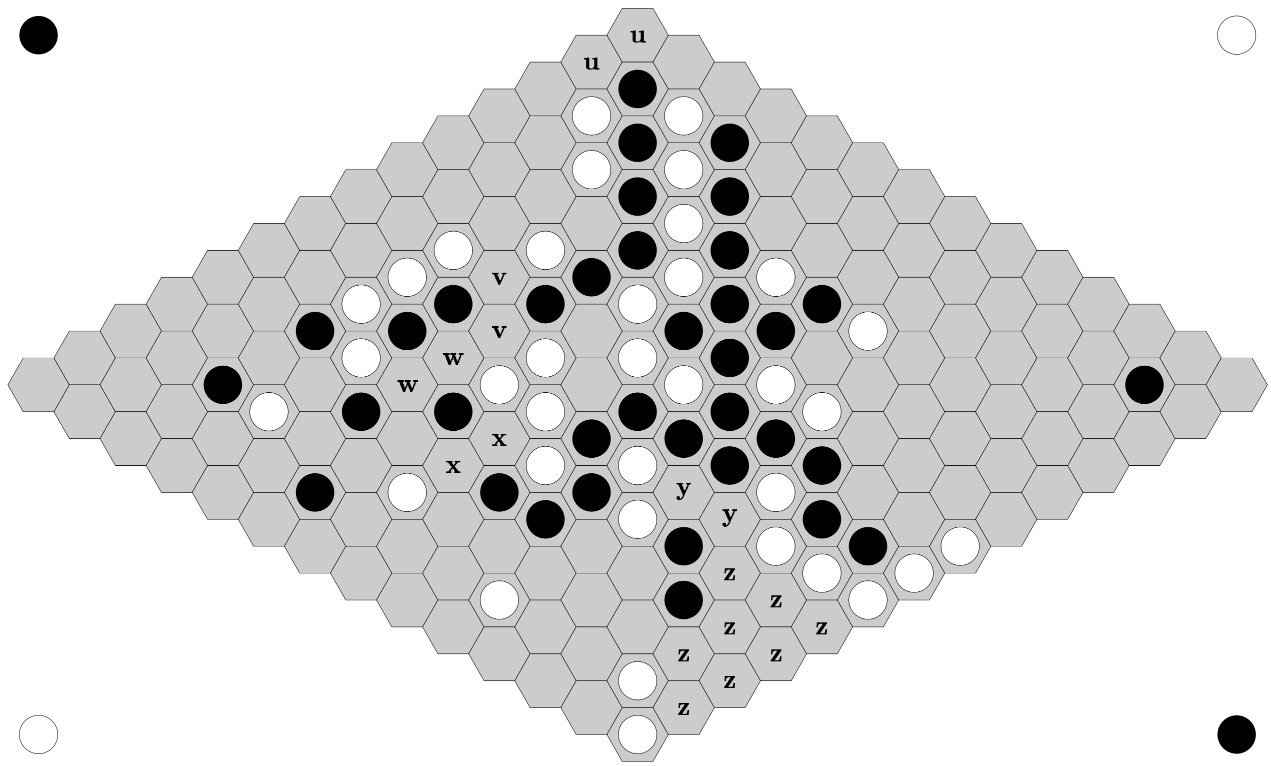
1977 Berge

"l'art subtil du Hex"



