Apply Heuristic Search to Discover a New Winning Solution in 8 × 8 Hex Game

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ABSTRACT

In certain games such as Hex, human experts are still much stronger than the best existing computer programs. By combining human Hex experts' excellent "sense" of good and bad opening moves and our newly developed search algorithms, knowledge representations, and detailed rules, this paper describes a new Hex solver.

As the result, a new winning solution on 8×8 Hex board has discovered with the first move at F3.

1. INTRODUCTION

Hex is a two-player strategy board game. A Hex board is composed of hexagons that form a parallelogram shape shown in Figure 1. Sometimes, the Hex board is also described as a diamond shape illustrated in Figure 2. In fact, these two Hex boards are identical and each board is merely a rotated version of the other. In this paper, we use the parallelogram version of the Hex board. The size of a Hex board, denoted by n×n that indicates n rows and n columns, can vary. The larger the Hex board is, the more complex and difficult the game will be. As shown in Figure 1, corresponding to their location on the parallelogram, four borders of the Hex board are referred to as "top", "bottom", "left", and "right", respectively, Two players take turns placing black and white game pieces on empty hexagons. The player playing black color pieces will start the game. The player with black pieces begins the game. The player playing black pieces tries to make a connected chain from "top" to "bottom", and the player playing white pieces tries to make a connected chain from "left" to "right". The first player to make a connected chain from one border to another wins the game. For example, Figure 3 illustrates the middle stage of a Hex game and it is black's turn. If the player plays a black piece at hexagon "A", the player playing black will win the game. However, if a black piece is placed at any empty hexagon other than "A", the other player will play a white piece at hexagon "A" and win the game.

Since Hex was invented independently by Piet Hein and John Nash in 1940's [3], several important research papers have been published. Shannon and Moore developed a Hex playing machine in 1953 [8], and their electrical charge distribution approach still is applied in

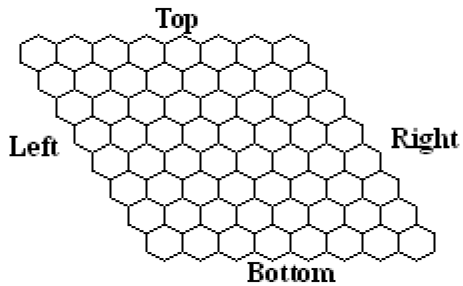

Figure 1: An 8x8 Hex board

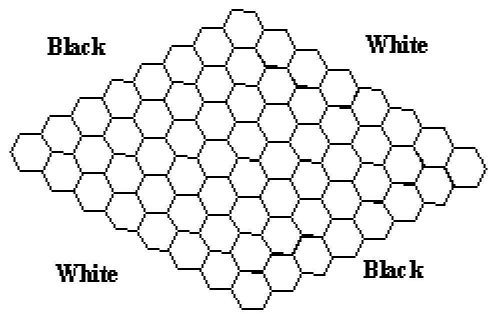

Figure 2: A diamond shape 8x8 Hex board

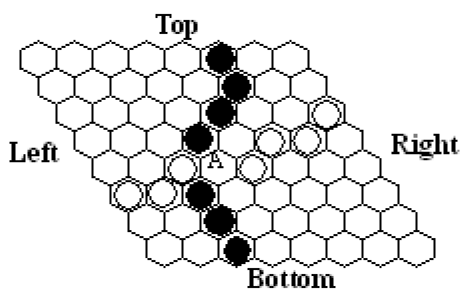

Figure 3: The first player plays at cell "A" wins the game

Hexy, one of the strongest computer Hex programs nowadays [2]. Since a white connected chain and a black connected chain cannot co-exist, there is no draw in Hex. A strict mathematical proof was given by David Gale in late 1970's [4]. Based on this fact and strategy stealing proof technique, John Nash proved that the first player will win, if optimal moves are made [5], though he did not provide a detailed solution on how to win. Usually, solving a game means that a detailed account of achieving the optimal result is provided. Stefan proved that Hex is a

