alternating linear clobber

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thank you organizers!   MMüller!

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clobber

- *intro to clobber* 2001 AGNW

  Albert Grossman Nowakowski Wolfe

  https://webdocs.cs.ualberta.ca/~hayward/papers/AGNW.pdf

  this talk:

  https://webdocs.cs.ualberta.ca/~hayward/talks/cgtcv.pdf

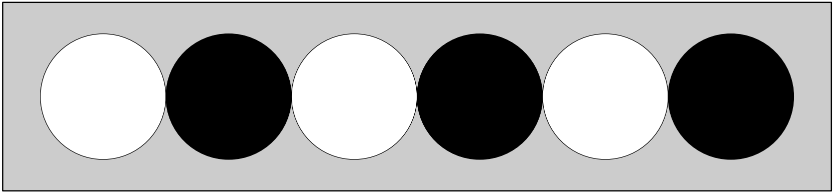
alternating linear clobber

- *linear clobber* clobber on a path
- *alternating linear clobber*

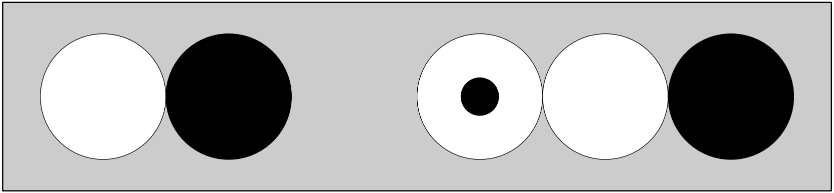
  linear clobber, starting from one of

  `ox, oxox, oxoxox, oxoxoxox, ...`

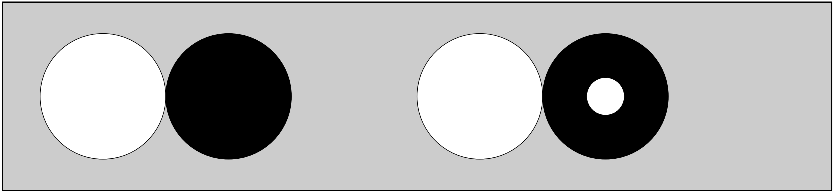
example ALC game



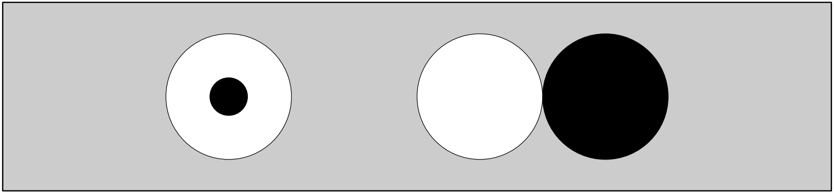
example ALC game



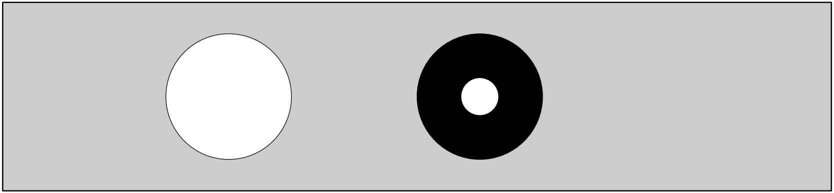
example ALC game



example ALC game



example ALC game



now White has no legal moves, White loses

conjecture

conjecture 3.2 AGNW 2001:

for every ALC start position except `oxoxox`,

first player can win

we prove this

proof sketch

$\mathcal{A}$ is set of starting ALC positions

K is some games reachable from $\mathcal{A}$, all 1p-win

for each opponent move on a game in K,

find a move that returns to K

details: what subgames can occur?

: 6 sets of parts

: what is K?

: what is the 1p-win strategy?

: what moves can the opponent make?

