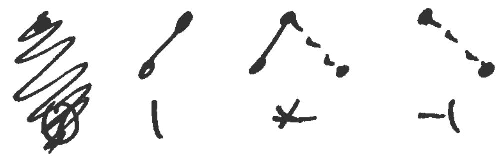
SOME ~~SHALLOW~~ SHALLOW GAMES

WHO WAR?
0

GAMES BORN ~~BY~~ ON DAY 0 • 0

ON DAY 1

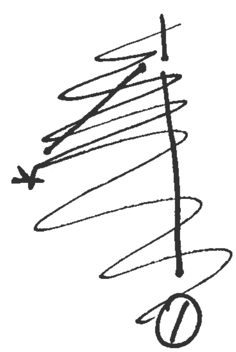


ON DAY 2: ?

L OPTIONS: · ANY SUBSET OF {} 0 1 * -1

· BUT....

~~HASSE DIAGRAM~~



DOWNLOAD CGSUITE

· BUT... SOME OF THESE ARE COMPARABLE

<u>E.G.</u> $0 < 1$

$* < 1$

. . . .

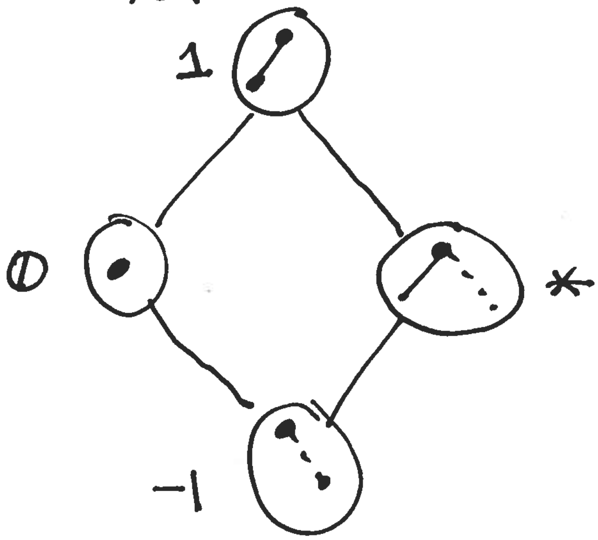
SO ONLY L OPTION SETS ARE

{} {1} {-1} {0, *} {0} {*}

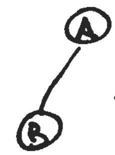
SO AT MOST 36 DIFF'T GAMES BORN ON DAY 2