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PSPACE-complete game [6], which indicates that searching for a winning strategy in an arbitrary game state can be a computation-consuming task. By John Tromp's estimation of possible game states for Hex games [3], when the Hex board is 8×8, 9×9, 11×11, and 14×14, the possible game states are 4.18×10^{29}, 2.4×10^{37}, 2.38×10^{56}, and 1.14×10^{92}. Performing a brute force search on such search spaces with the existing technologies and computers will be impossible. Therefore, finding winning strategies for larger Hex boards is a very challenging task to researchers. In 2001, a winning strategy for a 7×7 Hex board was first accomplished [9]. Then, the winning strategies for 8×8 and 9×9 Hex boards were also conducted [11][12]. In 2003, a team of researchers from the University of Alberta solved all first moves on a 7×7 Hex board [7].

In this paper, we will describe a Hex solver that discovers a new winning strategy on an 8×8 Hex board.

2. THE SEARCH ALGORITHMS

Hex is one of the a few games that human experts are still much stronger than the best existing computer programs. The human Hex experts are "smarter" in two perspectives: they can apply general problem solving techniques to reduce the search space significantly, and human players have an excellent "sense" of good and bad moves, which leads to limit the search space to good moves only.

A greedy decomposition method was introduced in 2001 [9], which lead to the discovery of the first winning strategy to solve the 7×7 Hex game. The decomposition method divides a Hex game into several "local games" with sub-goals. After all sub-goals are achieved, the original game will be solved. A sub-goal is associated with several empty positions called the "influence region" or "carrier". When the opponent's move is not played within the influence region, the sub-goal is secured. A Hex game can often be decomposed in this way. One of the benefits of the decomposition method is that the influence regions can lead to cut-offs in a search. For example, in Figure 4, if white's 1 is not played among positions A through N and position black 2, black can play at 2 to win the game. Other moves for white would not be further searched because they are worse than or equal to the previous move 1.

In a depth first search, the region cut-off takes effect when the search process has reached a leaf of the tree. In Figure 5, assume node L is a leaf, and the score of this node is 1 (black wins). In step 1, the influence region, marked as IA, can be calculated. Then, in step 2, IA is used to cut white's nodes on the above level (nodes A, B, C, D, ...). These nodes A, B, C, D, …, may not be further explored if they are located outside of the influence region. Assume that nodes A, B, and C have not been cut, then,

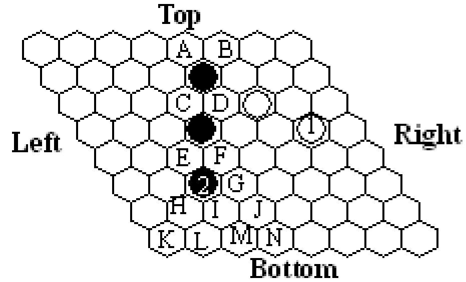

Figure 4: Positions A to N and black 2 form an influence region for black's win.