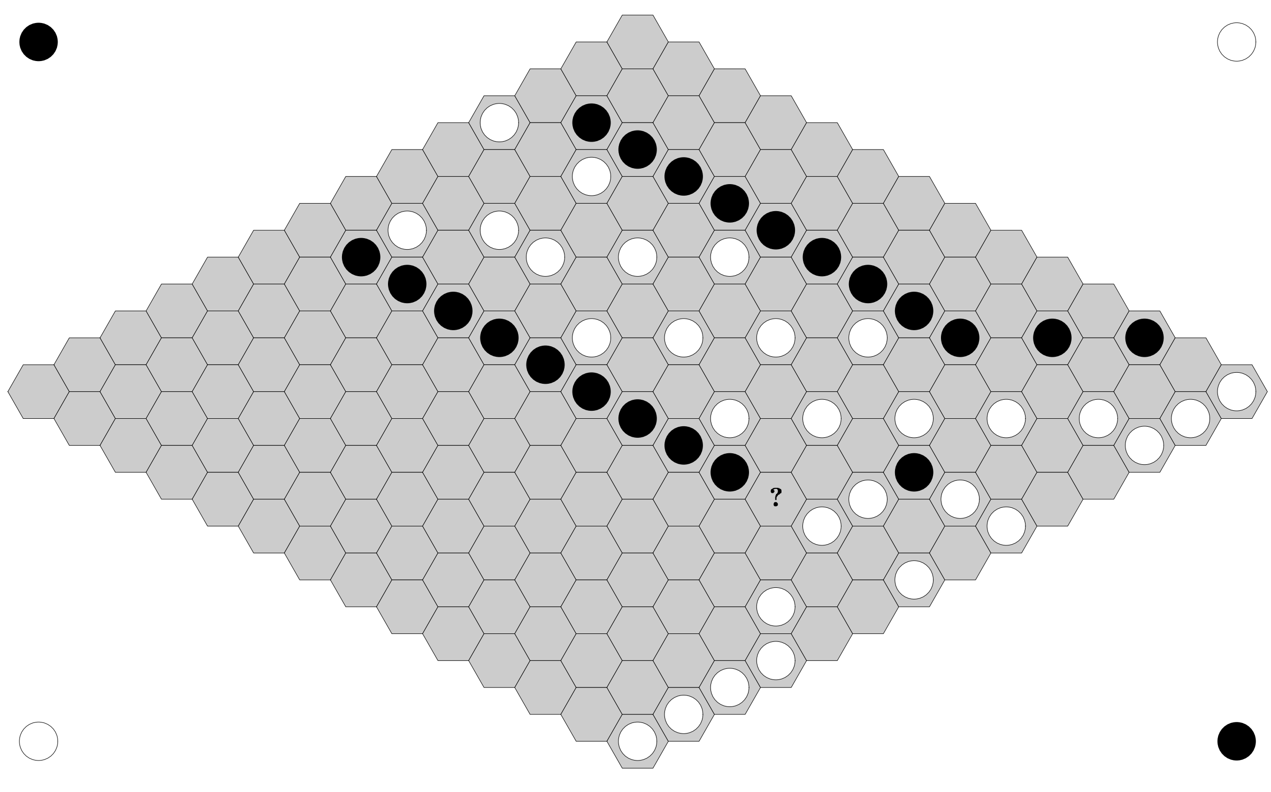


virtual connections, mustplay



1981 Berge

"...to solve some Hex problem by using nontrivial theorems about combinatorial properties of sets ..."



1976 Even & Tarjan

generalization of Hex PSACE-complete

1981 Reisch

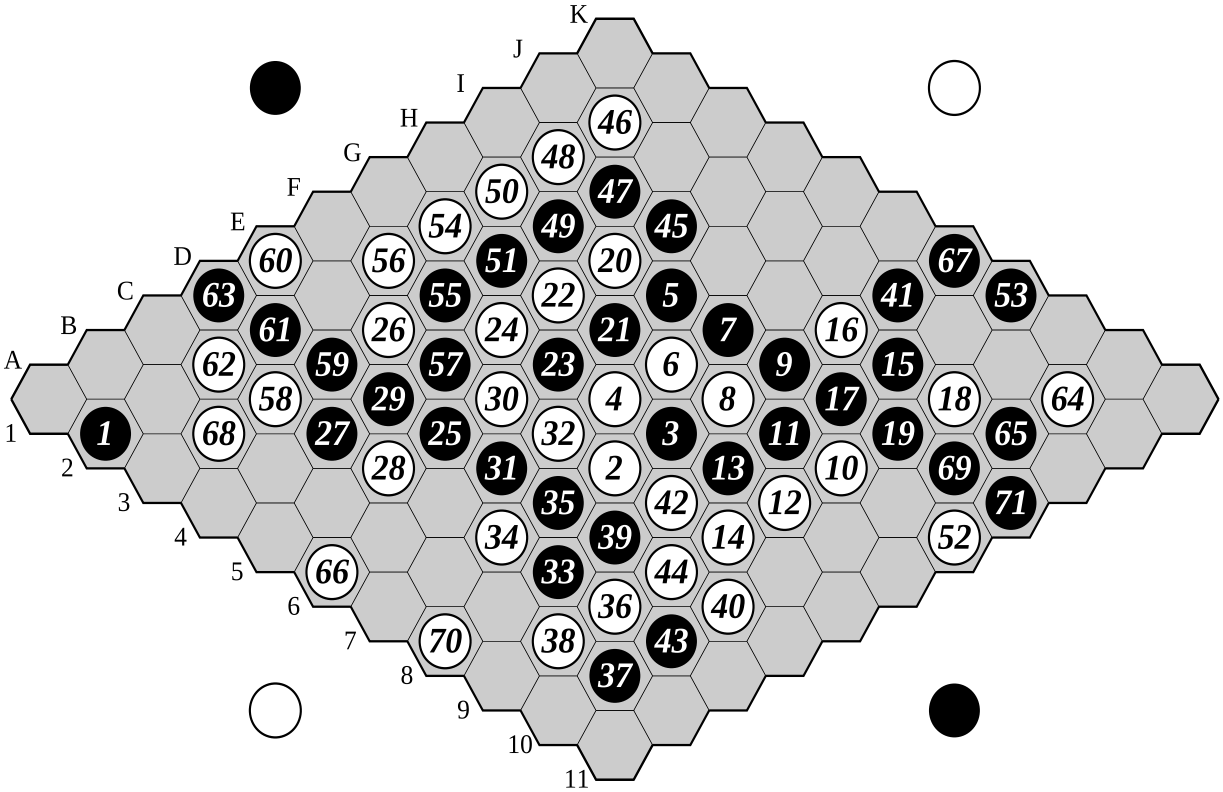
solving arbitrary Hex states is PSPACE-complete

