

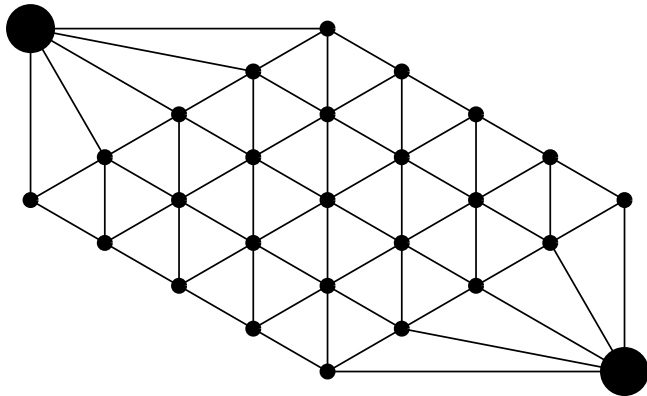
Hex: centermost opening wins?

ryan hayward@ualberta.ca

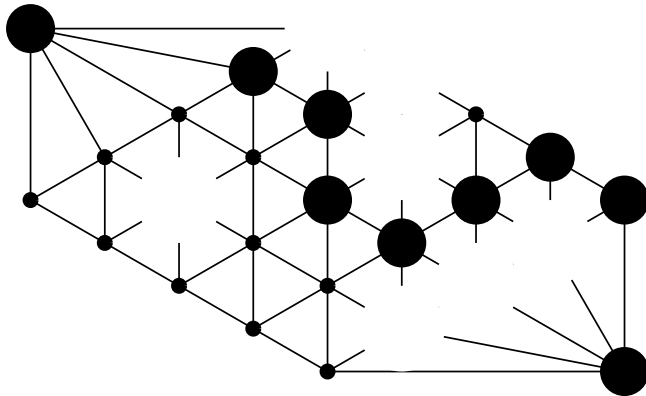
NCM29 2023 Jul 3-7 Aalborg DK

a game on a graph

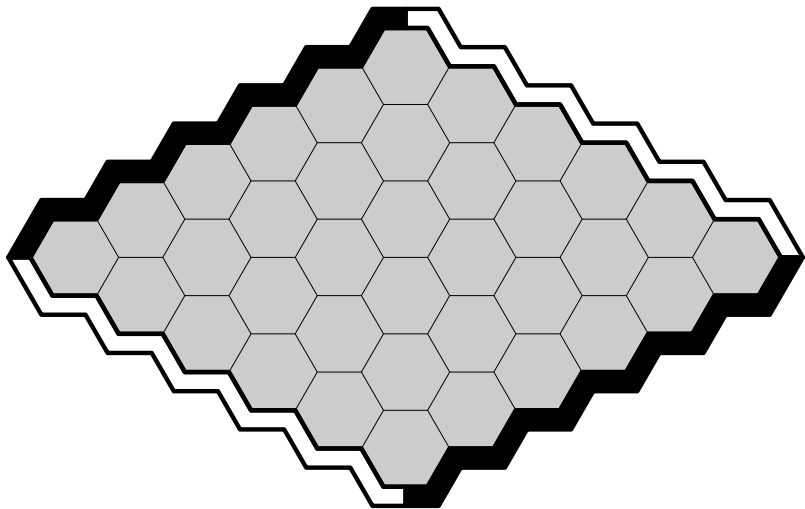
a game on a graph



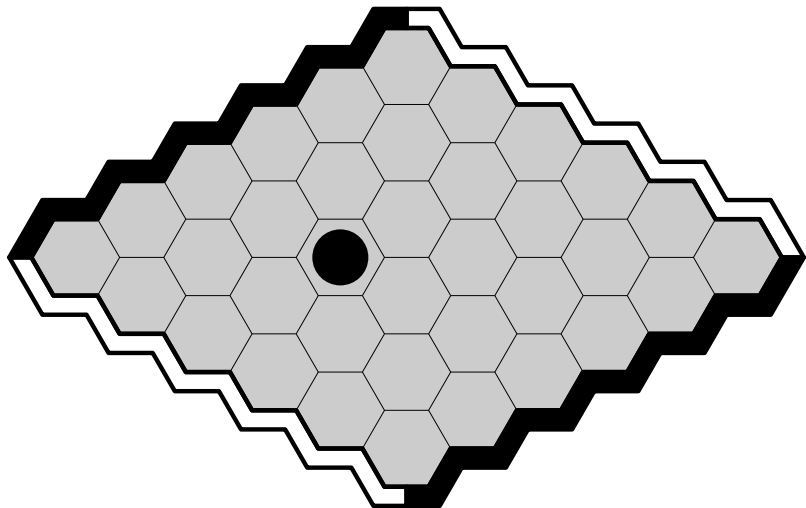
a game on a graph



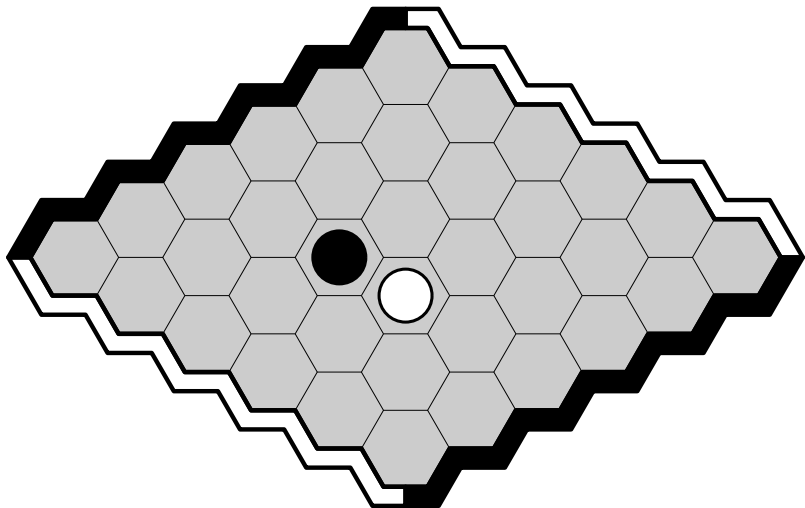
Hex (polygon) Hein-Lindhard 1942



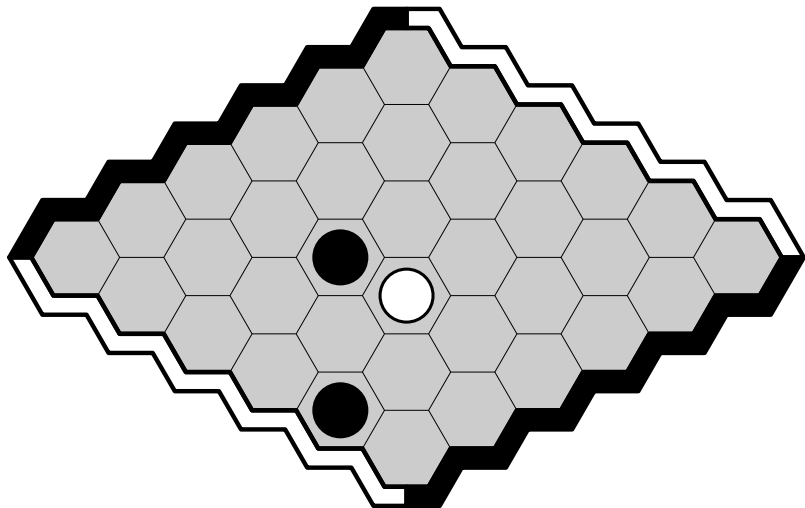
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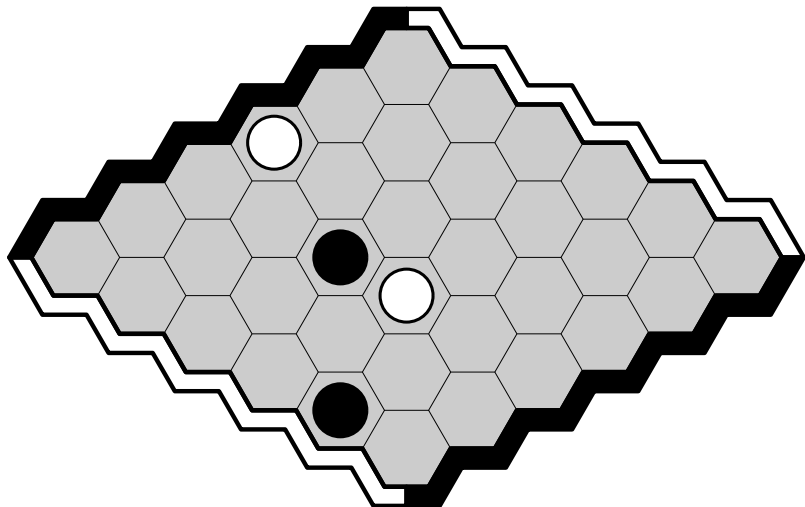