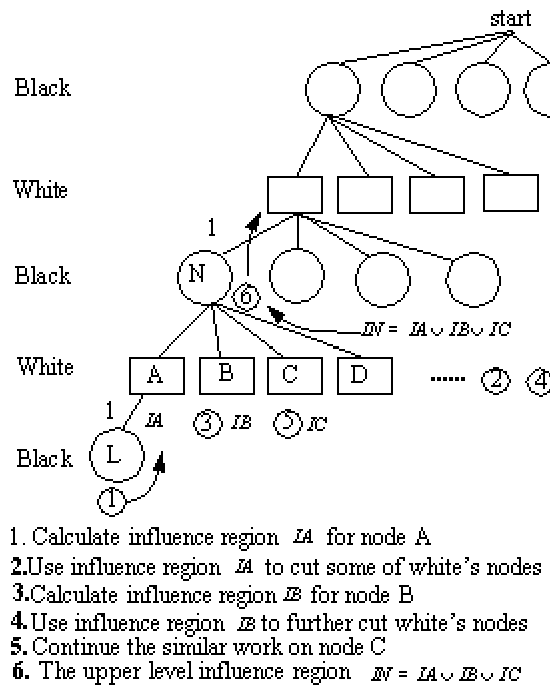

Figure 5: Region cut-off details

step 3 will search down the node B. If the result of searching down node B is also a black's win, the step 4 will be conducted, in which a new influence region IB is calculated again and used to cut rest of nodes similar to that of step 2. If there is no more node to be cut further, step 5 will be carried out is to search down node C. Assume the score of node C is also 1 and the new influence is IC, since all nodes on this level are either checked or cut, the score of node N will be 1 (black wins) and the influence region is the union of IA, IB, and IC. This bottom-up process can further propagate to the upper levels.

During the process of searching for a winning solution, all of the opponent's possible moves should be considered. Although the region cut-off idea can greatly reduce the number of defense moves to consider, however, it is still not enough to lead to a winning strategy on an 8×8 Hex board. Human Hex experts can understand a board situation with logical insights. These insights will tell them that playing a certain move may lead to a win and playing another move will make a win impossible. Based on this advantage, Hex experts only need to consider a few moves on each level of a search. They may make a mistake sometimes, but for the most part this insight will lead to the desire result.

Searching a subset of possible moves is called a selective search, which is a popular technique in computer game programs. If one applies this approach to solving a game when the heuristics are not perfect, the solution maybe missed. There is an interesting concept named symmetric property in Hex game, which indicates that a winning opening may reveal some losing openings. This idea was applied in our previous work to find losing openings in a 7×7 Hex board [10]. The same idea was also utilized in the process of finding a winning strategy on an 8×8 Hex board [11]. Figure 6 shows black's first move in this winning strategy for 8×8 Hex. Based on the region cut-off algorithm, the positions marked by a cross in Figure 6 do not have any effect to disallow black 1 to win the game. Due to the symmetric property, if black's first move is played at anyone of the positions shown in Figure 7, white can play at 2 and use the same winning strategy to win.

The selective search attempt will not be able to find a winning solution if there is no winning strategy for the player under the current game situation. The symmetric property can further be used to determine the reason of not being able to find a winning move. To do this, a symmetry game needs to be created and the move against black's solving in the original game will be the testing move in the symmetry game. Figure 8 shows the symmetry game of an original game displayed in Figure 9 by changing original position (x,y) to (y,x) and swapping the color. If the symmetry game is solvable, there will be no winning solution for the original game. If the symmetry game is not solvable, it will lead to the conclusion that the heuristic set needs to be improved further. For example, the proposed Hex solver did not find a solution for the original game shown in Figure 9, but solved the symmetry game displayed in Figure 8. Therefore, the final conclusion is that there is no winning solution for black in the case.

3. KNOWLEDGE REPRESENTATION

Human Hex experts understand a game based on groups, shapes, and the relationships among groups. They can view a game on both abstract and physical levels. On an abstract level, unimportant information is filtered out. The understanding procedure is a step by step communicating process from physical (local) level to abstract (global) level. Local information will be reflected up to the global level and can affect the global view. Modification of the global view will also provide a guide

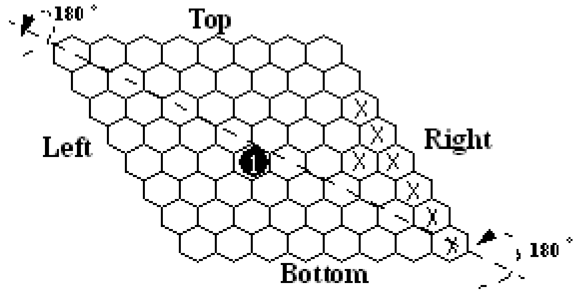

Figure 6: Black 1 is a winning move and those positions with a cross mark do not have any influence on the black's winning. This can be proved by rotating board 180 degree along its long diagonal.