1984 Berge

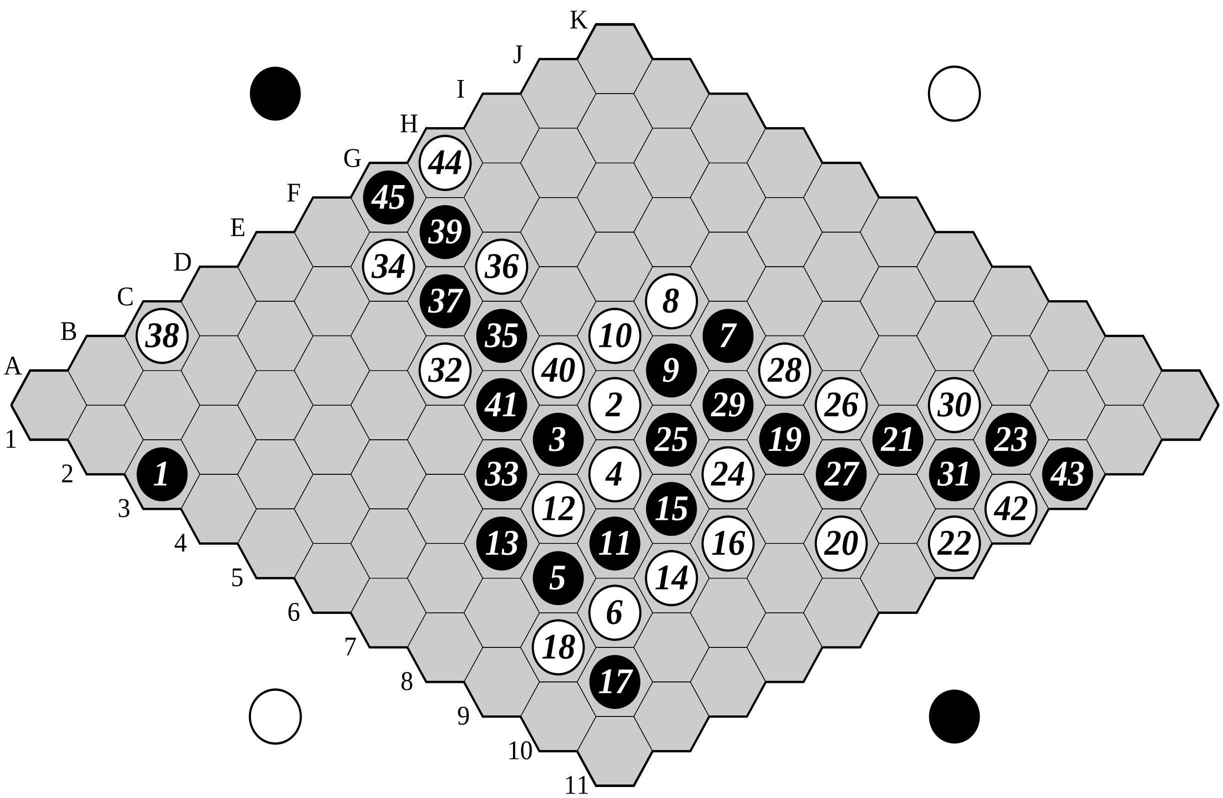
"computers will never beat humans at Hex"

Computer Games Olympiad

- **Hexy-Queenbee-KillerBee 8-4-0 London 2000**
- **Six-Mongoose 6-2 Graz 2003**
- **Six-Mongoose 5-1 Bar-Ilan 2004**

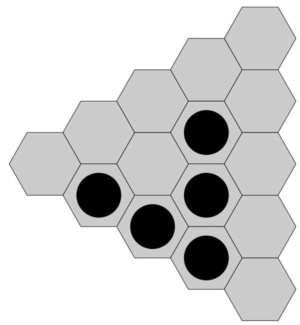


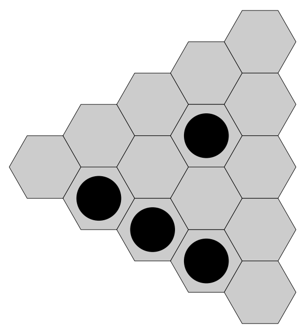
Bar-Ilan Game 3. Six (black) wins.