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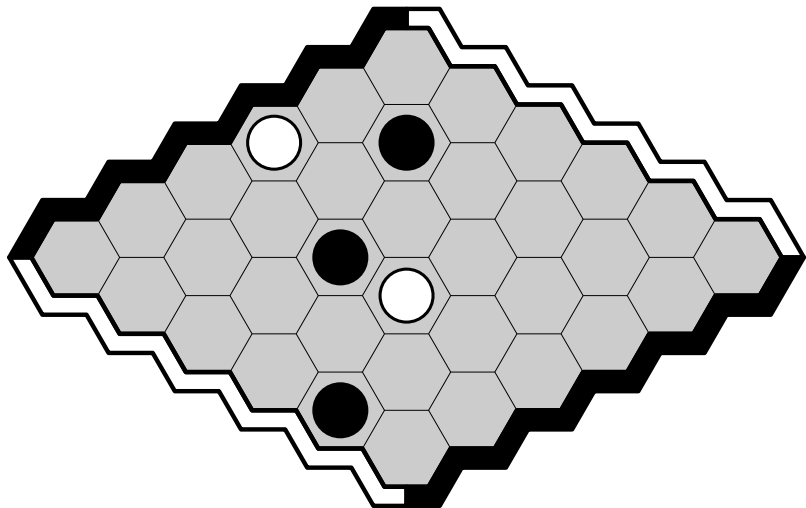
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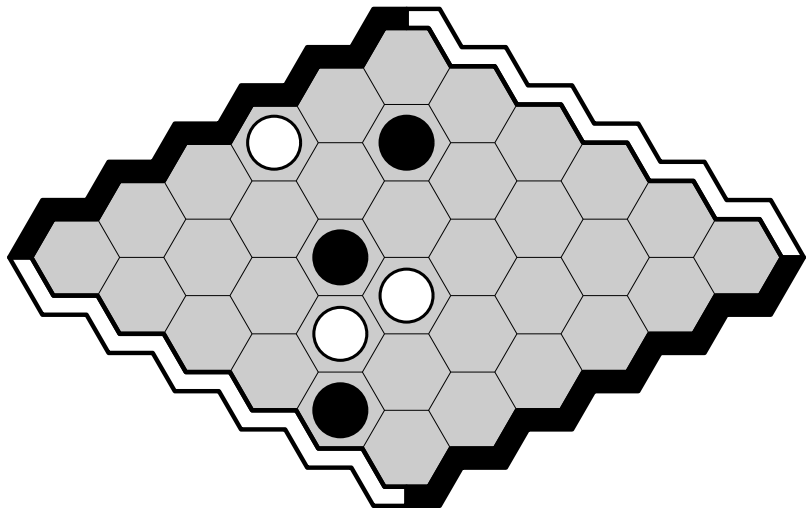
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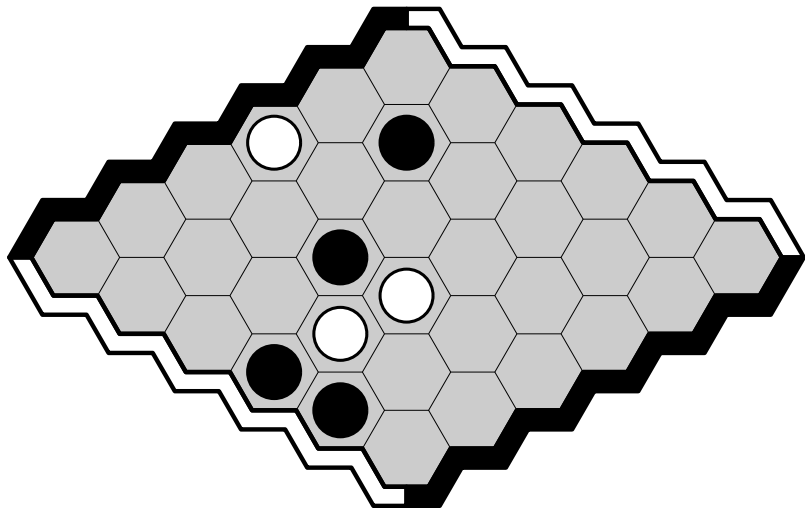
Hex (polygon) Hein-Lindhard 1942



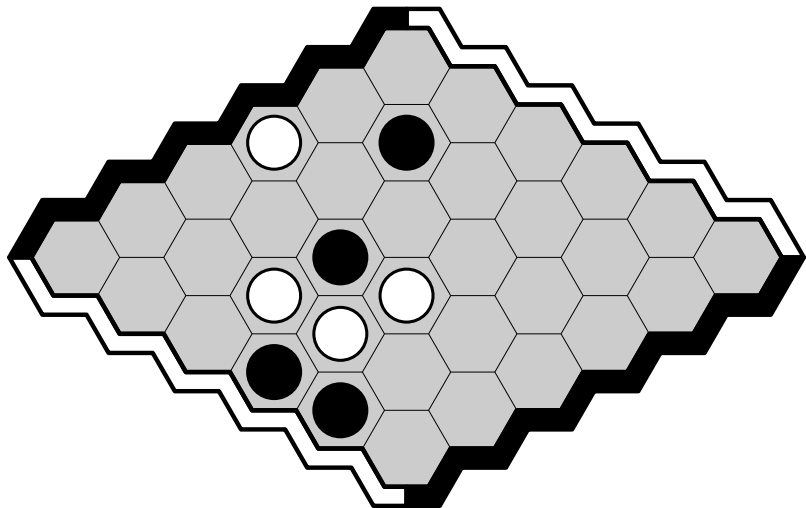
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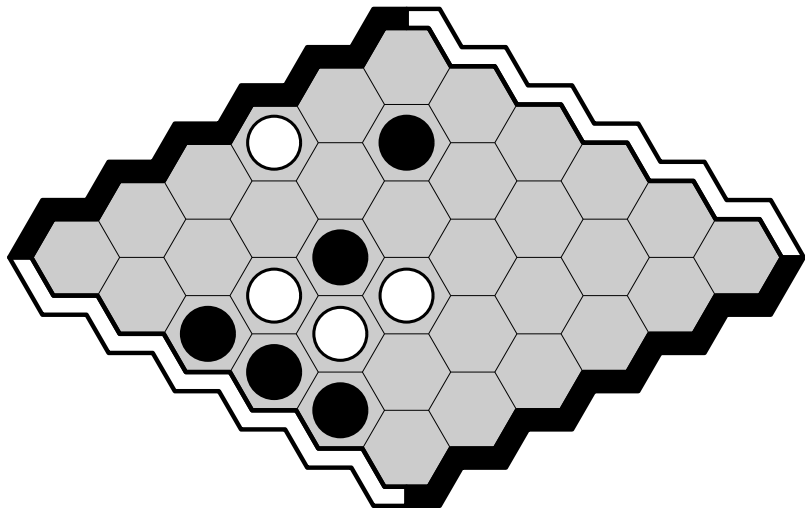
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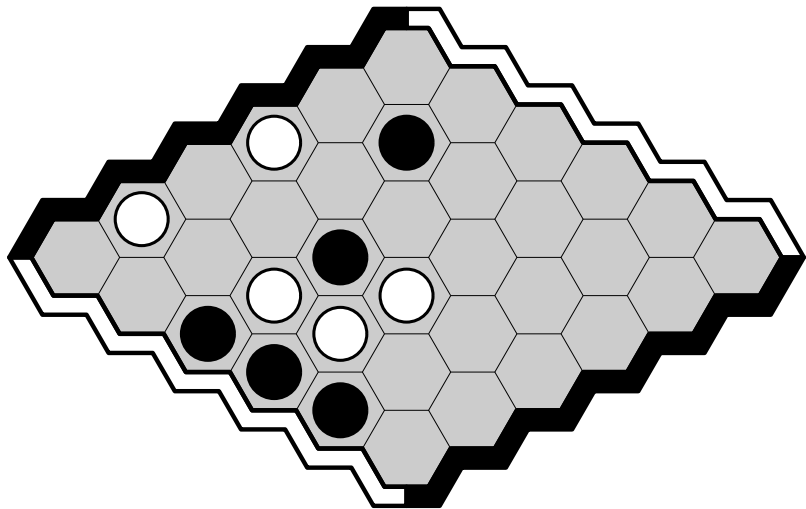
Hex (polygon) Hein-Lindhard 1942