IN FACT, ONLY 22

GAMES w DEPTH ≤ 1



HASSE DIAGRAM



$B < A$

TRANSITIVE

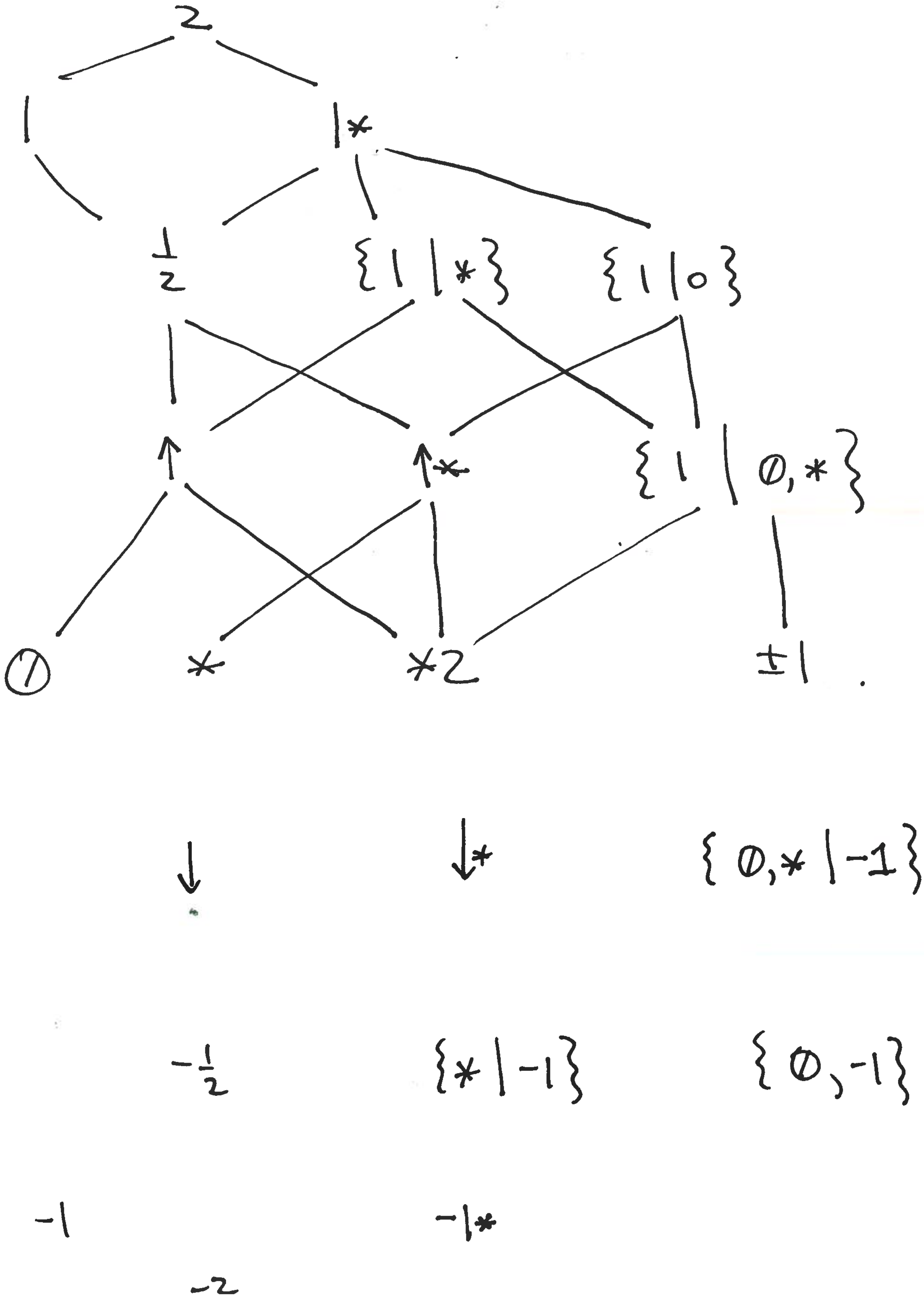
$\exists$ TOTAL ORDER $\leq$ ON INTEGERS