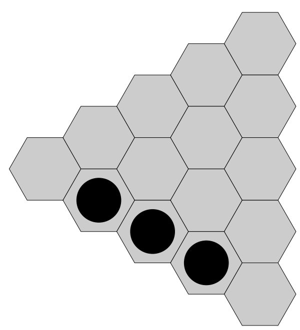


Bar-Ilan Game 4. Mongoose (black) wins.

dead cell analysis (H-vR-B-J)

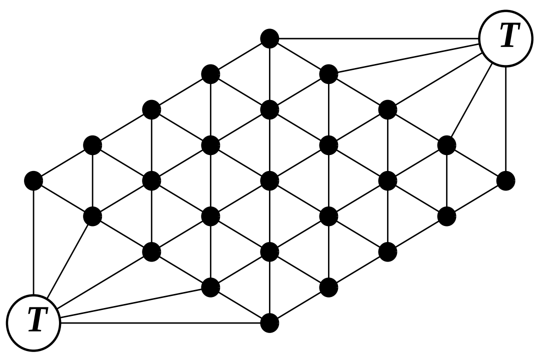


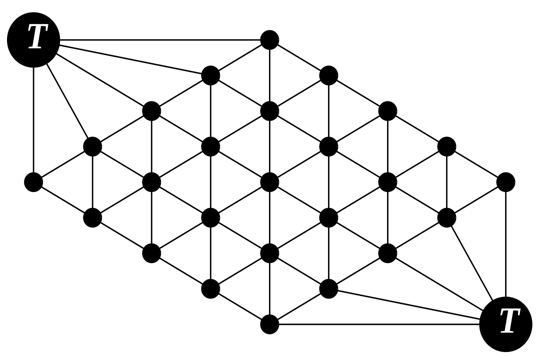


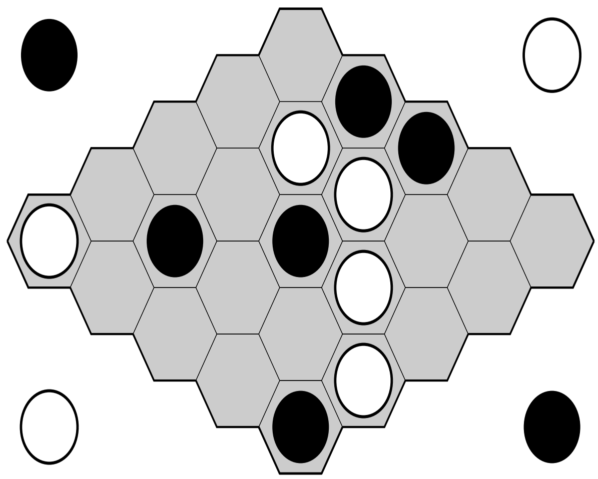


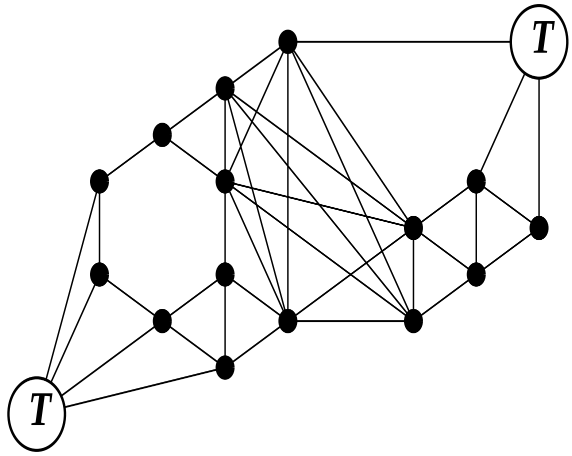
monophonic intervals

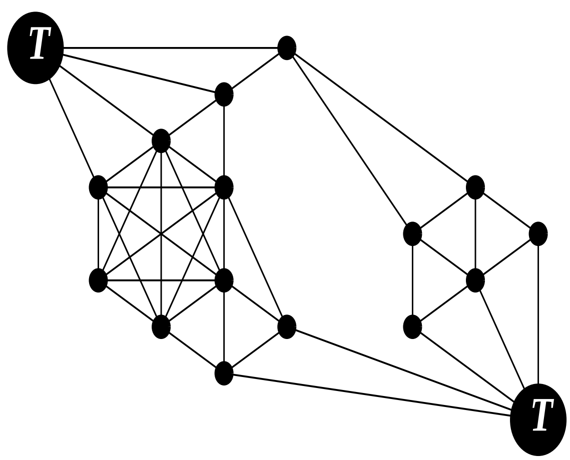
- **node** v **is** *dead* **if,**
  for every completion of $G - v$**,**
  colour of v **does not change winner**
- *live* **iff not dead**
- **Theorem: live iff on terminal-terminal monophonic interval of reduced graph**