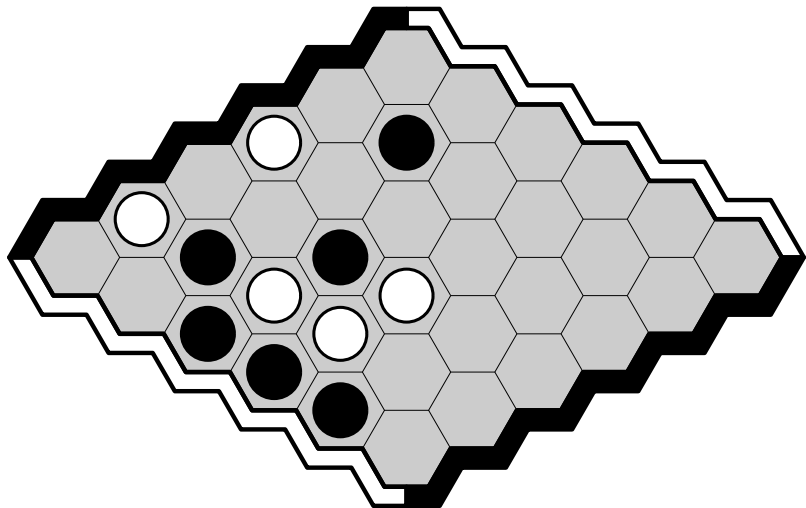
Hex (polygon) Hein-Lindhard 1942



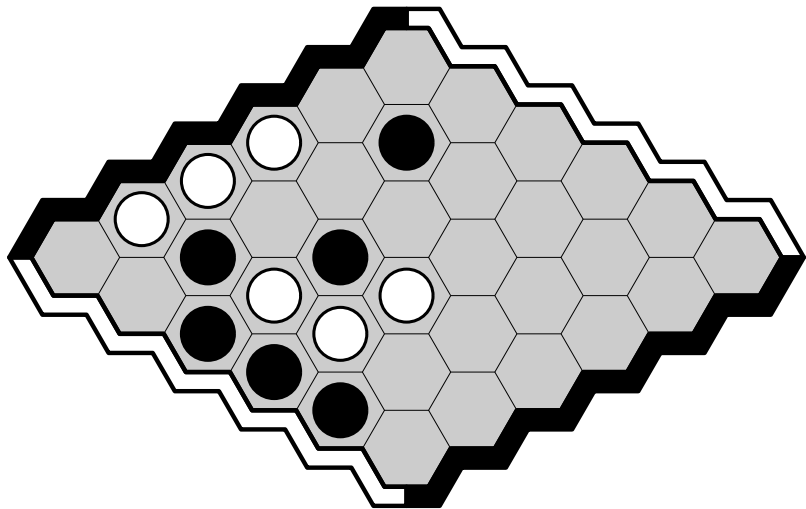
Hex (polygon) Hein-Lindhard 1942



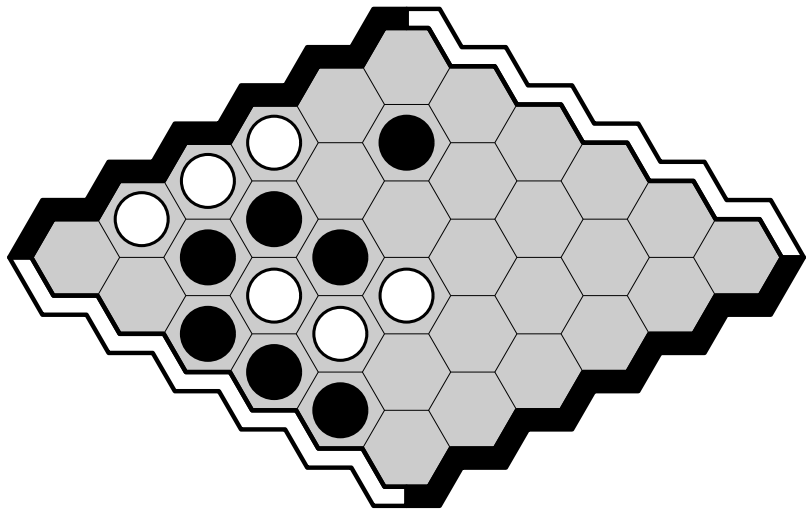
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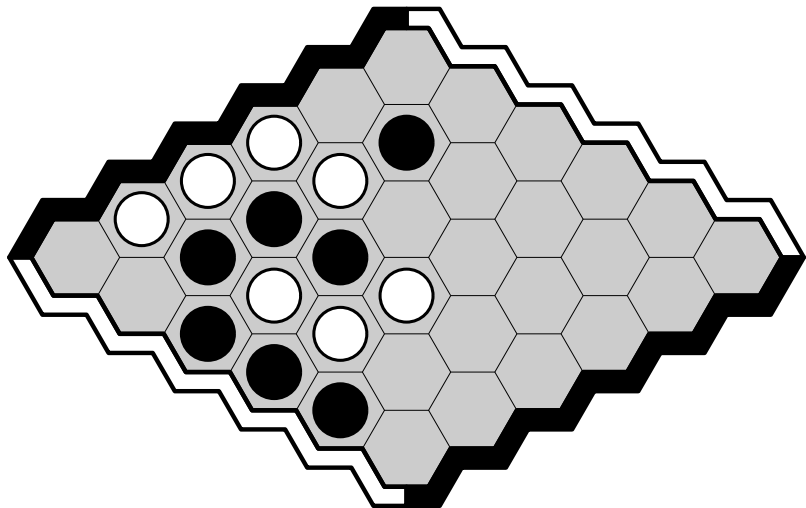
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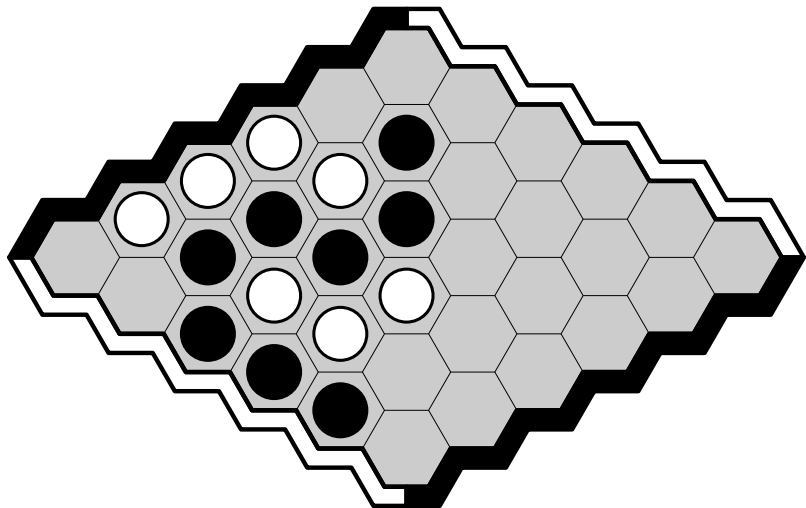
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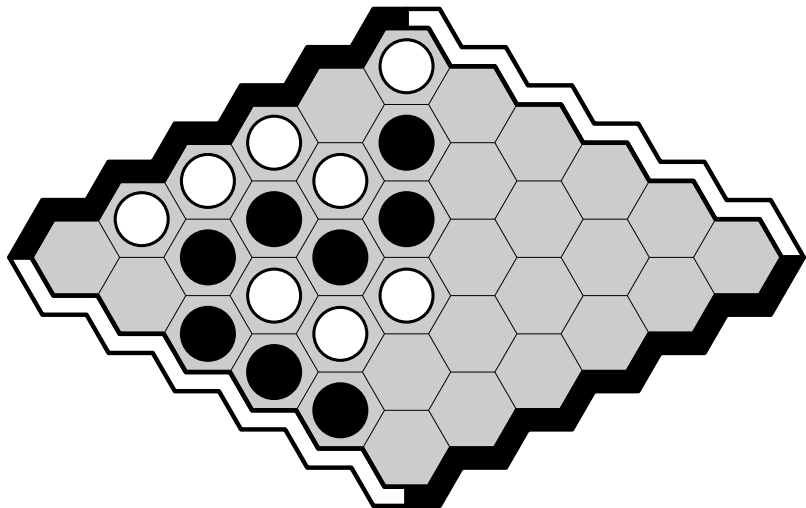
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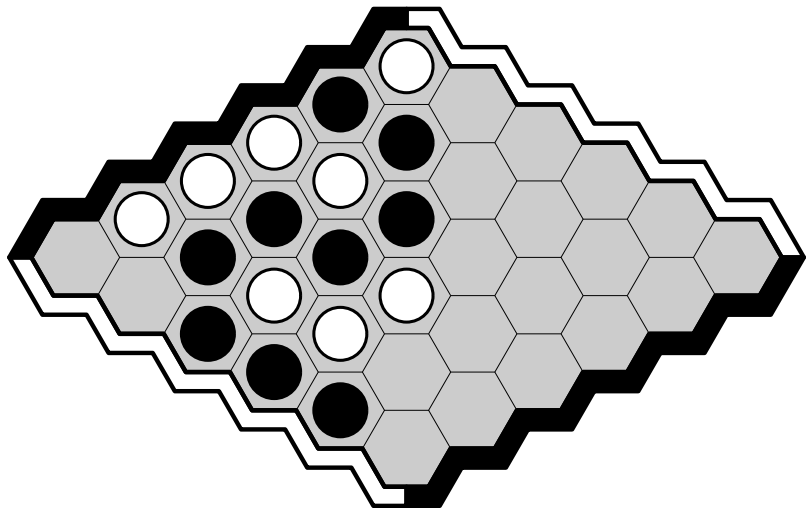