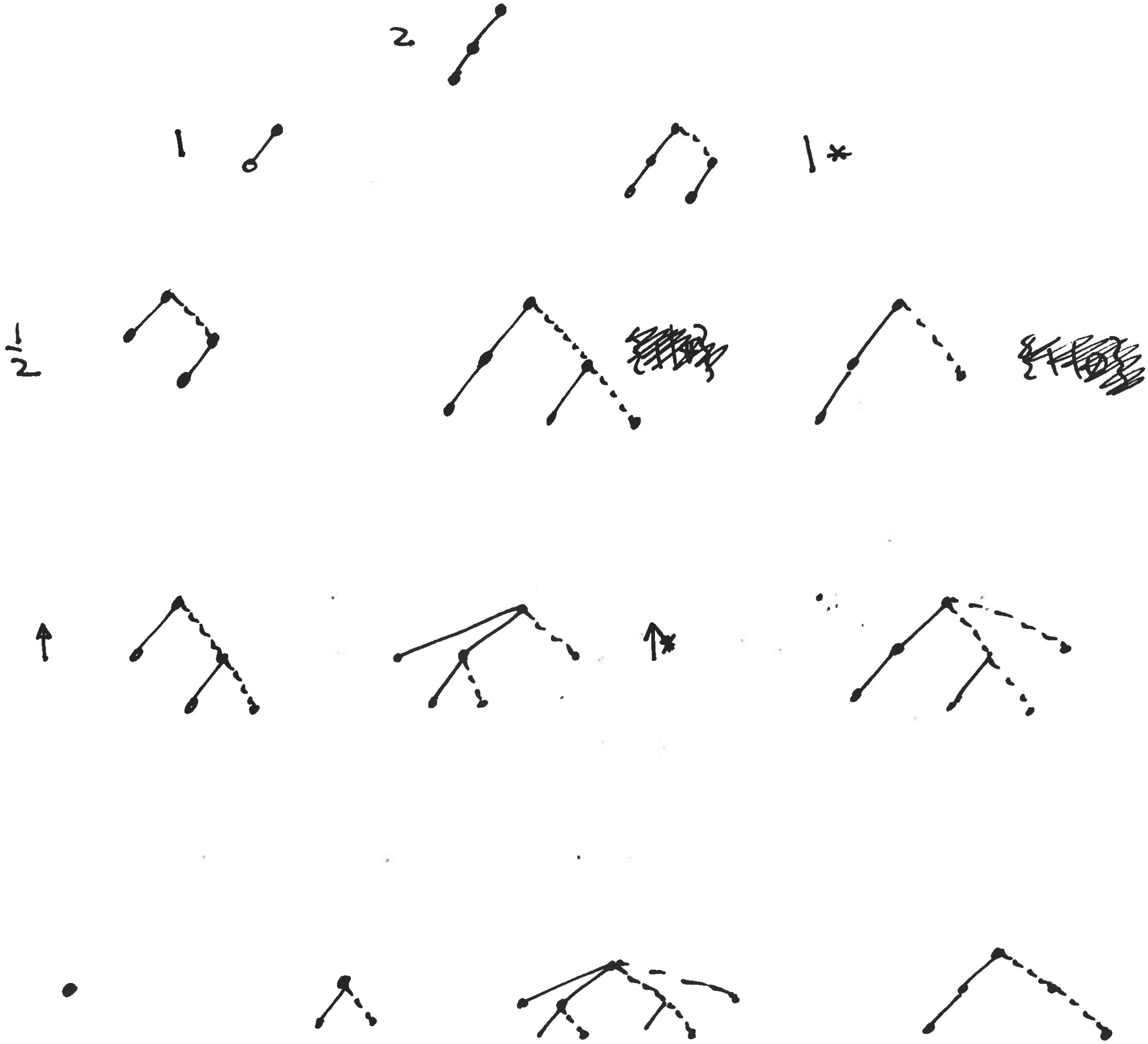
$\exists$ PARTIAL " $\leq$ ON GAMES

(SOME GAMES A, B HAVE A || B)