- **computing m. i. NP-hard (Fellows)**
- **dead nodes often simplicial**

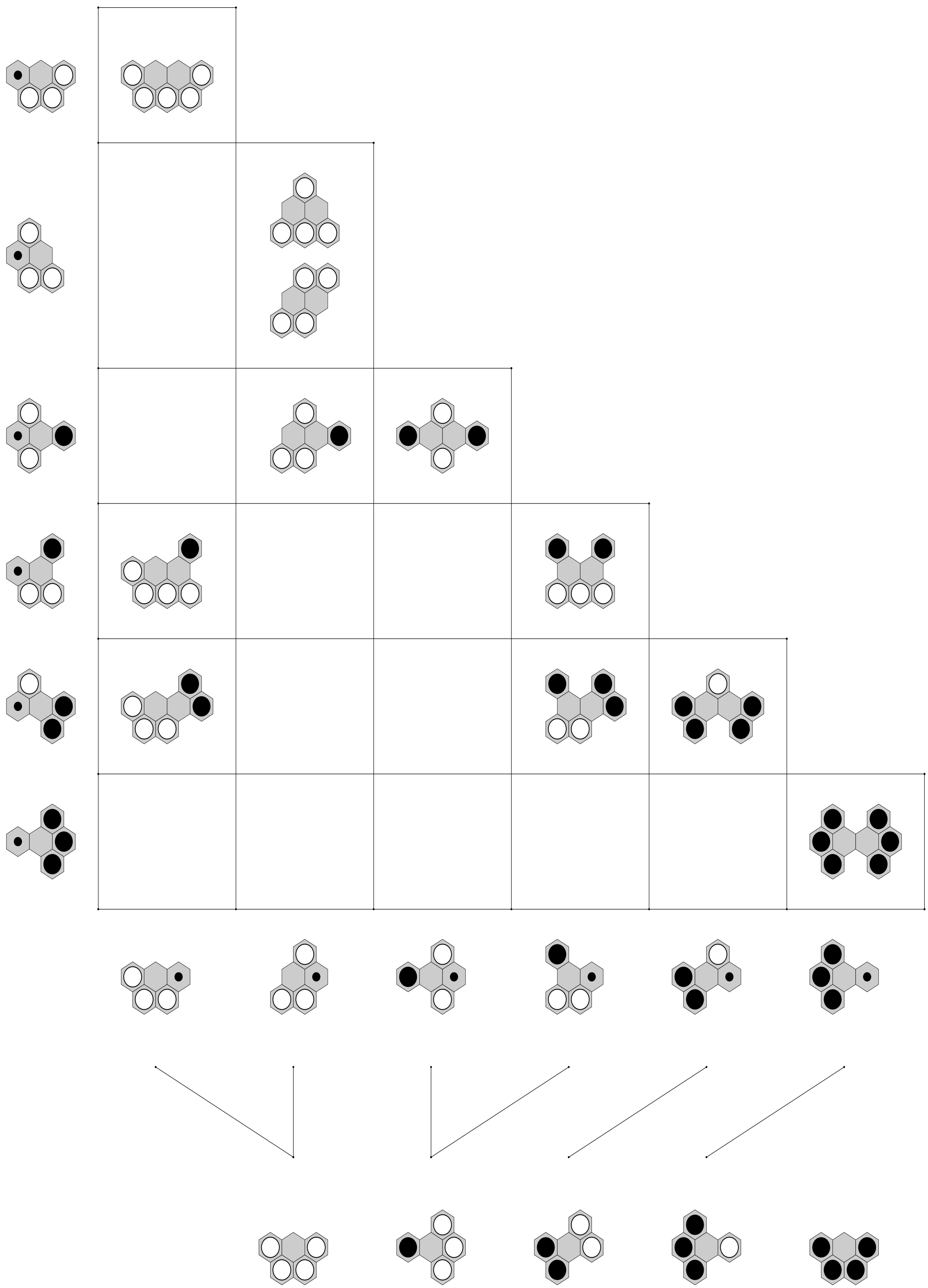
death has consequences

- **P-captured set of nodes:**