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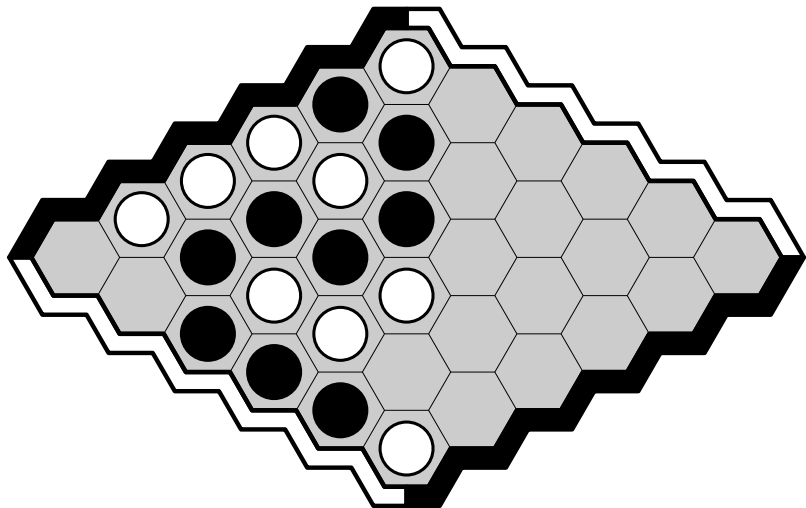
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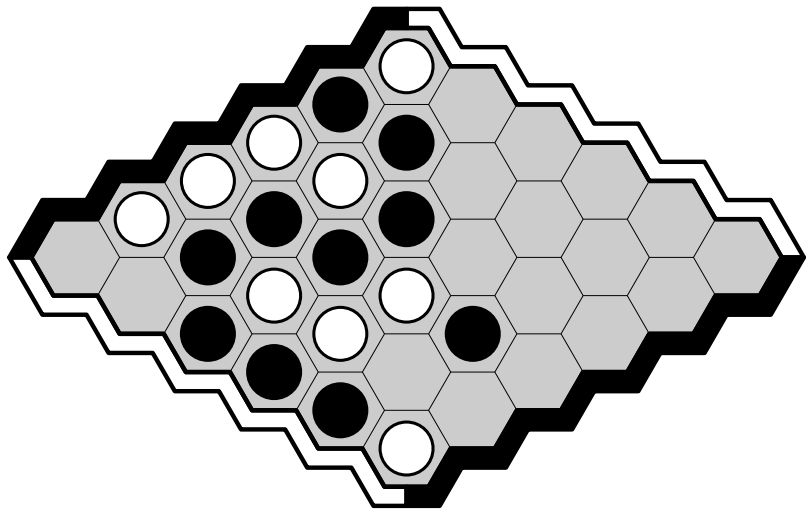
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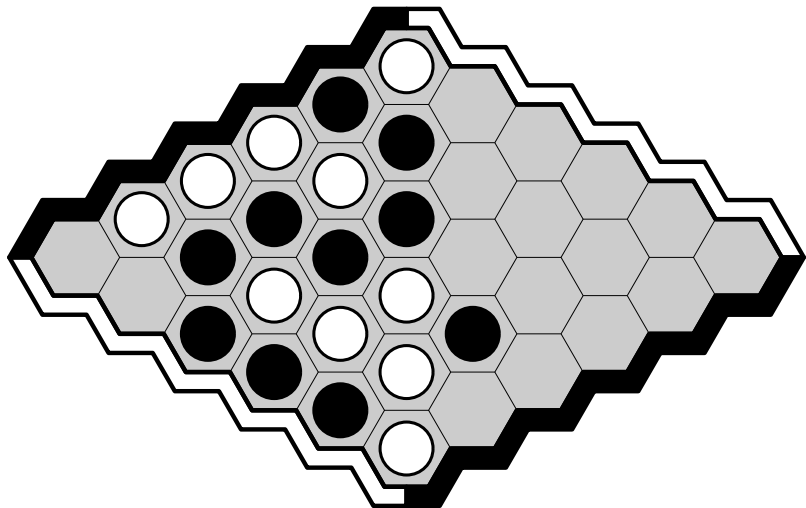
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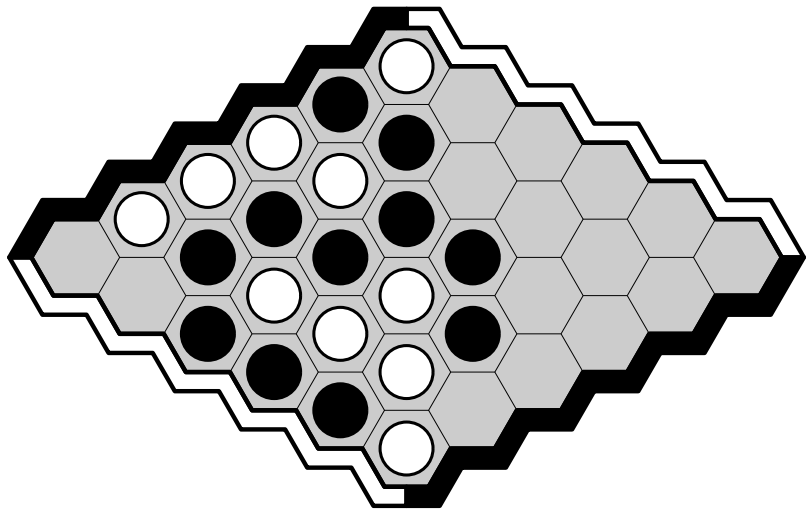
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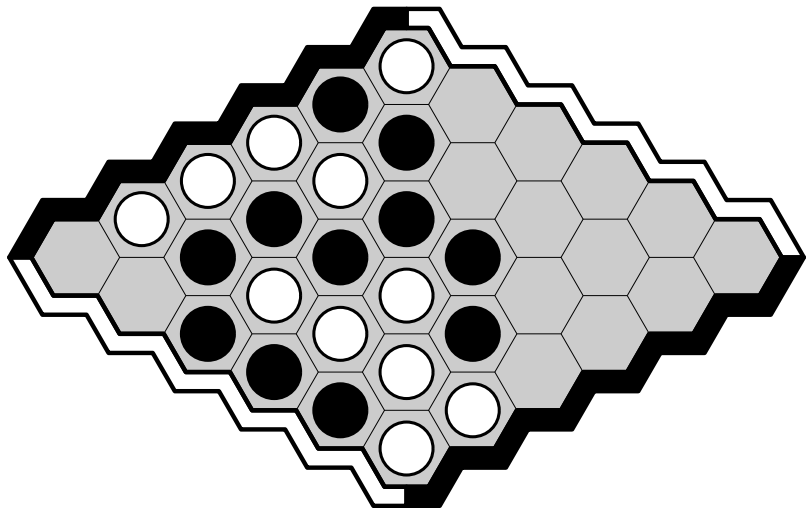
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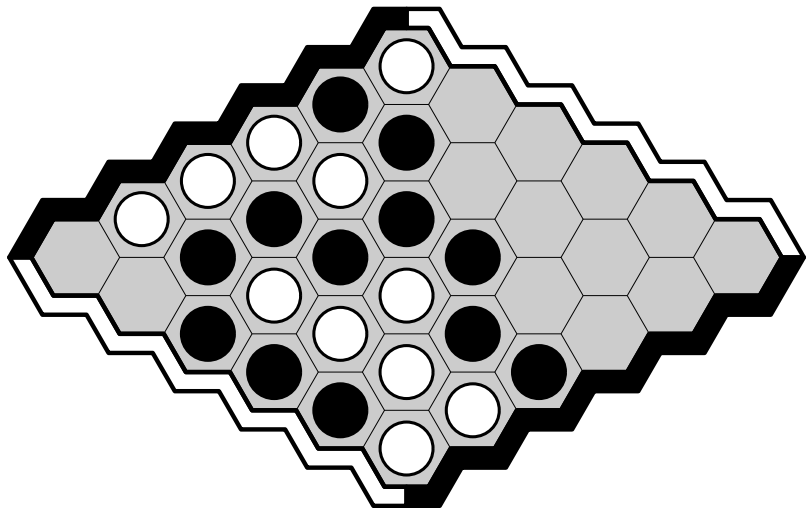
Hex (polygon) Hein-Lindhard 1942