2020.3.3 (2

22 GAMES w DEPTH ≤ 2




2
1
1*
1/2
↑
↑*

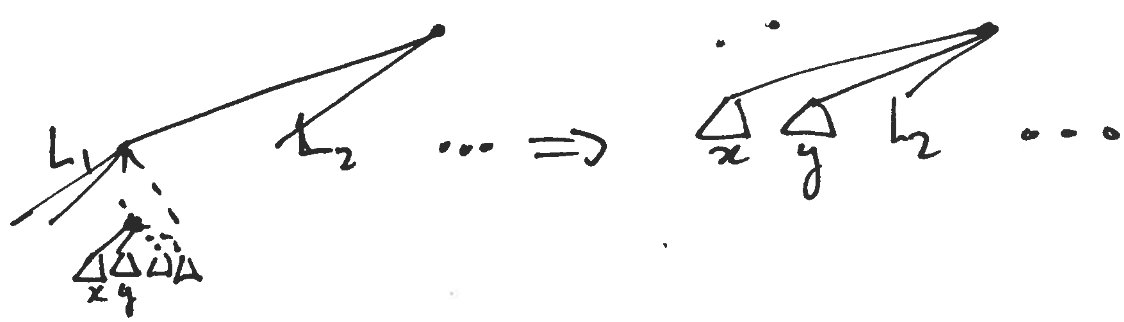
SIMPLIFYING: CAN. FORM II

LET $G = \{ L_1 \; L_2 \; \ldots \mid R_1 \; R_2 \; \ldots \}$

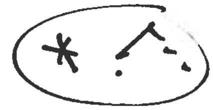
IF L_1 HAS A R-OPTION β_1 S.T. $\beta_1 \leq G$

THEN

$$G = \left\{ \begin{matrix} \text{ALL L OPT'NS OF } \beta_1 \\ \cancel{L_1} \end{matrix} \quad L_2 \; \ldots \;\middle|\; R_1 \; R_2 \; \ldots \right\}$$

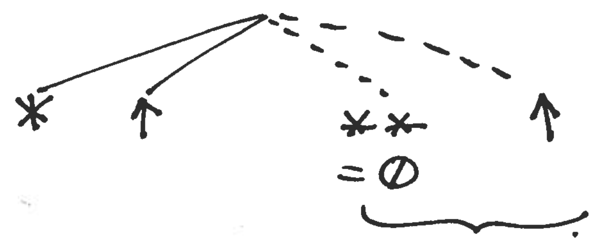


E.G. SIMPLIFY ~~*~~ $\uparrow *$



2020MAB: 5

$\uparrow *$



- PRUNE DOMINATED R-OPT

- REVERSE LEFT ↑-OPT

IF L PLAYS TO ↑

R CAN PLAY ON ↑ TO *

AND $* \le G = \uparrow *$

PROOF $* \le \uparrow *$

IFF ~~0~~ $** \le \uparrow * *$

IFF $0 \le \uparrow$

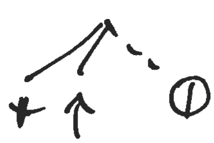
✓

SO REPLACE LEFT ↑-OPT

WITH SET OF ALL L-OPTIONS IN *

NAMELY $\{0\}$

SO



=

=

MORE CAN. FORM EXAMPLES (HAFF & GARNER)

SIMPLIFY

$\{1,2 \mid 3\}$ $\quad\quad$ $\{2 \mid 3\} = \frac{5}{2}$

$\{*, 1* \mid -5\}$ $\quad\quad$ $\{1* \mid -5\}$

$\{*, \uparrow, 1 \mid -1\}$ $\quad\quad$ $\{1 \mid -1\}$

$\{0, * \mid 0, \Downarrow\}$ $\quad\quad$ $\{0, * \mid \Downarrow\}$

$\{* \mid \uparrow, *\}$ $\quad\quad$ 0

$\{0, * \mid *\}$ $\quad\quad$ $\uparrow$

SINCE L $*$ OPTION IS REVERSED, ∴ REPLACE $*$ WITH $\{\}$

$\{\uparrow \mid \downarrow\}$ $\quad\quad$ $*$

REVERSE L: $\uparrow$ TO 0
R: $\downarrow$ TO 0

NOTICE $\uparrow\uparrow \equiv \Uparrow = \{\uparrow \mid \uparrow *\}$

$\Uparrow * = \{\Uparrow, \uparrow * \mid \Uparrow, \uparrow\}$ $\quad\quad$ $\{0 \mid \uparrow\}$

- R REVERSES $\Uparrow$ TO $\uparrow *$ AND $\uparrow * < \Uparrow *$ ☺
- SO REPLACE L: $\Uparrow$ WITH $\uparrow * . L = \{*, 0\}$

← REPLACE W/ $\{\}$

CANONICAL FORM

A.K.A. SIMPLEST FORM

"RECALL"

THM: ~~LET~~ $G = \{L_1, L_2 \ldots | R \ldots\}$

~~AND~~ **IF** $L_1 \le L_2$

THEN $G = \{\cancel{L_1}, L_2 \ldots | R \ldots\}$

PART I: DOMINATED OPTIONS

ALL SIMPLEST GAMES DEPTH ≤ 2

DEPTH 0 • 0

DEPTH 1

1 -1 *

DEPTH 2 ?

