HEXBOTS POST ALPHAAGO
HAYWARD PAWLEWICZ GAO YOUNG WENINGER

hayward@ualberta.ca

computing UAlberta

2017 oct 2
THANK YOU

- invitation Yngi, Magnus
- solving $10 \times 10$ Hex joint with Jakub Pawlewicz
- builds on work with B Arneson, P Henderson
- machine Martin Müller
- photo courtesy MIT Museum, MIT, Cambridge MA
- Natural Sciences and Engineering Research Council of Canada
ALPHAGO: 2015 - 2017

https://www.youtube.com/watch?v=l-GsfyVCBu0&t=77m40s Mar 2016
https://www.youtube.com/watch?v=OCEvCII1zo0&t=1m23s Mar 2017
1. HEX
2. KNOWLEDGE
3. SEARCH
4. 10x10
5. POST ALPHAGO
1942 HEX

RULES

- 2 players, alternate moves
- win: connect your two sides
1942 HEX

RULES

- 2 players, alternate moves
- win: connect your two sides
**N x N Hex: 1st-Player Win**

**Proof**

- lemma: extra X-cell ok for player X
- lemma: no draws
- suppose P2 has win strategy S2
- then P1 can move anywhere, forget move, and follow S2
- thus P1 has win strategy, contradiction □
NO-DRAW
NO-DRAW
**n x n+1 Hex: longer-side win**

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Hexbots post AlphaGo
**N x N+1 Hex: longer-side win**
1951 Shannon Machine
1951 Shannon Machine

- play on any graph
- two marked vertices
- black move: ‘short’ any vertex (make nbrs clique)
- white move: ‘cut’ any vertex (delete)
- black wins iff two marked vertices are shorted (connected)

- generalizes Hex
1951 Shannon machine
1951 Shannon Machine
1951 Shannon machine
PROVABLY HARD

- 1975 Even & Tarjan
  Shannon v-switching: PS-c
- 1981 Stefan Reisch
  Hex: PS-c
- 2000 Clay Math Inst
  P vs NP: $1,000,000
# HUMANS

## SOLVED OPENINGS

<table>
<thead>
<tr>
<th>Year</th>
<th>Player</th>
<th>Board Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>Yang</td>
<td>17/49 7x7</td>
</tr>
<tr>
<td>2002</td>
<td>Yang</td>
<td>8x8</td>
</tr>
<tr>
<td>2003</td>
<td>Yang</td>
<td>9x9</td>
</tr>
<tr>
<td>2004</td>
<td>Noshita</td>
<td>7x7</td>
</tr>
<tr>
<td>2005</td>
<td>Noshita</td>
<td>8x8</td>
</tr>
<tr>
<td>2006</td>
<td>Mishima</td>
<td>8x8</td>
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</table>
## SOLVED OPENINGS

<table>
<thead>
<tr>
<th>Year</th>
<th>Authors</th>
<th>Game Size</th>
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</thead>
<tbody>
<tr>
<td>1995</td>
<td>Enderton</td>
<td>6x6</td>
</tr>
<tr>
<td>2000</td>
<td>van Rijswijck</td>
<td>6x6</td>
</tr>
<tr>
<td>2003</td>
<td>H Bjö Joh Kan Po vRij</td>
<td>5d 7x7</td>
</tr>
<tr>
<td>2007</td>
<td>Rasmussen et al.</td>
<td>7x7</td>
</tr>
<tr>
<td>2009</td>
<td>Arneson H Henderson</td>
<td>4d 8x8</td>
</tr>
<tr>
<td>2010</td>
<td>A H H</td>
<td>25d some 9x9</td>
</tr>
<tr>
<td>2012</td>
<td>Pawlewicz H</td>
<td>110d x 24 thread 9x9</td>
</tr>
<tr>
<td>2013</td>
<td>Pawlewicz H</td>
<td>63d x 24 thread centre 10x10</td>
</tr>
<tr>
<td>COMPUTERS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Hex Knowledge Search**

Shannon Machine

Provably Hard

Humans

Computers

Post AlphaGo

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Hexbots Post AlphaGo
<table>
<thead>
<tr>
<th>HEX</th>
<th>PROPERTIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>KNOWLEDGE</td>
<td>SHANNON MACHINE</td>
</tr>
<tr>
<td>SEARCH</td>
<td>PROVABLY HARD</td>
</tr>
<tr>
<td>10x10</td>
<td>HUMANS</td>
</tr>
</tbody>
</table>