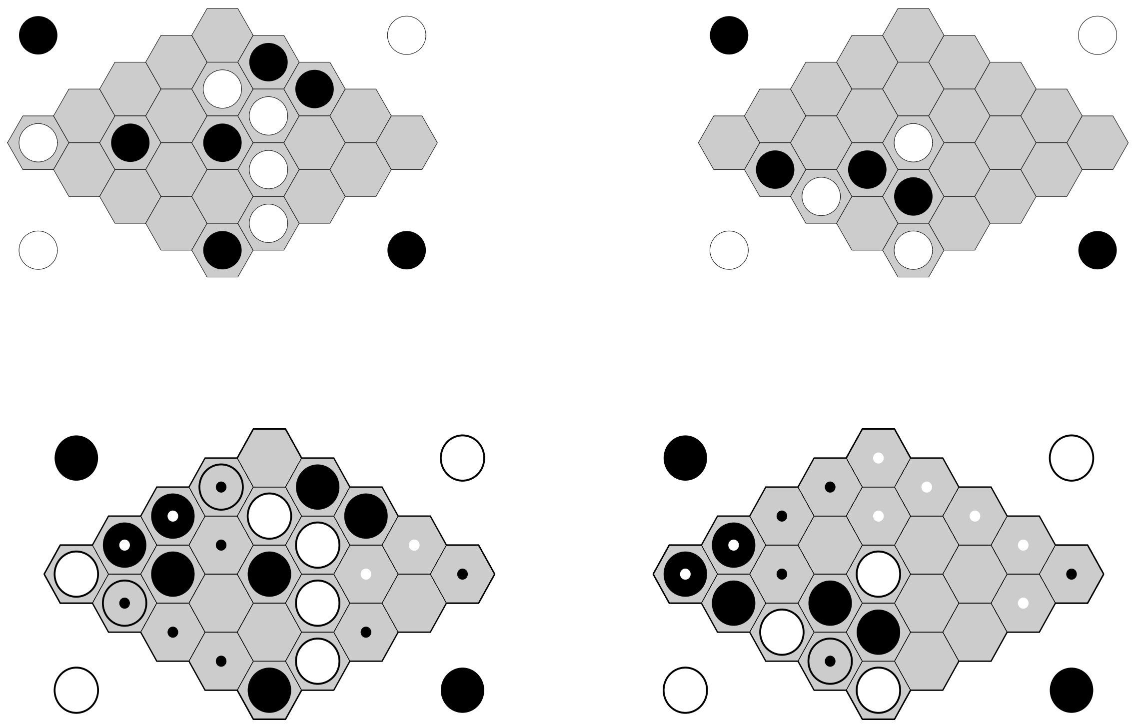
  adding P-stones doesn't change game

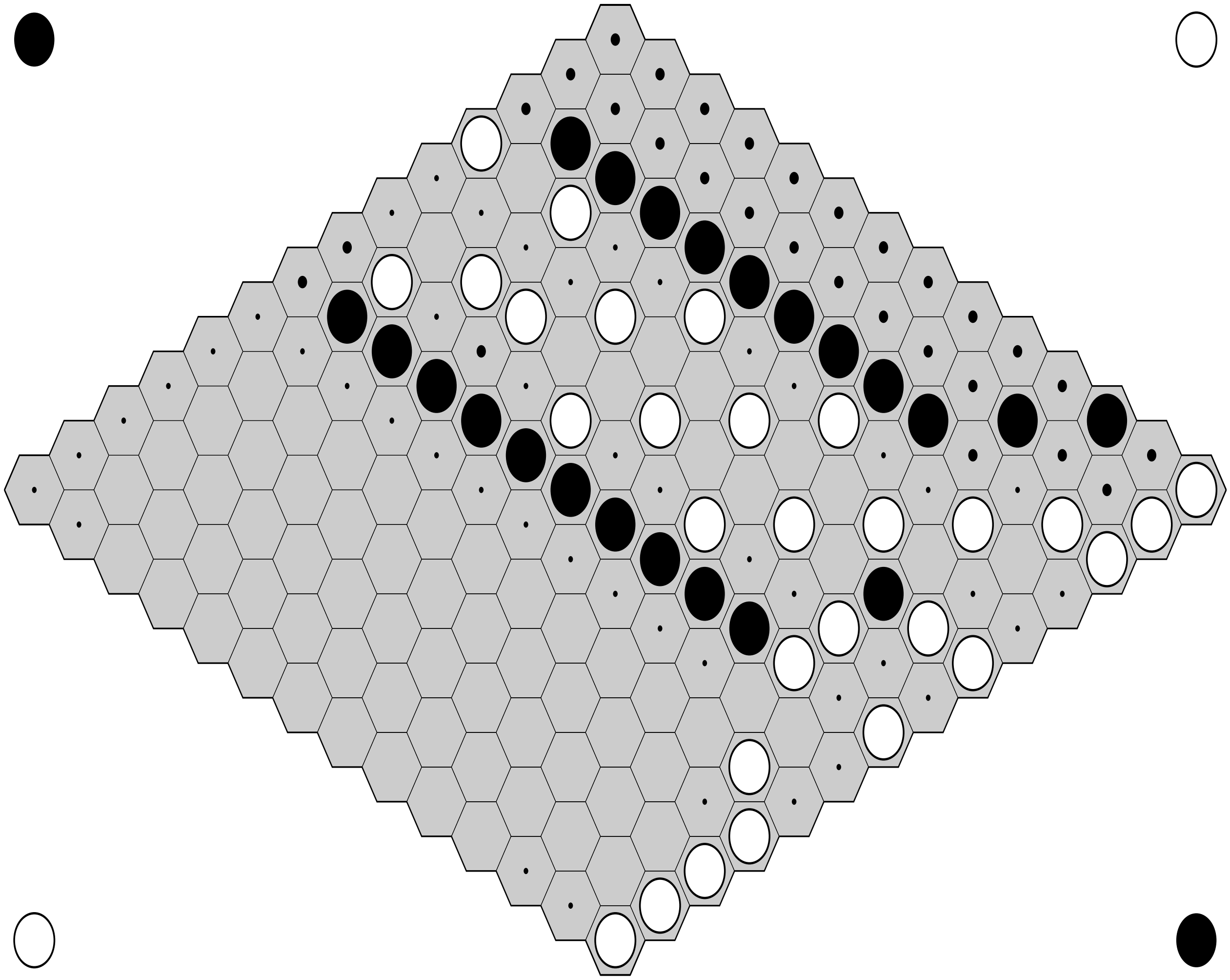
- **P-dominated set of nodes:**