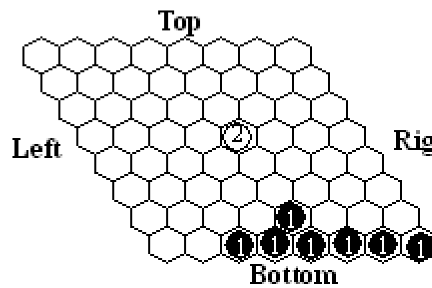

Figure 7: If black's first move is any 1s, white can play at 2 to win the game.

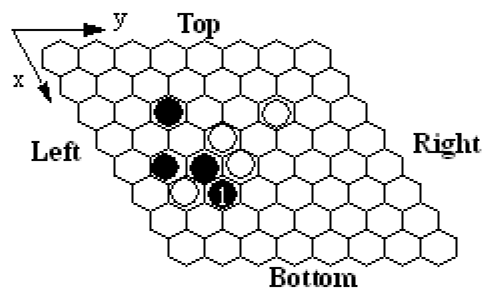

Figure 8: The symmetry game of the game in Figure 9

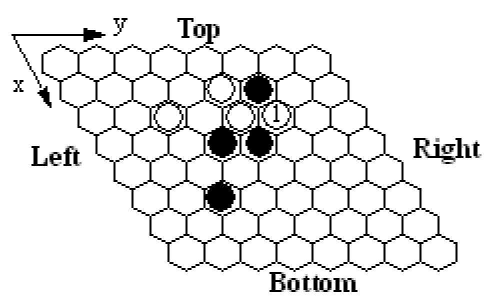

Figure 9: The original game.

for the further understanding in local levels. This communication process may be repeated continuously until a conclusion is reached. The proposed Hex solver is designed to match experts' understanding. It is an object

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oriented design and has a multiple-layer structure that passes information back and forth between layers. Each layer keeps different type of information in order to be retrieved efficiently. The lowest physical layer is an integer matrix that represents the Hex board, with 0 for unoccupied, 1 for black piece, and 2 for white piece in each cell. The same color pieces that form a string object are called a connected string.

Each connected string has a property "StringType" to indicate the string's relationship with borders. For example, "Top" means that the string pieces at the top row, and "TopBottom" means that the string is a winning string for black. Based on the region cut-off concept, two connected strings may virtually connect each other with an influence region. The proposed Hex solver uses pattern matching to figure out the virtual connection for each connected string. Two virtual connection examples are shown in Figure 10 and Figure 11. Each example has six rotations and a number of special border situations. A connected string and all of its virtually connected strings forms a group. A group also has a property "GroupType" that indicates the relationship of the group with borders. The proposed Hex solver considers two types of group relationships, "OneToConnect" and "OnePossibleConnect". "OneToConnect" means that playing one more move will make a connection between the two groups. For example, in Figure 12, position "c" is an "OneToConnect" move to connect the two groups. "OnePossibleConnect" means that playing one more move will have a good chance to make a connection with the two groups. For example, in Figure 13, position "a" is an "OnePossibleConnect" move to the bottom for black. Please note that the top and the bottom can be viewed as a long black string just beside the border, and so are the left and the right to be viewed as a long white string beside the border.

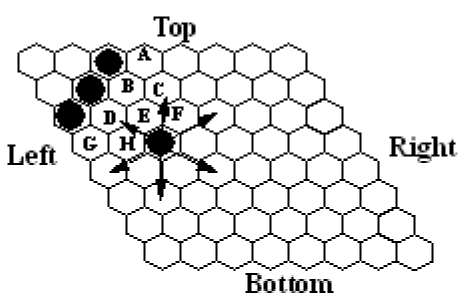

Figure 10: A virtual connection example, the arrows indicate the rotations.