Hex (polygon) Hein-Lindhard 1942



Hex (polygon) Hein-Lindhard 1942



hex book one – history, 2019 CRC

HEX

The Full Story

Ryan B. Hayward
with Bjarne Toft

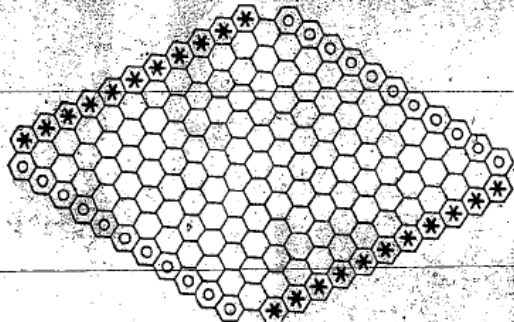


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Vil De lære Polygon?

Piet Hein har konstrueret et Spil, der med lige stor Glæde kan dyrkes af Skakeksperten og den, der blot kan holde en Blyant

„Politiken“ udskriver i Dag en Præmieopgave, der vil volds Hovedbrud for Begyndere



Saaledes ser Polygon-Spillebrættet ud.

ten kan afbryde Forbindelsen ved at besætte det mellemste gode Felt, dens Angrebsside bliver saa Blykærpens Placering i den Vidtre Omegn. I det hele taget vider det sig snart nødvendigt at lægge en stærk Del af Spillebrættet med i Betragtning.

En anden Erfaring, som kommer senere, men som man kan lætte Spillet Begyndelse ved at se, er, at det befaler sig at begynde i hvert Fald nogenlunde paa Midten. En rimelig, men paa ingen Maade nødvendig Aabning af Spillet er denne:

Paa Spillebrættet i Midten er Hvid begyndt i Midterfeltet. Sort sat i Kontaktfeltet til det ned imod Midten af Hvids Front og derved gjort to nyttige Felter, som staar i Vinkelstilling til Midterfeltet, usikre. Hvid har saa vaegt et Felt i Kontakt med sin første. Og nu svarer Sort med at besætte et Vinkelfelt, som vilde være meget nyttigt for Hvid. Hvor skal nu Hvid sætte? Der er forskellige gode Muligheder.

Saa dan er dette Spil nu begyndt. Nu kan enhver fortsætte. Det er ogsaa Hvids Tur. Man skal ikke være uspekuleret fra Begyndelsen. Der er ingen bedre Vej til at lære Spillet end at spille lek.

Det er nyttigt at se skiftevis offensivt og defensivt paa Situationen, d. v. s. skiftevis paa sine egne og Modspillernes Muligheder, for at føre en Forbindelse igen-

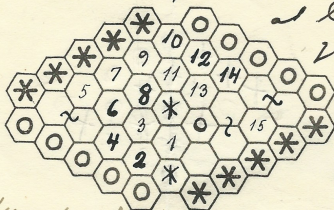
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POLYGON – SPILLEBRÆT

5 × 5

22. 1. 43.

Kæm J.L.! Er det muligt vand,
saa benyt det. Det er
jo ellers svært nok
at lave en i de
vennemæssige
opgave!



7

Denne Opgave begynder lidt-og vinder, men først efter
15 Træk og under skadige Træk fra sorte Side.
Jeg har prøvet adskillige (men ikke alle!) andre Mø-
ligheder for hvids første Træk, men har kun Lang
fundet et Modstrik fra sorte Side.

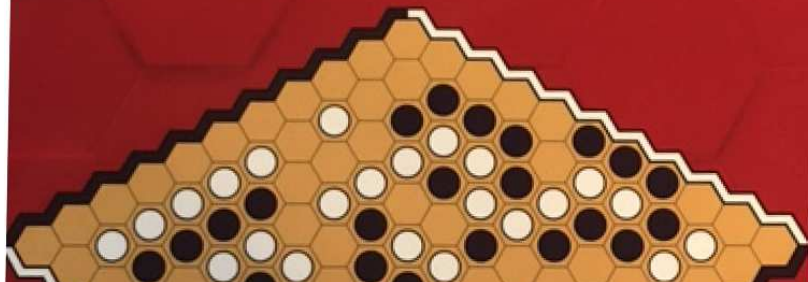
Venlig Hilsen. stard. mag. Klaus Thorsborg

<https://www.routledge.com/hex-Inside-and-Out->

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Hex

A Playful Introduction



<https://bookstore.ams.org/nml-54/>

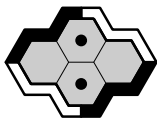
Hein-Lindhard 1942 Hex (polygon)

- draw not possible
- $n \times n$ board: exists 1st-player-wins strategy
- $n \times (n-1)$ board: simple closer-sides-wins strategy
- arbitrary positions Pspace-complete
- variants: reverse hex, dark hex, . . .