\# SIMPLES GAMES $\leq$?

SET OF L OPTIONS ?

\# " " ?

≤ 16

SUBSET OF $\{0, 1, -1, *\}$

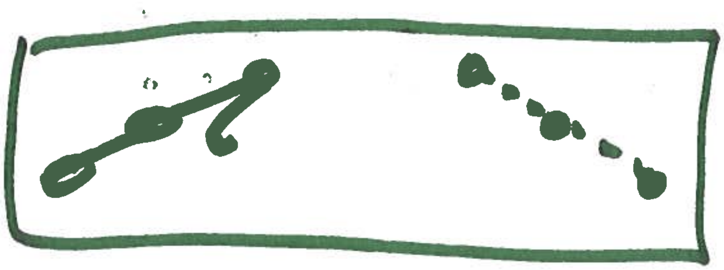
OF DEPTH 2 GAMES

$\leq 16 \times 16 = 256$

IN FACT $\underline{\underline{< 256}}$

BECAUSE SOME OF THESE 256 GAMES ARE EQUAL

CLAIM: $A = B$

PF: 

SET OF
L OPTIONS: SUBSET OF

$\{0, 1, -1, *\}$

$\{\ \}$

$\{0\}$ $\{0, 1\}$

$\{1\}$ $\{0, -1\}$

$\{-1\}$

$\{*\}$ $\{0, *\}$ . . . .

AFTER PRUNING

L OPTION SETS?