6 sets of parts: $\mathcal{A}$, $\mathcal{O}$, $o\mathcal{A}$

$\mathcal{A} = \{$ox, oxox, oxoxox, ...$\}$

$\{$a2, a4, a6, ...$\}$

$\mathcal{O} = \{$o, oxo, oxoxo, ...$\}$

$\{$o, o3, o5, ...$\}$

$o\mathcal{A} = \{$o, oox, ooxox, ... $\}$

$\{$o, oo3, oo5, ... $\}$

6 sets of parts: $o\mathcal{O}$, $o\mathcal{O}o$, $o\mathcal{A}x$

$o\mathcal{O}$ = {oo, ooxo, ooxoxo, ...}

{oo, oo4, oo6, ...}

$o\mathcal{O}o$ = {ooo, ooxoo, ooxoxoo, ...}

{ooo, oo5oo, oo7oo, ...}

$o\mathcal{A}x$ = {ooxx, ooxoxx, ooxoxoxx, ...}

{ooxx, oo6xx, oo8xx, ...}

warmup

$$\text{ox, oxo, ooxox, ooxoxoo} \quad = \quad *$$

$$\text{oxx} = \uparrow$$

$$\text{oxox} = \pm\{\uparrow, 0\}$$

$$\text{ooxo, oxoxoxoxo} \quad = \quad \uparrow*$$

$$\text{ooxoxx, oxoxoxoxoxox} \quad = \quad \text{oxox} \; *$$

$$\text{oxoxo} = \{\uparrow \mid *\}$$

standard form: apply after each move

o, oo, ooo, ooxx, oxoxox $\Rightarrow$ _ (game 0)

oxo, ooxox, ooxoxoo $\Rightarrow$ ox

oxoxoxoxo $\Rightarrow$ ooxo

ooxoxx, oxoxoxoxoxox $\Rightarrow$ oxox ox

ooxoo $\Rightarrow$ oox

ooxo $\Rightarrow$ xxo ox

standard form

also apply the negative rule forms

also apply: for any p,

$$p + -p \Rightarrow \quad \_ \quad \text{(game 0)}$$

repeatedly apply rules until no rule applies

theorem 1 (easy)

theorem 1:

every part that arises in an ALC game

is in one of $\mathcal{A}$, $\mathcal{O}$, $o\mathcal{A}$, $o\mathcal{O}$, $o\mathcal{O}o$, $o\mathcal{A}x$

or their negatives.

$\mathcal{A}'$ to $o\mathcal{O}o'$

$$
\begin{aligned}
\mathcal{A}' &= \mathcal{A} \setminus \{\text{a2, a4, a12}\} \\
\mathcal{O}' &= \mathcal{O} \setminus \{\text{o, o3, o9}\} \\
o\mathcal{O}' &= o\mathcal{O} \setminus \{\text{oo, oo4, oo6, oo8}\} \\
o\mathcal{A}' &= o\mathcal{A} \setminus \{\text{o, oo3, oo5}\} \\
o\mathcal{O}o' &= o\mathcal{O}o \setminus \{\text{ooo, oo7oo }\}
\end{aligned}
$$

$\mathcal{A}'$ to $o\mathcal{O}o'$

$\mathcal{A}'$	{		a8	a10		a14	...
$\mathcal{O}'$	{	o5	o7		o11	o13	...
$o\mathcal{O}'$	{			oo10	oo12	oo14	...
$o\mathcal{A}'$	{	oo7	oo9	oo11	oo13	oo15	...
$o\mathcal{O}o'$	{	oo5oo		oo9oo	oo11oo	oo13oo	...

definition of irregular games $\mathcal{I}$

$$\mathcal{I} = \{\texttt{a2},\ \texttt{a4},\ \texttt{oo6}\}$$

$$= \{\texttt{ox},\ \texttt{oxox},\ \texttt{ooxoxo}\}$$

definition of set K

$K =$ games g, each part in any of

$$\mathcal{O}' \quad o\mathcal{O}' \quad o\mathcal{O}o' \quad \mathcal{I} \quad \{\text{xxo}\} \quad \{\text{oo8}\} \quad \mathcal{A}' \quad o\mathcal{A}'$$

count vector $\mu(g)$

(a, b, c, d, e, f, y, z)

definition of games to be avoided Q

- $Q = \{\text{o5} + \text{a2}, \text{o7} + \text{a4}, \text{o15} + \text{a4} + \text{a2}\}$
- thm: $\mathcal{O}' \subseteq \mathcal{L}$
- thm: sum $\mathcal{O}' +$ ox $\setminus Q \subseteq \mathcal{L}$
- thm: sum $\mathcal{O}' +$ oxox $\setminus Q \subseteq \mathcal{L}$
- thm: sum $\mathcal{O}' +$ oxox $+$ ox $\setminus Q \subseteq \mathcal{L}$