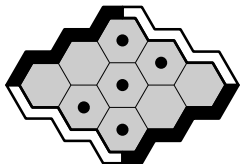
Hex: winning black-1st-moves



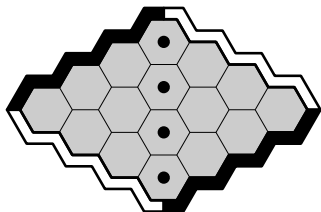
Hex: winning black-1st-moves



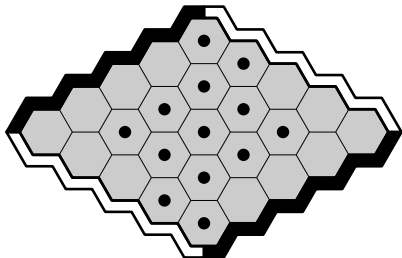
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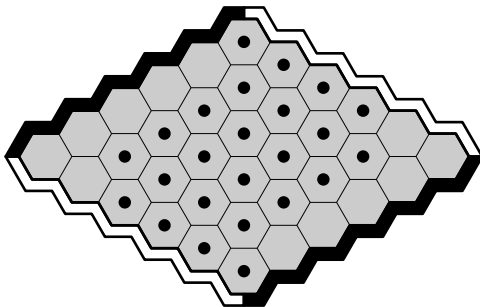
Hex: winning black-1st-moves



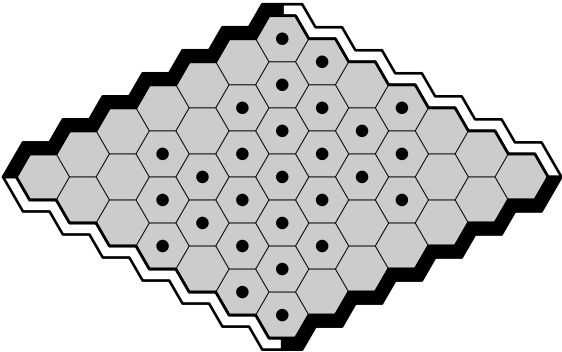
Hex: winning black-1st-moves



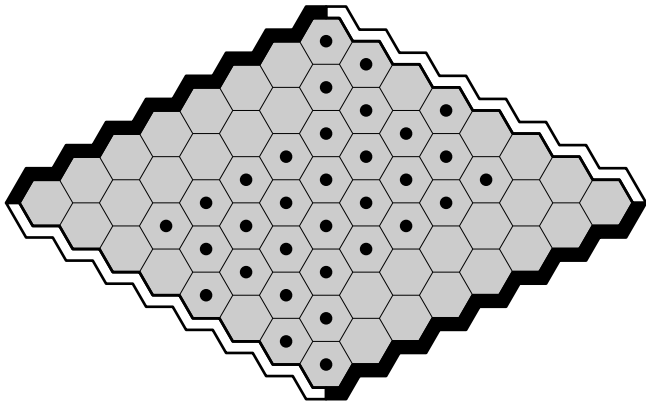
winning 1st-moves 1995 Enderton



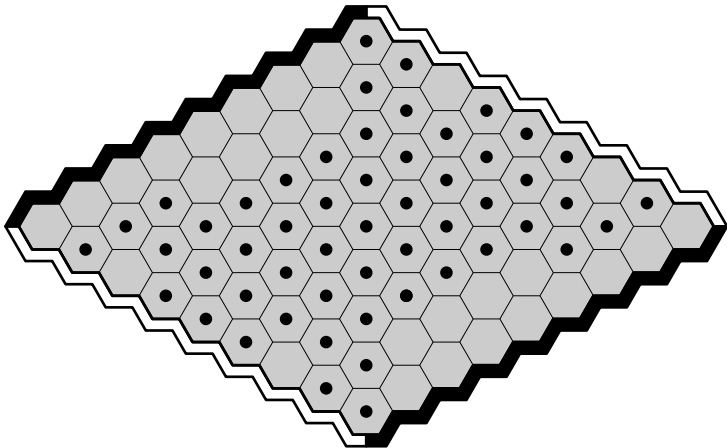
winning 1st-moves 2004 HBJPvR



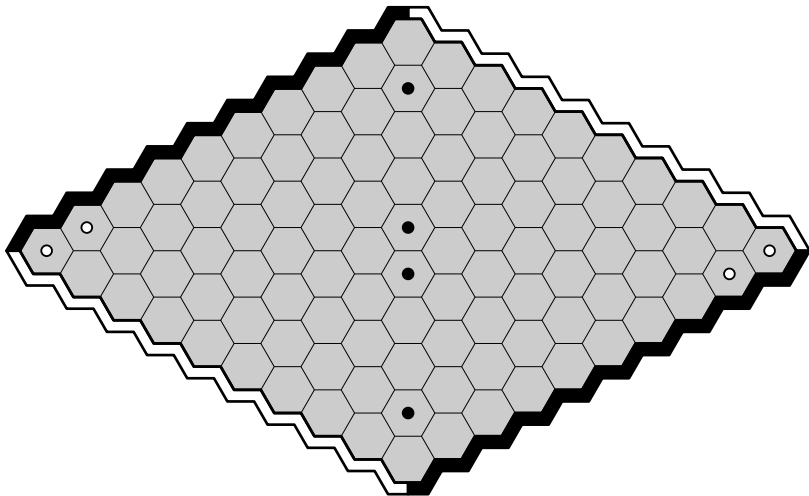
winning 1st-moves 2009 HAH



winning 1st-moves 2013 AHHP



winning 1st-moves 2014 PH



Hex conjectures: winning 1st moves

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- short diagonal: centermost cell

Hex conjectures: winning 1st moves

- short diagonal: centermost cell
- short diagonal: each cell

Hex conjectures: winning 1st moves

- short diagonal: centermost cell
- short diagonal: each cell
- short diagonal: obtuse corner barely wins

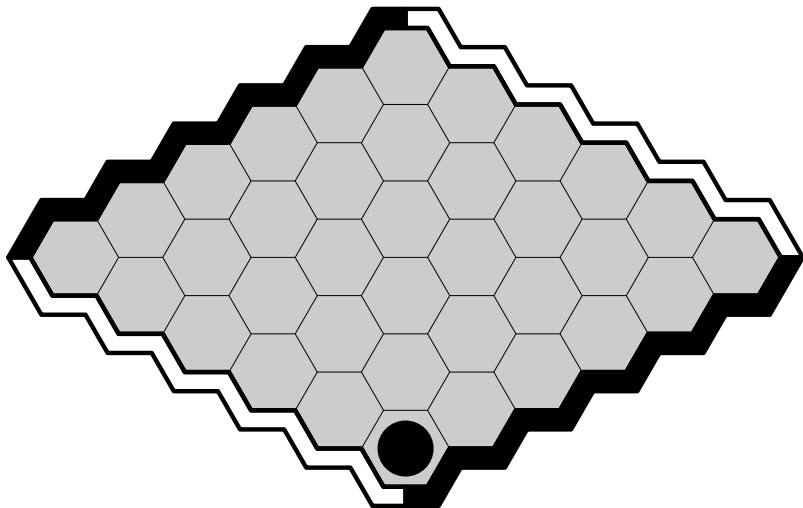
strategy needs all empty cells

Hex conjectures: winning 1st moves

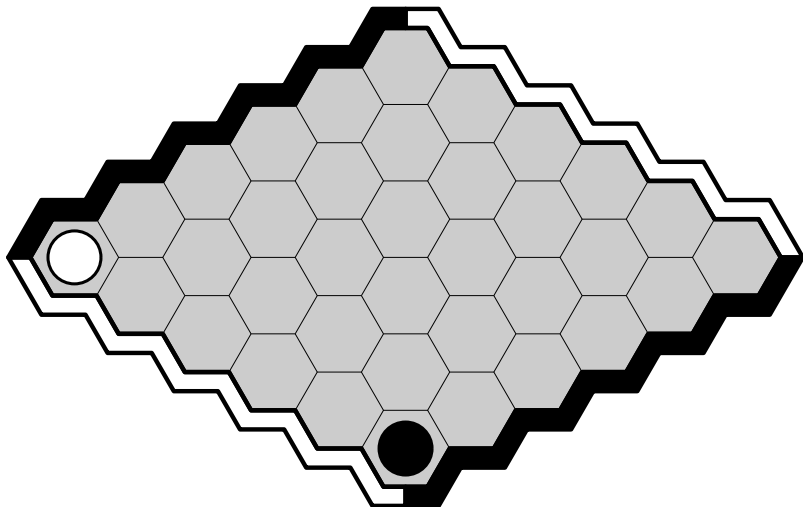
- short diagonal: centermost cell
- short diagonal: each cell
- short diagonal: obtuse corner barely wins
strategy needs all empty cells
- short diagonal: centermost cell strongly wins
1st row: strategy needs only obtuse corner