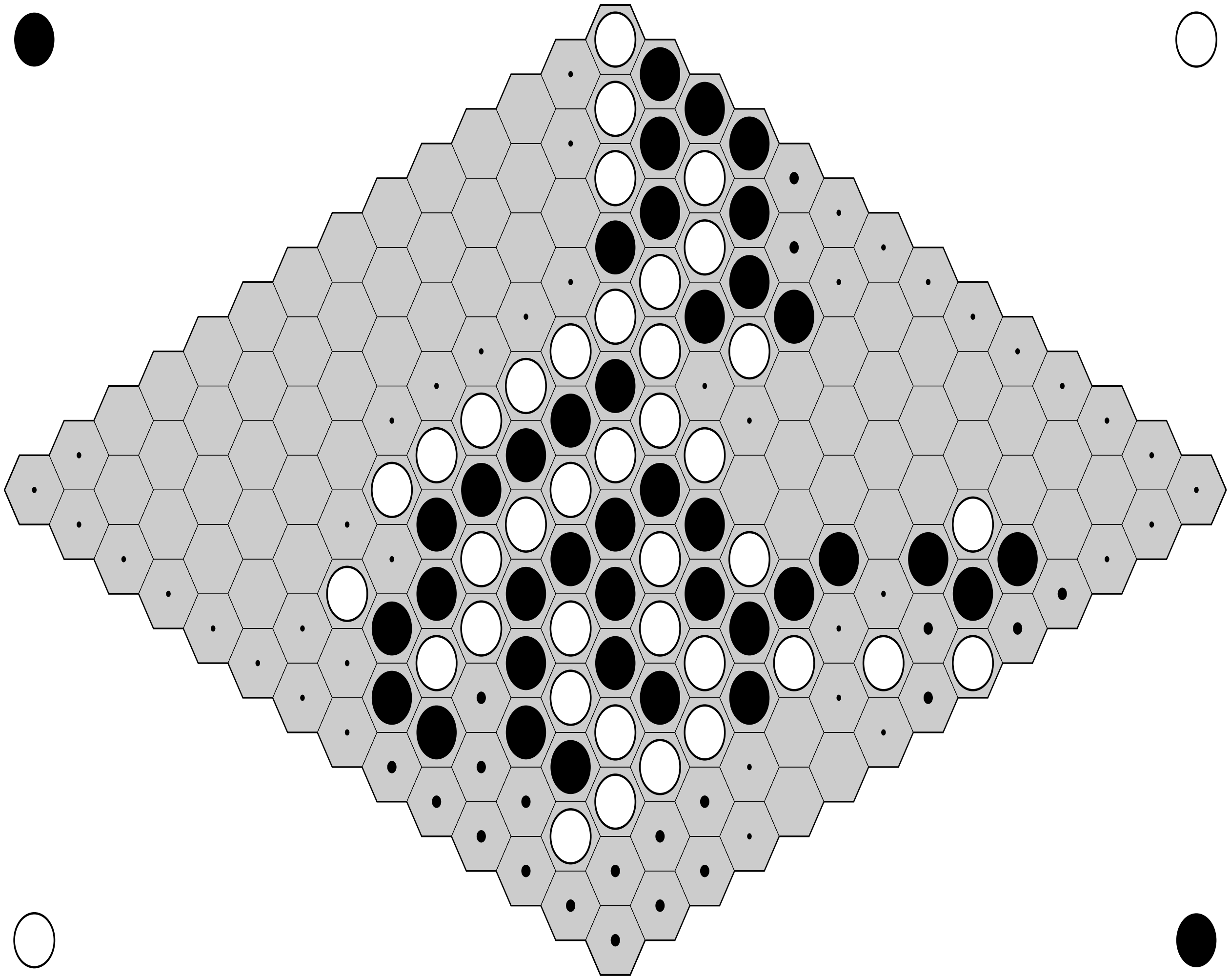
  some P-move in set P-captures the rest;
  such a move is P-dominating;
  P can ignore all other moves into the set



