Departments

BOOKS BEAT

Bewitching Puzzles

—Stephen Kennedy

Hex: A Playful Introduction by Ryan B. Hayward.
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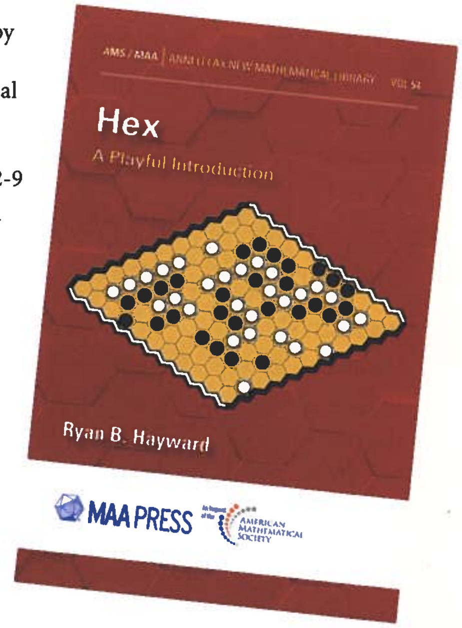

Hex is a two-player game invented in the 1940s, independently and nearly simultaneously, by Danish polymath Piet Hein and mathematician John Nash. The game is played on an $n \times n$ hexagonal grid. Players, we'll call them White and Black after the color of their playing pieces, alternately place stones in the cells of the grid. Black wins if they construct a connected path from the top border to the bottom of the board. White wins if they connect the left to the right.

The best thing you could do to learn more right now is to draw a grid, find some black and white stones, and recruit a friend to play a few games. The second-best thing you could do would be to attempt to solve the puzzles in Figure 1. In the top six boards of the figure, Black is to play and win. In the bottom two, White should play and win. (The third-best thing you could do would be to buy Ryan Hayward's engaging *Hex: A Playful Introduction* and read it.)

If you solved the first puzzle in Figure 1 you probably discovered the strategy Hayward calls bridging. Let's label the rows from top to bottom with the integers from 1 to 4 and the (slanted) columns from left to right a to d. Notice that if Black plays in b3, White cannot prevent Black from connecting to the bottom. The pair of cells {a4, b4} form a *bridge*. If White plays in either one, Black should immediately play in the other. Similarly, the pair {b2, c2} form a bridge to the top. This is not Black's only winning play, did you find the other? (If it were White's turn, they also have two winning plays. Can you find them?)

The strategy, as you might imagine, gets quickly complicated as the board size increases. Perhaps surprisingly, and both Hein and Nash knew this, the game cannot end in a draw and the first player has a winning strategy. The first claim is a consequence of the topology of the game board. If the game ended in a draw, then every cell would be occupied by either a black stone or a white stone and there would be no connected winning path. Imagine such a board and adjoin to the top and bottom of the grid rows of black stones (n on the top, $n+1$ on the bottom). Similarly, adjoin to the left and right sides columns of white stones (n on the left, $n+1$ on the right). I claim we can construct a directed path along the edges of the extended hexagonal grid that always keeps a black stone on its left and a white stone on its right. Start in the northwest corner between the abutting black and white stones and proceed to the vertex of cell a1. Turn left or right according to whether the stone occupying a1 is white or black, respectively. Notice that you will preserve the black on left and white on right property. Continue, making the same decision every time you get to a new vertex of the grid. At this point, if you haven't already, you might want to look at Figure 2.

Notice that you will never revisit a vertex you have already visited. If you did, there would have to be a first such

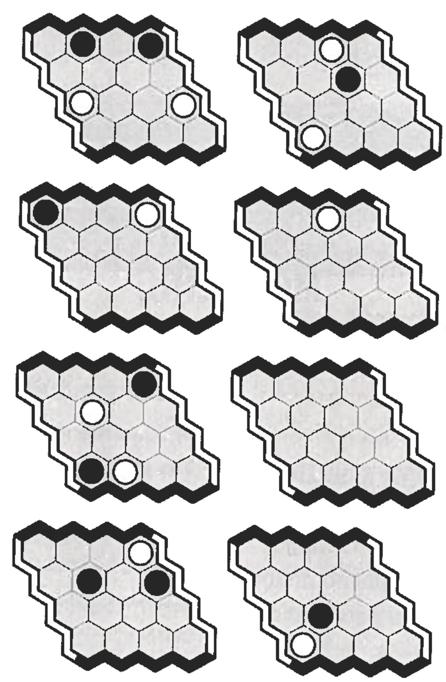

Figure 1. *On the top six grids, Black is to play and win. On the bottom two, White should play and win.*

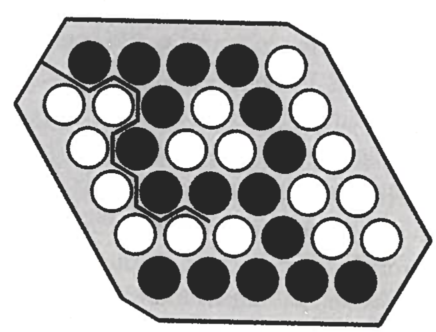

Figure 2. *This directed path keeps black on its left and white on its right until it exits the grid.*