COMPUTERS

hexbots post alphago

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COMPUTERS
COMPUTERS

HEX
KNOWLEDGE
SEARCH
10x10
POST ALPHAGO

PROPERTIES
SHANNON MACHINE
PROVABLY HARD
HUMANS
COMPUTERS

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HEXBOTS POST ALPHAGO
COMPUTERS
COMPUTERS
COMPUTERS
virtual connections: combining rules, mustplay
inferior cells: dead, captured, etc.
A VIRTUAL CONNECTION
A VIRTUAL CONNECTION
COMBINING RULE: AND (FULL)
COMBINING RULE: AND (FULL)
COMBINING RULE: AND (FULL)
COMBINING RULE: AND (FULL)
COMBINING RULE: AND (FULL)
COMBINING RULE: AND (FULL)
COMBINING RULE: AND (SEMI)
COMBINING RULE: AND (SEMI)
COMBINING RULE: AND (SEMI)
COMBINING RULE: OR
COMBINING RULE: OR
COMBINING RULE: OR
COMBINING RULE: OR
WHERE MUST WHITE PLAY?
WHERE MUST WHITE PLAY?
WHERE MUST WHITE PLAY?
WHERE MUST WHITE PLAY?
DEAD
BLACK-DOMINATED (DOT SUPERIOR)
BLACK-CAPTURED
BLACK-DOMINATED (DOT SUPERIOR)
BLACK-_CAPTURE-REVERSIBLE (TO WHITE DOT)
BLACK FILL DECOMPOSITION
STAR DECOMPOSITION
BLACK STAR DECOMP DOMINATION
and/or combining rules + capture
PROOF NUMBER SEARCH

NEGAMAX P,D VALUES

3,5

1,4

2,3

2,4

1,3

1,2

1,2

2,2

1,3

1,2,1,1,1,1,1,1,4

1,1,1,1,1,1,3

1,2

1,1,1,1,1,1,1,1

1,1,1,1,1,1,1,1

1,1,1,1,1,1,1,1

1,1,1,1,1,1,1,1

1,1,1,1,1,1,1,1

1,1,1,1,1,1,1,1
PROOF NUMBER SEARCH

```
2,5
/   \
1,4   2,2
|     |
|     |
|     |
1,3 1,1 1,1 1,1 1,1 1,2

1,3
1,1 1,1 1,1 1,1 1,1 1,1 1,4 1,1 1,1 1,1 1,1 1,3

1,2
1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1 1,1
```

Hexbots Post AlphaGo

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PROOF NUMBER SEARCH

```
3,4
  /   \\  \\
1,4   1,3   2,4
 |
1,3
 |
1,2
 |
1,1
```

```
1,4
 |
1,3
 |
1,2
 |
1,1
```

```
1,3
 |
1,2
 |
1,1
```

```
1,1
```

```
1,1
```

Hayward@ualberta.ca  Hexbots Post Alphago
F-DFPNS

- PNS  Allis et al
F-DFPNS

- PNS  Allis et al
- DFPNS  Nagai
F-DFPNS

- PNS    Allis et al
- DFPNS  Nagai
- DFPNS in Hex?
F-DFPNS

- PNS  Allis et al
- DFPNS  Nagai
- DFPNS in Hex ?
- ...requires non-incremental H-search :(
F-DFPNS

- PNS   Allis et al
- DFPNS  Nagai
- DFPNS in Hex ?
- ...requires non-incremental H-search :( 
- ...uniform branching factor :(
F-DFPNS

- PNS  Allis et al
- DFPNS  Nagai
- DFPNS in Hex?
- ... requires non-incremental H-search :(
- ... uniform branching factor :(
- idea: move ordering + DFPNS = F-DFPNS
F-DFPNS (1)

- expand node
- consider first $b + \left\lceil f \times 6 \right\rceil = 4$ (of 6) live children
F-DFPNS (2)

- discover move 3 loses
- consider first $b + \lceil f \times 5 \rceil = 4$ (of 5) live children
• discover move 5 loses
• consider first \( b + \lceil f \times 4 \rceil = 3 \) (of 4) live children
F-DFPNS (4)

- discover move 2 wins, so ...
- ... root solved without exploring 6th move
SOLVING 10x10

- stronger VC computations
- scalable parallel DFPN S
Pawlewicz: stronger VC computations

- faster and/or-rule VC computation
- limit form of new VCs, so never redundant
- find fewer VCs, but solve 2 to 10 times faster
EXAMPLE: VCS TO SIDE
EXAMPLE: SEMIS
EXAMPLE: SEMIS
EXAMPLE: SEMIS
EXAMPLE: SEMIS
GREEDY UNION SEMIS TO GET FULL
BLOCK CELL TO GET ANOTHER VC
BLOCK CELL TO GET ANOTHER VC
BLOCK CELL TO GET ANOTHER VC
BLOCK CELL TO GET ANOTHER VC
Pawlewicz: scalable parallel DFPNS

- Parallel PNS: keep tree in memory? e.g. I-Chen Wu connect6
- Hex: leaf computations fast, so tree too big
- How to assign jobs to processors?
  - Jobs too long: computation redundant
  - Jobs too short: too much client/server traffic
- Solution: MaxWorkPerJob
SP DFPN S features

- $1+\varepsilon$ variant of DFPN
- Advanced TT resolution: upon collision, search next $k$ (say 4) cells for empty location; if none found, overwrite location with smallest work job
- Once node computation assigned to leaf, use virtual win/loss so new threads go elsewhere
- ... so compute virtual (dis)proof numbers
- Shared TT: many-read / 1-write locks
- Tune MaxWorkPerJob
- For Hex: use Focussed DFPN S
SP DFPN S PERFORMANCE

