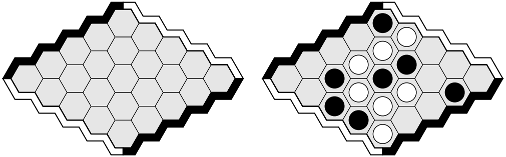
A Handicap Hex Strategy

Philip Henderson

Department of Computing Science
University of Alberta
Edmonton, Alberta, Canada

Joint work with Broderick Arneson and Prof. Ryan Hayward

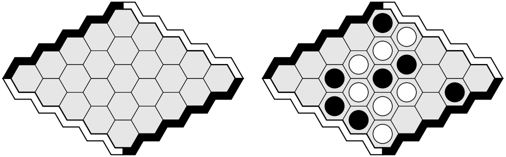
Hex Basics



Rules

- Two players alternate turns playing on any empty cell
- Stones are permanent (no moving or capturing)
- Goal is to connect your two sides of the board

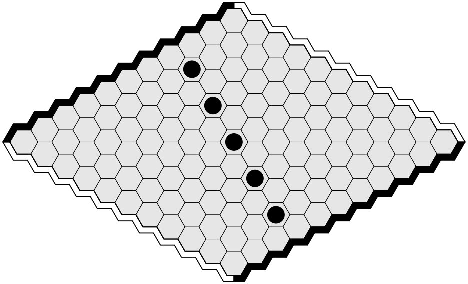
Hex Basics



Theoretical Properties

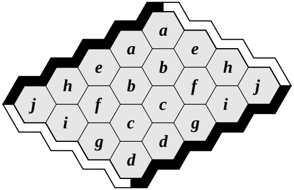
- An extra stone of your color is never a disadvantage
- Hex cannot end in a draw
- First-player win: strategy-stealing argument
- PSPACE-complete to determine winner in arbitrary position

Motivation



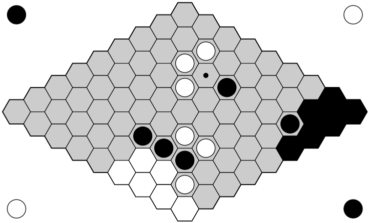
- Know only that there exists a winning first-player strategy
- How many stones do we need to place initially to guarantee a win, and where should these handicap stones be placed?
- Claude Berge would give three stone handicaps on the 11×11 board

Irregular Hex



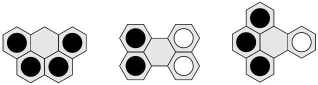
- On irregular Hex boards, player traversing shorter distance can win even as second-player using a simple pairing strategy
- Idea: use handicap stones to essentially reduce a regular Hex board to an irregular one

Fill-in



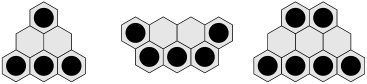
- Using graph-theoretic properties, can determine stones that can be added to Hex positions without changing its value
- Two types of fill-in: dead cells and captured cells

Dead Cells



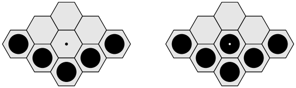
- A cell is dead if it is not on any minimal winning path (for either player)
- Dead cells can be filled-in with stones of either colour

Captured Cells



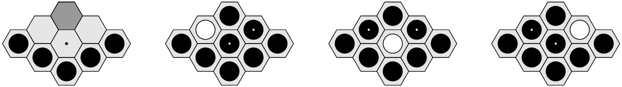
- A set of cells S is P-captured if player P has a second-player strategy to make all $\overline{P}$-claimed S-cells dead
- P-captured sets of cells can be filled-in with P-coloured stones

Permanently Inferior Cells



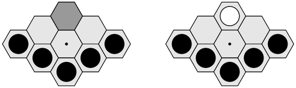
- Have identified a new type of inferior cell: permanently inferior cells
- Adds a single stone for one particular player *P*
- Unlike fill-in, the strategy set is larger than the filled-in set

Permanently Inferior Strategy (1)



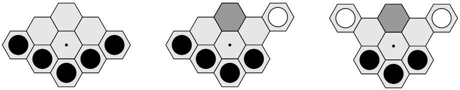
- If $\overline{P}$ moves first in pattern, claim $\overline{P}$ must play at shaded cell
- Any other move allows P to capture all four cells