definition of sets $\mathcal{S}_0$, $\mathcal{S}_1$, $\mathcal{S}_2$

$\mathcal{S}$ games of $K \setminus Q$, $a + \ldots + z \geq 1$,

$\mathcal{S}_0$: $a \geq c, \quad y \leq 1, \quad z = 0$ or

$a \geq c + 1, \quad y = 0, \quad z = 1$

$\mathcal{S}_1$: $y, z = 0$, $a \geq c + 1$

$\mathcal{S}_2$: $a, b, c, y, z = 0$, $e + f \geq 1$ and

$d = 0$ or $d + e \geq 3$

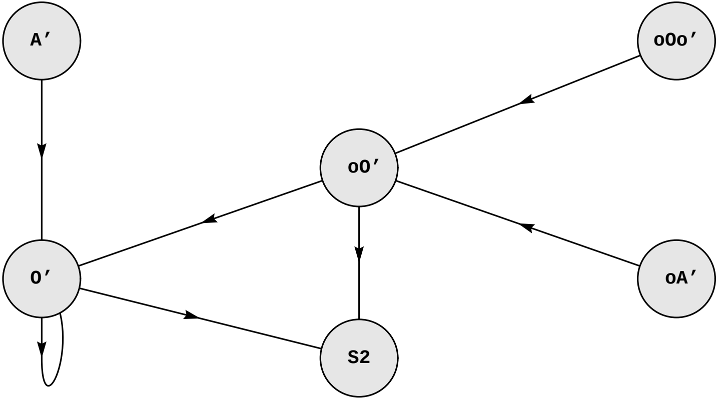
main results

- $g \in \mathcal{S}_1 \cup \mathcal{S}_2$: each R-move stays in $\mathcal{S}_0$
- $g \in \mathcal{S}_0$: L can move into $\mathcal{S}_1 \cup \mathcal{S}_2 \cup \{0\}$
- $\mathcal{S}_0 \subset \mathcal{L} \cup \mathcal{N}$
- $(\mathcal{S}_1 \cup \mathcal{S}_2) \subset \mathcal{L}$
- $\mathcal{A} \setminus \{\texttt{oxoxox}\} \subset \mathcal{N}$

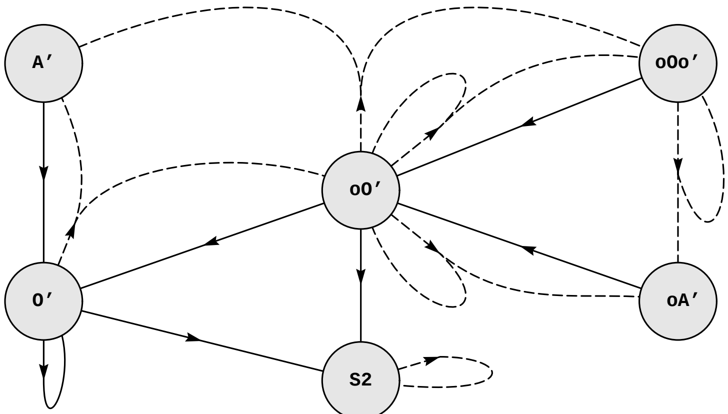
how Left wins on $\mathcal{A} \setminus \{$oxoxox$\}$: outline

- play any part in $\mathcal{A}' \cup o\mathcal{O}'$ to $\mathcal{O}'$
- play any part in $o\mathcal{O}o' \cup o\mathcal{A}'$ to $o\mathcal{O}'$
- avoid Q
- ignore $\mathcal{I}$ until the end

how Left wins



how Left and White play



where is the devil?

where is the devil?

in the details

how L wins

rule	{game} or subgame	result
1.	g in $\mathcal{S}_0$	
	{ a8 a2}	a4 ooxo (avoid o5 a2)
	{a10 a4}	o5 xxox a4 (avoid o7 a4)
	{a18 a4 a2}	a14 xxo a4 a2 (avoid o15 a4 a2)
	a8, a10, a14, ...	o5, o7, o11, ...

how L wins

rule	$\{$game$\}$ or subgame	result
2.	$\mathcal{A}'$ empty	
	oo7, oo9, . . .	oo4, oo6, . . .
	$\{$o5 oox$\}$	xxo oox
	o5 a4 a2 oox	o5 a4 a2 a2
	a4 oox	xxo oox
	oox	a2
3.	$\mathcal{A}', o\mathcal{A}'$ empty	
	oo9oo, oo11oo, . . .	oo4, oo6, . . .