conj.: obtuse corner barely wins

conj.: obtuse corner barely wins

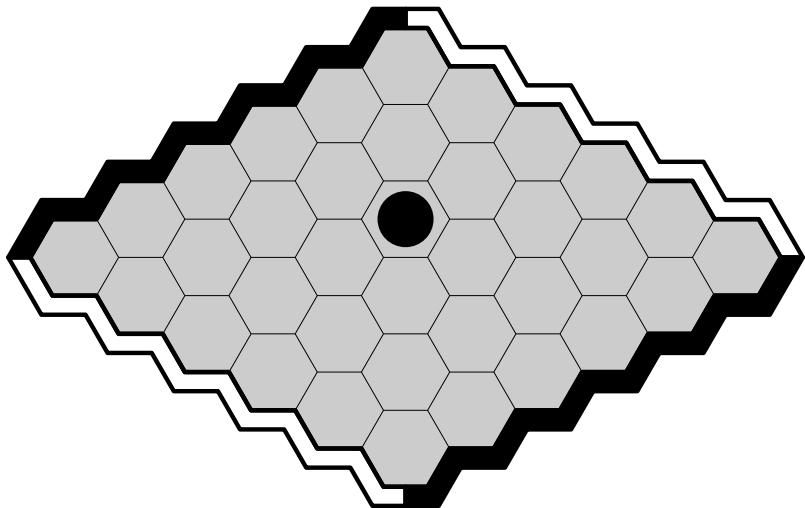


conj.: obtuse corner barely wins

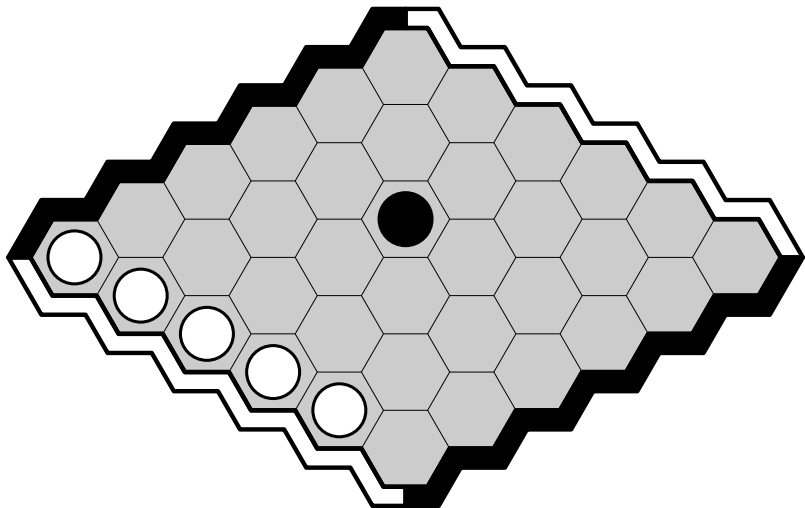


conj.: centermost cell strongly wins

conj.: centermost cell strongly wins

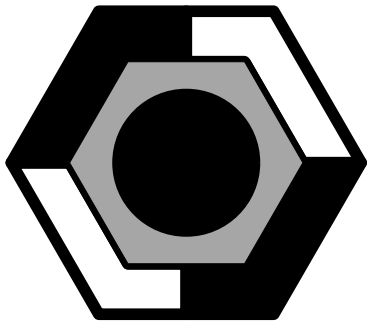


conj.: centermost cell strongly wins

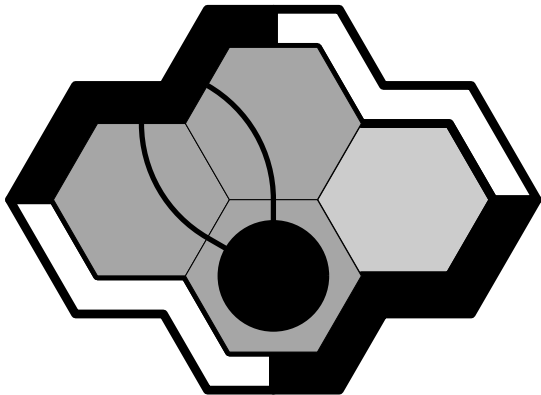


conj.: centermost cell wins

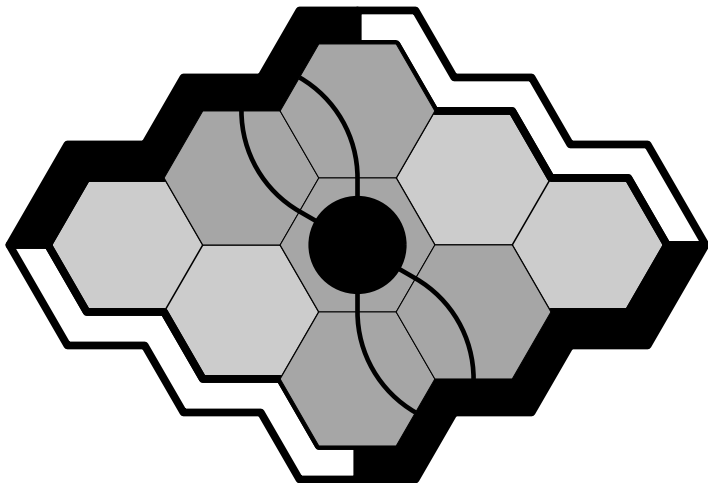
conj.: centermost cell wins



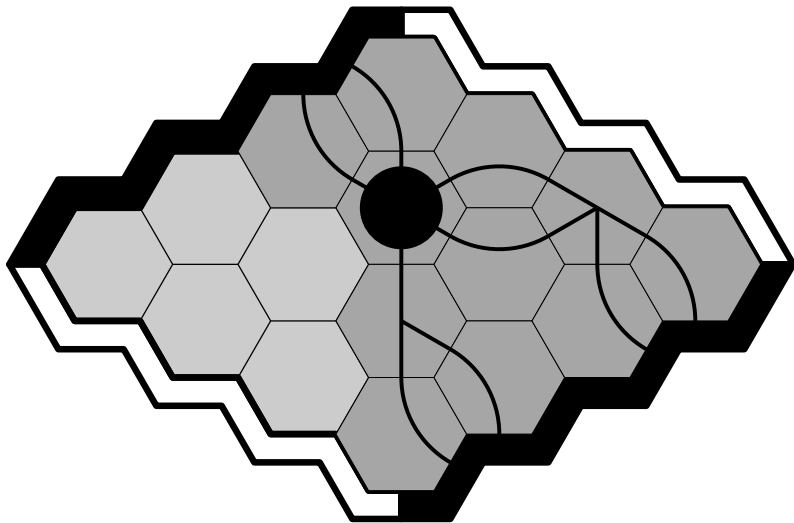
conj.: centermost cell wins



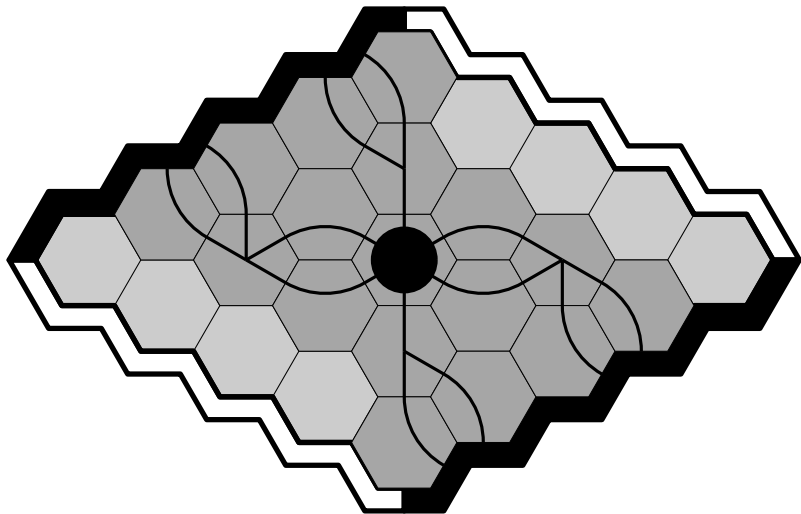
conj.: centermost cell wins



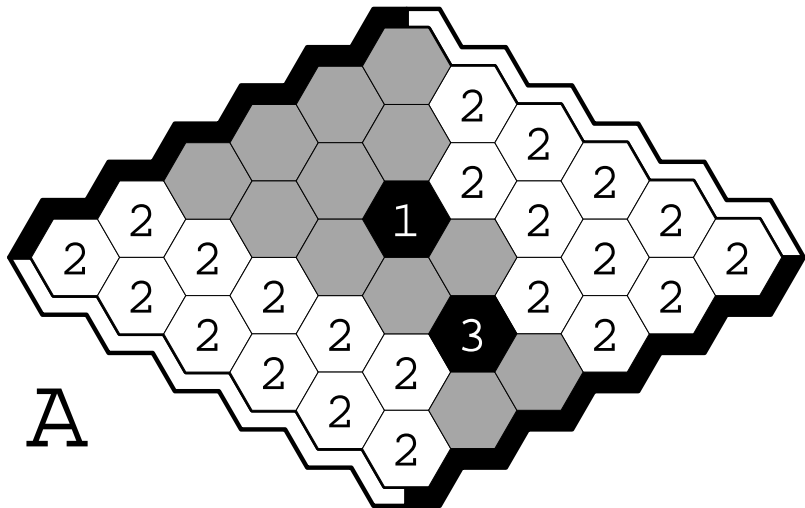
conj.: centermost cell wins