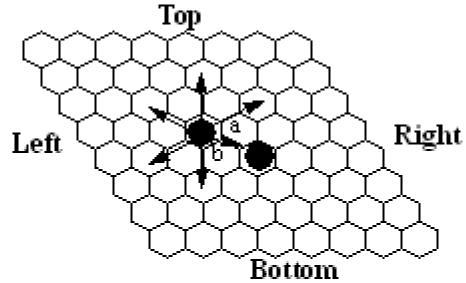

Figure 11: Another virtual connection example, the arrows indicate the rotations.

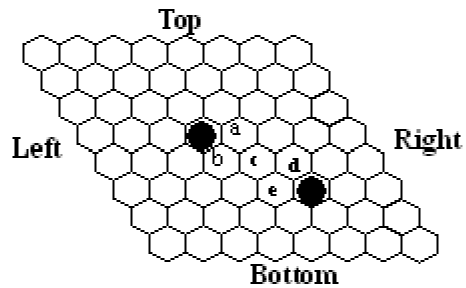

Figure 12: An OneToConnect case: 'c' is an OneToConnect move for black

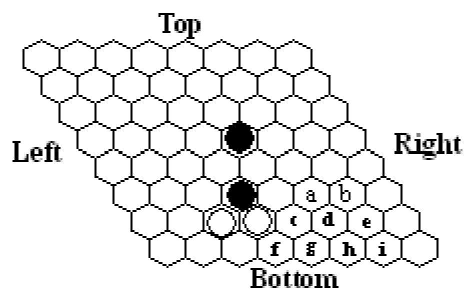

Figure13: An OnePossibleConnect case: 'a' is an OnePossibleConnect move for black

4. DETAILED RULES

The candidate moves are generated based on following heuristics: (listed by priority)

1. IF a black group has group type = "TopBottom" AND carrier overlap number >0 THEN select the overlap positions as generated moves
2. IF white group has group type = "LeftRight" AND carrier overlap number >0 THEN select the overlap positions as generated moves
3. IF a group has group type = "Top" THEN the group's "OneToConnect" to Bottom moves are generated moves.
4. IF a group has group type = "Bottom" THEN the group's "OneToConnect" to Top moves are the generated moves.
5. IF a group has group type = "Top" AND the group has an "OneToConnect" move to another group whose group

type = “Bottom” THEN the “OneToConnect” move is a generated move.

6. IF a group has group type = “Bottom” AND the group has an “OneToConnect” move to another group whose group type = “Top” THEN the “OneToConnect” move is a generated move.

7. IF a group has group type = “Top” THEN the group’s “OnePossibleConnect” to Bottom moves are generated moves.

8. IF a group has group type = “Bottom” THEN the group’s “OnePossibleConnect” to Top moves are the generated moves.

9. IF a group has group type = “Top” AND the group has an “OnePossibleConnect” move to another group whose group type = “Bottom” THEN the “OnePossibleConnect” move is a generated move.

10. IF a group has group type = “Bottom” AND the group has an “OnePossibleConnect” move to another group whose group type = “Top” THEN the “OnePossibleConnect” move is a generated move.

11. IF a group has “OneToConnect” moves to “Top” AND the group has “OneToConnect” moves to “Bottom” THEN all the “OneToConnect” moves in both sides are generated move. The moves in the side that have more “OneToConnect” moves have less preference value than that of moves in the other side.

12. IF a group has group type = “Top” AND the group has “OneToConnect” moves to another group which has “OneToConnect” moves to “Bottom” THEN all “OneToConnect” moves in both sides are generated moves. The moves in the side that have more “OneToConnect” moves have less preference value than that of moves in the other side.

13. IF a group has group type = “Bottom” AND the group has “OneToConnect” moves to another group which has “OneToConnect” moves to “Top” THEN all “OneToConnect” moves in both sides are generated moves. The moves in the side that have more “OneToConnect” moves have less preference value than that of moves in the other side.

Based on the described search algorithm and move generation mechanism, our Hex solver found a new winning solution for black on 8×8 Hex board. In order to show a position on Hex board more clearly, a coordinate system associated with letters and numbers is taken. For example, in Figure 14, black 1 is denoted as F3, which is the first winning move in the new winning solution.