how L wins

rule	{game} or subgame	result
4.	$\mathcal{A}', o\mathcal{A}', o\mathcal{O}o'$ empty	
	{oo10 a2}	o7 a2 a2 (avoid o5 a2)
	{oo12 a4}	oo8 xxo a4 (avoid 7 4)
	oo10	o5
	oo12	o7
	oo14	o11 a2
	oo16	o11
	oo18	o13 (avoid o15 a4 a2)
	oo20	o17 a2 (avoid o15 a4 a2)
	oo22, oo24, . . .	o17, o19, . . .

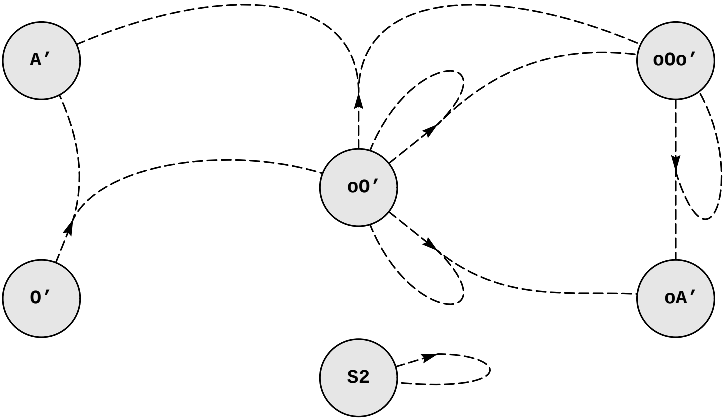
how L wins

rule	{game} or subgame	result
5.	$\mathcal{A}'$, $o\mathcal{A}'$, $o\mathcal{O}o'$, $o\mathcal{O}'$ empty	
	oo6 oo6 a4	xxo oo6 a4
	oo6 oo6 a2	xxo oo6 a2
	oo6 oo6	xxo a2 oo6
	oo6 a4 a2	xxo a4 a2
	oo6 a4	xxo a4 a2
	oo6 a2	xxo
	a4 a2	a2 a2
	oo6	xxo
	a4	xxo
	a2	0

how L wins

rule	{game} or subgame	result
6.	$\mathcal{A}'$, $o\mathcal{A}'$, $o\mathcal{O}o'$, $o\mathcal{O}'$, $\mathcal{I}$ empty	
	o13, o15, . . .	o11, o13 . . .
	o11	o7 xxo
	o7	o5
	o5	xxo
7.	$\mathcal{A}'$, $o\mathcal{A}'$, $o\mathcal{O}o'$, $o\mathcal{O}'$, $\mathcal{I}$, $\mathcal{O}'$ empty	
	oo8	ooxo xxo
	xxo	0

how Right can move



e.g. how Right can move on oxoxoxoxo

can assume R clobbers to right

(oxoxoxoxo has left-right symmetry)

oxoxoxoxo $\longrightarrow$ _ ooxoxoxo

oxoxoxoxo $\longrightarrow$ ox _ ooxoxo

oxoxoxoxo $\longrightarrow$ oxox _ ooxo

oxoxoxoxo $\longrightarrow$ oxoxox _ oo

example game from a26

a26	ox. . . oxoxoxoxoxoxox
L $\rightarrow$ o23	ox. . . oxoxoxoxoxo xx
R $\rightarrow$ a12 oo10	oxoxoxoxoxox ooxoxoxoxo
st.form a4 a2 oo10	oxox ox ooxoxoxoxo
L $\rightarrow$ a4 a2 o7	oxox ox oxoxoxo
R $\rightarrow$ o3 a2 o7	oox ox oxoxoxo
L $\rightarrow$ a2 a2 o7	ox ox oxoxoxo

example game from a26 (cont.)

L $\rightarrow$ a2 a2 o7	ox ox oxoxoxo
st.form o7	oxoxoxo
R $\rightarrow$ a4	oxox oo
L $\rightarrow$ xxo	o xxo
R $\rightarrow$ a2	xo
L $\rightarrow$ 0	x

thank you

questions?

email hayward@ualberta.ca

to arrange video chat