Monophonic Intervals
and the Game of Hex

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Thankyou ...

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UofA Games Research Group
the game of Hex
’42 Piet Hein Copenhagen
’48 John Nash (& David Gale) Princeton
’53 Claude Shannon & E.F. Moore Murray Hill
’57 Martin Gardner ? Hendersonville NC
’69 Anatole Beck Madison WI
’75 Craige Schensted & Ch’ Titus Peaks Is. ME
’76 Shimon Even & Robert Tarjan
’77, ’81 Claude Berge Paris
’79 David Gale Berkeley CA
’81 Stefan Reisch ? Germany
’91 Sid Sackson & Klutz Press Palo Alto CA
’00 Jack van Rijswijck Edmonton AB
’00 Cameron Browne Brisbane
’00 Vadim Anshelevich Richardson TX
’01 Jing Yang Winnipeg MB
’02 H Edmonton AB
’03 H B J K P vR Edmonton AB
’04 H vR Edmonton AB
Claude Berge

• *L’Art Subtil du Hex* ’77
• *Some remarks about a Hex problem...* ’81

*It would be nice to solve some Hex problem by using nontrivial theorems about combinatorial properties of sets.*
Berge puzzle: Black to play and win
Berge and the Art of Hex

mystery of the missing stone
(and other stories)

White to play and win
monophonic intervals

- node $v$ is dead if,
  for every completion of $G - v$,
  colour of $v$ does not change winner
- live iff not dead
- Theorem: live iff on terminal-terminal monophonic interval of reduced graph
- computing m. i. NP-hard (Fellows)
- dead nodes often simplicial
death has consequences

- $P$-captured set of nodes:
  adding $P$-stones doesn’t change game

- $P$-dominated set of nodes:
  some $P$-move in set $P$-captures the rest;
such a move is $P$-dominating;
$P$ can ignore all other moves into the set
dead cell analysis
A set $S$ of unoccupied nodes is

$P$-captured:

if $S$ is empty, or
for each opponent-move to $m$ in $S$

- $S - m$ is $P$-dominated, and
- filling $S - m$ with $P$-stones makes $m$ dead

$P$-dominated:

if $S$ is empty, or
there is some $P$-move to $m$ in $S$ so that

- $S - m$ is $P$-captured