- speedup test: 8 hardest 8x8 openings, 8 11x11 positions
- speedup performance: 11.8 on 16 threads (.74)
- solved all 9x9 openings
- solved centre 10x10 opening
SP DFPN S PERFORMANCE
SP DFPN S PERFORMANCE

Pawlewicz: stronger VC engine
Pawlewicz: SPDFPNS

hexbots post alphago

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## SP DFPN S Performance

<table>
<thead>
<tr>
<th>opening</th>
<th>#threads</th>
<th>time</th>
<th>winner</th>
</tr>
</thead>
<tbody>
<tr>
<td>a2</td>
<td>8/24</td>
<td>68d09:40:18</td>
<td>black</td>
</tr>
<tr>
<td>a3</td>
<td>8</td>
<td>80d08:37:34</td>
<td>white</td>
</tr>
<tr>
<td>a4</td>
<td>8</td>
<td>33d14:06:03</td>
<td>black</td>
</tr>
<tr>
<td>a5</td>
<td>8</td>
<td>65d04:14:52</td>
<td>black</td>
</tr>
<tr>
<td>a6</td>
<td>24</td>
<td>110d14:35:06</td>
<td>black</td>
</tr>
<tr>
<td>a7</td>
<td>24</td>
<td>4d08:56:03</td>
<td>white</td>
</tr>
<tr>
<td>a8</td>
<td>24</td>
<td>6d14:21:30</td>
<td>black</td>
</tr>
</tbody>
</table>
# SP DFPN S Performance

<table>
<thead>
<tr>
<th>opening</th>
<th>#threads</th>
<th>time</th>
<th>winner</th>
</tr>
</thead>
<tbody>
<tr>
<td>b2</td>
<td>8</td>
<td>53d15:18:21</td>
<td>black</td>
</tr>
<tr>
<td>b4</td>
<td>8</td>
<td>29d23:53:14</td>
<td>black</td>
</tr>
<tr>
<td>b6</td>
<td>8</td>
<td>1d21:52:28</td>
<td>black</td>
</tr>
<tr>
<td>b7</td>
<td>8</td>
<td>4d17:19:13</td>
<td>black</td>
</tr>
<tr>
<td>c2</td>
<td>24</td>
<td>1d08:42:57</td>
<td>black</td>
</tr>
<tr>
<td>i1</td>
<td>24</td>
<td>6d00:51:25</td>
<td>black</td>
</tr>
<tr>
<td>10x10:f5</td>
<td>24</td>
<td>63d20:44:30</td>
<td>black</td>
</tr>
</tbody>
</table>
### How Long Until 11x11?

<table>
<thead>
<tr>
<th>Size</th>
<th>States (approx)</th>
<th>Center Cell: Solver Fn Calls</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x2</td>
<td>9.0 e 0</td>
<td>0</td>
</tr>
<tr>
<td>3x3</td>
<td>5.5 e 1</td>
<td>0</td>
</tr>
<tr>
<td>4x4</td>
<td>7.6 e 5</td>
<td>0</td>
</tr>
<tr>
<td>5x5</td>
<td>4.0 e 9</td>
<td>0</td>
</tr>
<tr>
<td>6x6</td>
<td>4.0 e 14</td>
<td>2</td>
</tr>
<tr>
<td>7x7</td>
<td>1.5 e 20</td>
<td>68</td>
</tr>
<tr>
<td>8x8</td>
<td>1.0 e 27</td>
<td>19 554</td>
</tr>
<tr>
<td>9x9</td>
<td>2.7 e 34</td>
<td>912 352</td>
</tr>
<tr>
<td>10x10</td>
<td>1.2 e 43</td>
<td>5 821 097 789</td>
</tr>
<tr>
<td>11x11</td>
<td>2.2 e 52</td>
<td>???</td>
</tr>
</tbody>
</table>
UAlbertaconnection

- Martin Müller, Rich Sutton: David Silver
- Martin Müller, Ryan Hayward: Aja Huang
- Csaba Szepesvari
- Michael Bowling
http://webdocs.cs.ualberta.ca/~hayward/670gga/jem/go.html

computer go

https://gogameguru.com/i/2016/01/Fan-Hui-vs-AlphaGo.jpg AG-FH 5-0

https://www.youtube.com/watch?v=l-GsfyVCBu0&t=77m40s shoulder hit

https://www.youtube.com/watch?v=0CevCII1zo0&t=1m23s Ke Jie moment

https://webdocs.cs.ualberta.ca/~hayward/talks/hex.deepQ.pdf

invitation Yngi, Magnus
solving $10 \times 10$ Hex joint with Jakub Pawlewicz
builds on work with B Arneson, P Henderson
machine Martin Müller
photo courtesy MIT Museum, MIT, Cambridge MA
Natural Sciences and Engineering Research Council of Canada