Permanently Inferior Strategy (2)



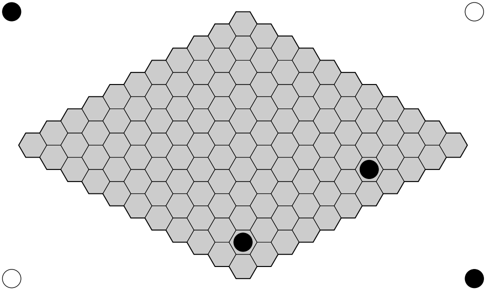
- If $\overline{P}$ plays at shaded cell, dotted cell is dead
- If P plays first, captures all four cells
- In all cases, P can claim the dotted cell
- Can assign P the dotted cell without changing position's value

Permanently Inferior Cell Patterns



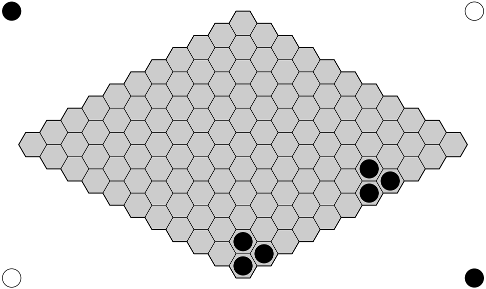
- All three permanently inferior patterns

Handicap Stone Placement



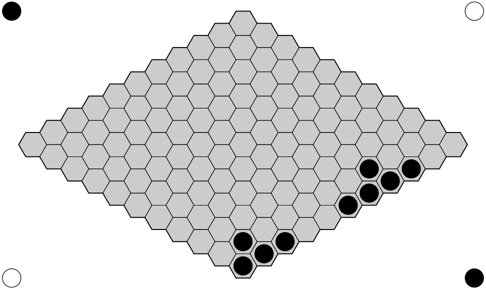
- Placement of stones begins on second row, main diagonal
- Stones placed every 6 spaces until last stone is at most distance four from the edge
- On an $n \times n$ Hex board, this requires $\lceil \frac{n+1}{6} \rceil$ stones

Handicap Reduction (1)



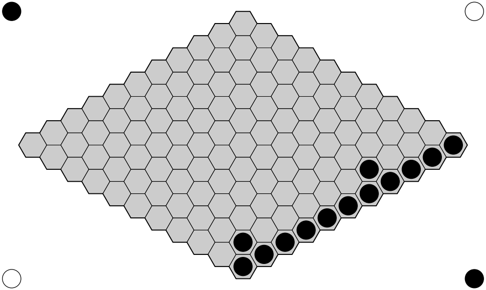
- Capture cells below handicap stones

Handicap Reduction (2)



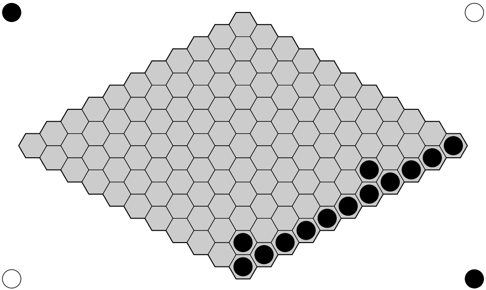
- Apply permanently inferior cell patterns next

Handicap Reduction (3)



- Fill-in remaining gaps via dead and/or captured

Handicap Strategy



- Essentially reduced to $n - 1 \times n$ board, so proven existence of winning strategy
- Can easily make strategy explicit by enforcing inferior cell strategies

Summary

- Identified new form of inferior cell: permanently inferior cells
- Developed efficient and explicit handicap strategy for $n \times n$ Hex

Open Questions:

- Are there more permanently inferior cell patterns?
- Can the number of handicap stones be further reduced while maintaining an explicit strategy?
- Can the number of handicap stones be reduced if we only desire an existence proof (while still specifying initial stone placement)?

Any Questions?

Thanks to:

- NSERC and iCORE for funding support
- Michael Johanson and Morgan Kan for helpful conversations