When solving an opening for an 8×8 Hex board, the Hex solver still needs human experts to provide the beginning moves. The solution tree paths in the Appendix have shown part of the “opening book” assigned by human experts. In the representation of the winning solution tree, a path means that one possible winning way corresponds to different white’s defense moves. Positions separated by commas and surrounded inside a rectangle box means that the move can be played at any position inside of the rectangle box. Inside a rectangle box, a “-” indicates all positions between the given range. Outside a rectangle box, a “-” or a line leads to the next move position. A path marked with “...” means that the further details are not shown in this paper.

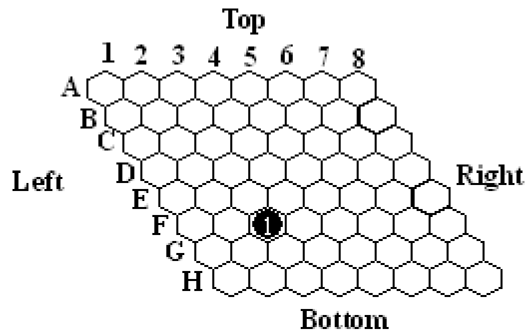

Figure 14: New Winning Solution for 8x8 Hex. F3 is the first winning move.

5. FUTURE RESEARCH

The proposed Hex solver can be further improved. For example, machine learning techniques can be used to grow the pattern database of the solver. More memorized patterns will help to detect early ending and to get more accurate board information. In a Hex opening, there exist good and bad moves, though both of them sometimes are very difficult to describe and quantify. How to find all of these good and bad opening moves remains an open question.

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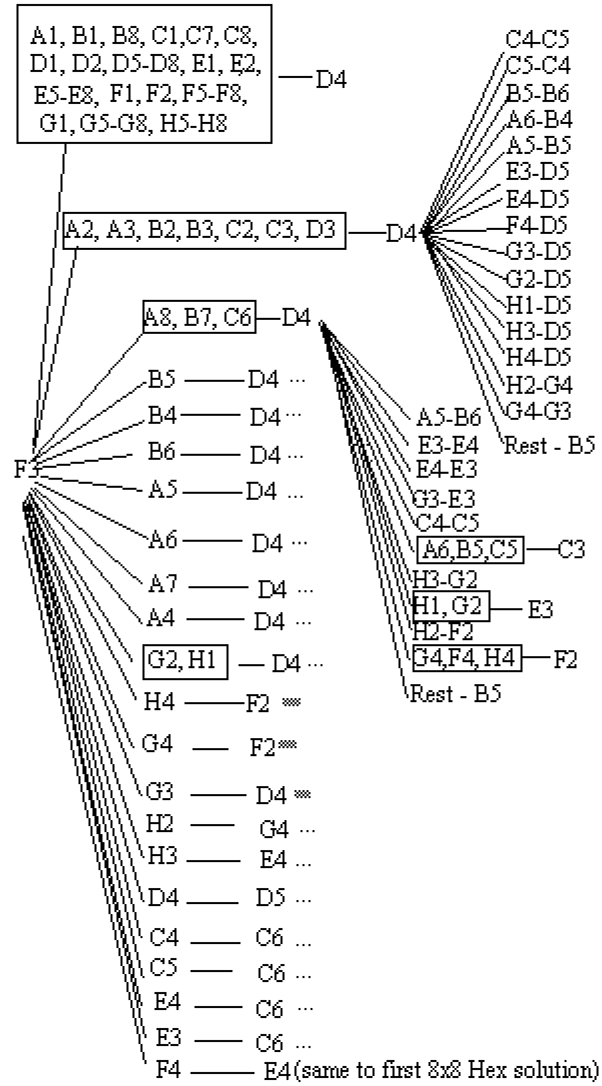
7. APPENDIX: THE SOLUTION TREE



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