dead cell analysis







A set S of unoccupied nodes is

P**-captured:**

if S is empty, or
for each opponent-move to m in S

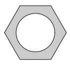
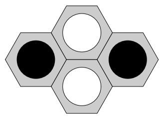
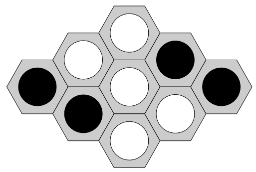
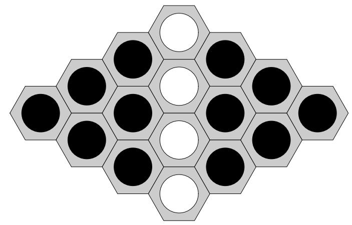
- $S - m$ **is** P**-dominated, and**
- **filling $S - m$ with P-stones makes m dead**

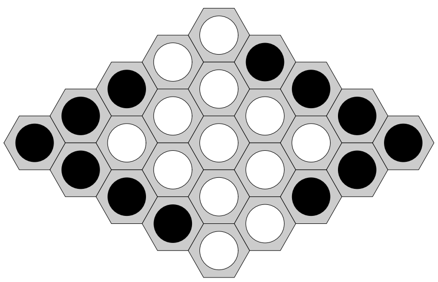
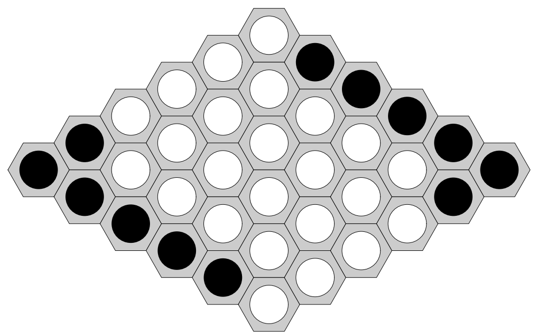
P**-dominated:**

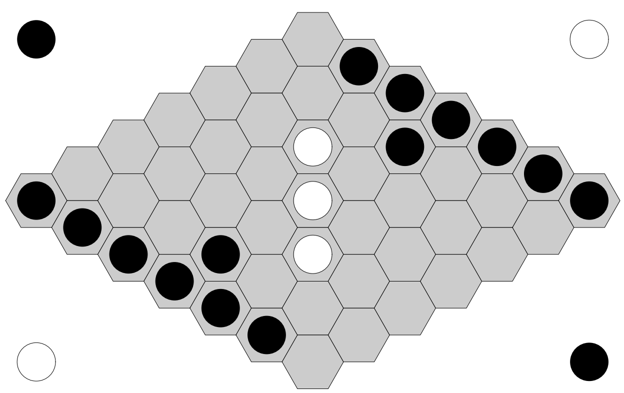
if S is empty, or
there is some P-move to m in S so that

- $S - m$ **is** P**-captured**

opening winning moves: previous results



new results