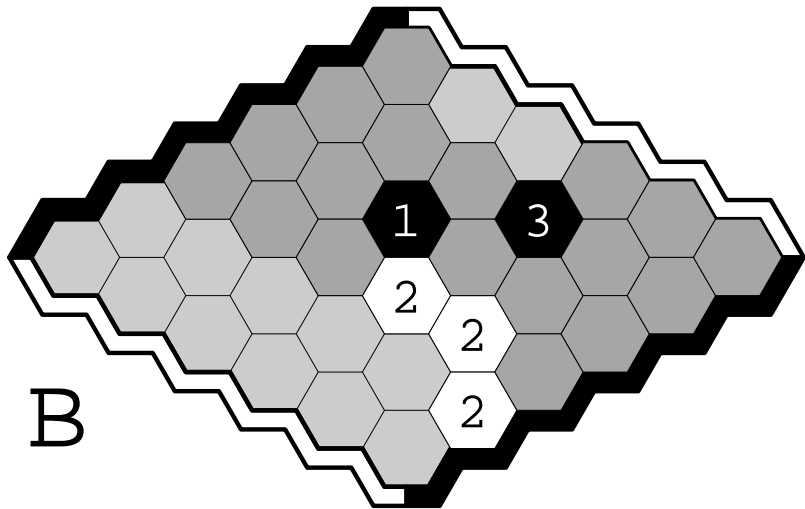


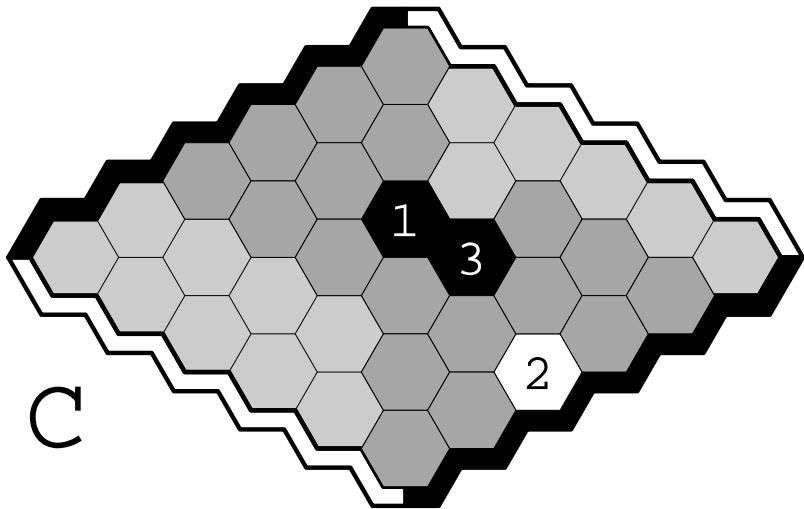
conj.: centermost cell wins

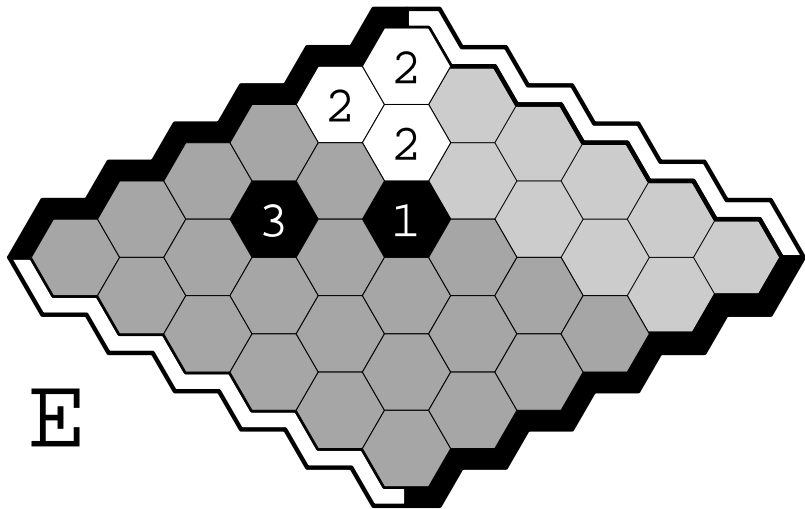


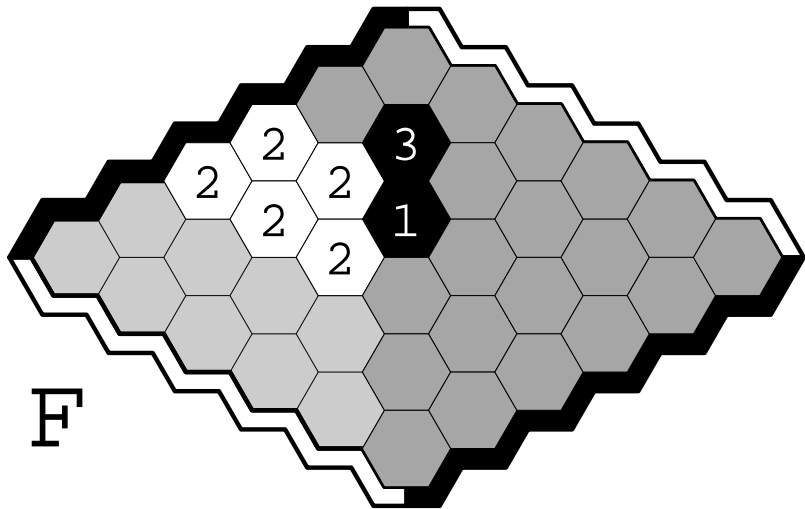
conj.: centermost Lindhard



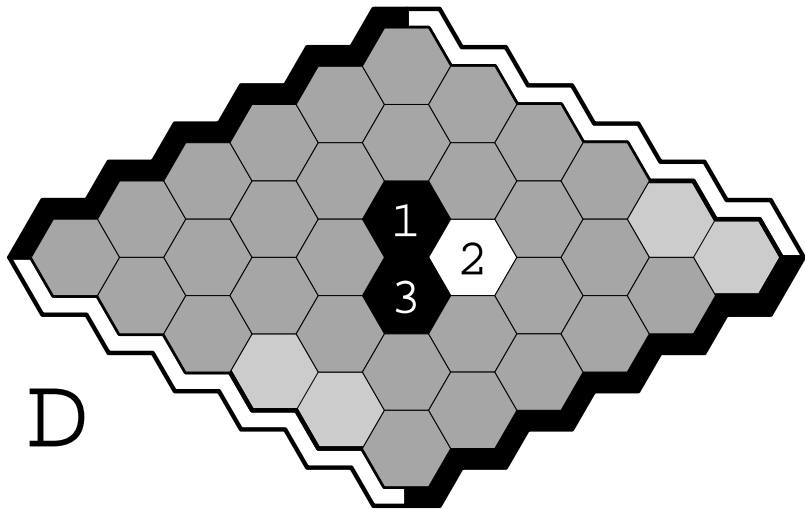








F



<http://webdocs.cs.ualberta.ca/~hayward/355/asn/hexviz/>

conj.: centermost Jing Yang



conj.: centermost Jing Yang



conj.: centermost Jing Yang



conj.: centermost Jing Yang



conj.: centermost Jing Yang



conj.: centermost Jing Yang



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conj.: centermost Jing Yang



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thank you

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