$\{\ \}$ $\{0, *\}$

$\{-1\}$ $\{0\}$

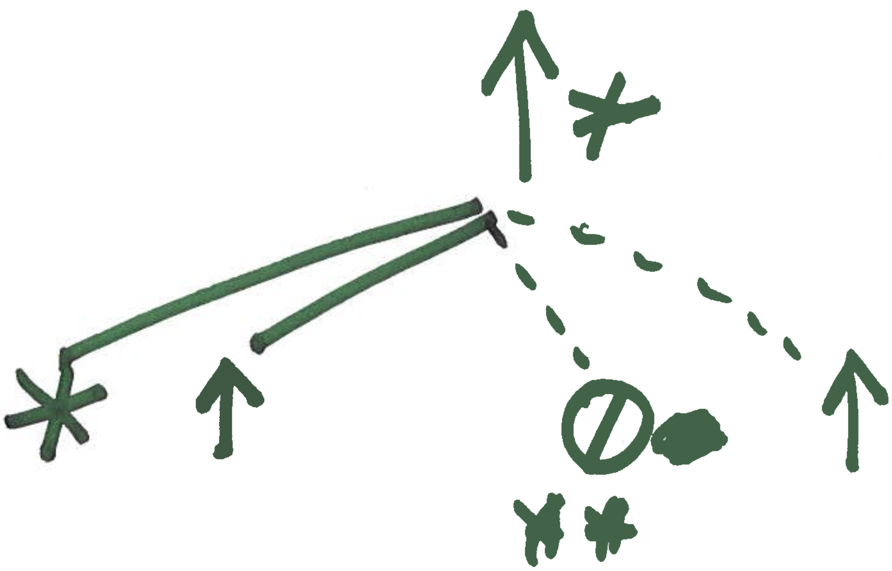
$\{1\}$

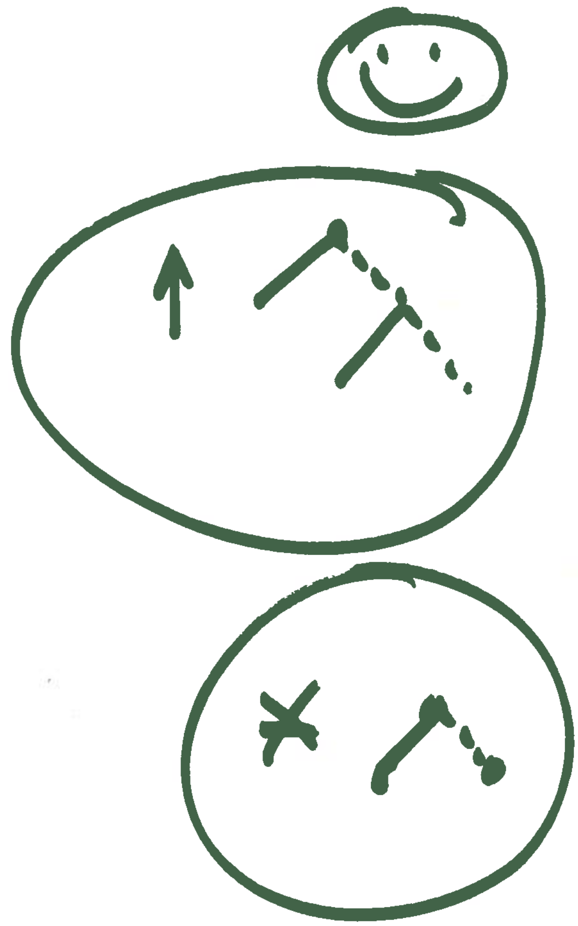
$\{*\}$

So ≤ 36 GAMES w DEPTH 2

MISSING 14 ?.

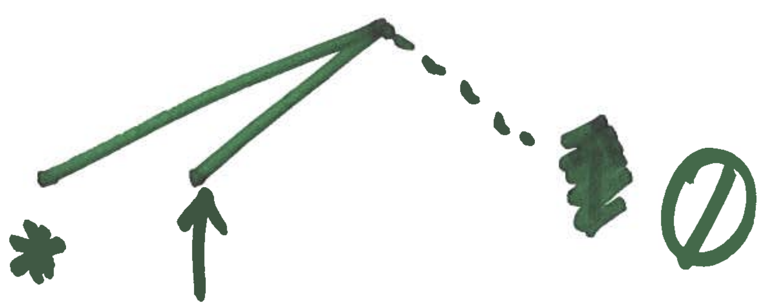
SIMPLIFIED





SIMP. ?

CAN. FORM : PART I



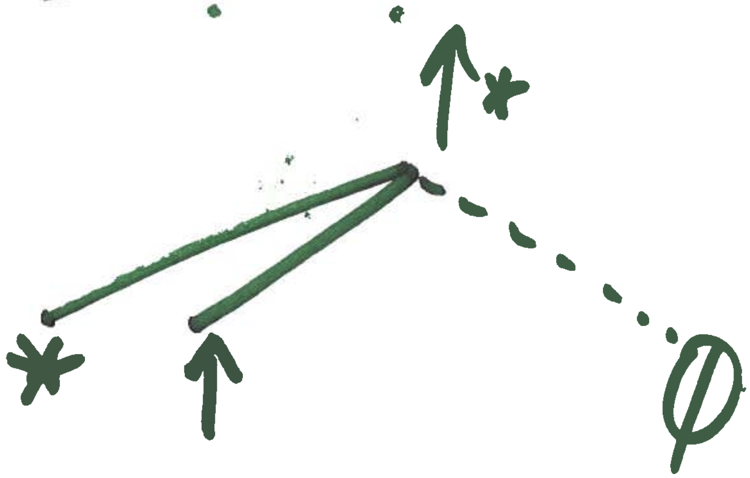
PROVE $\uparrow \parallel *$

$\uparrow \parallel *$

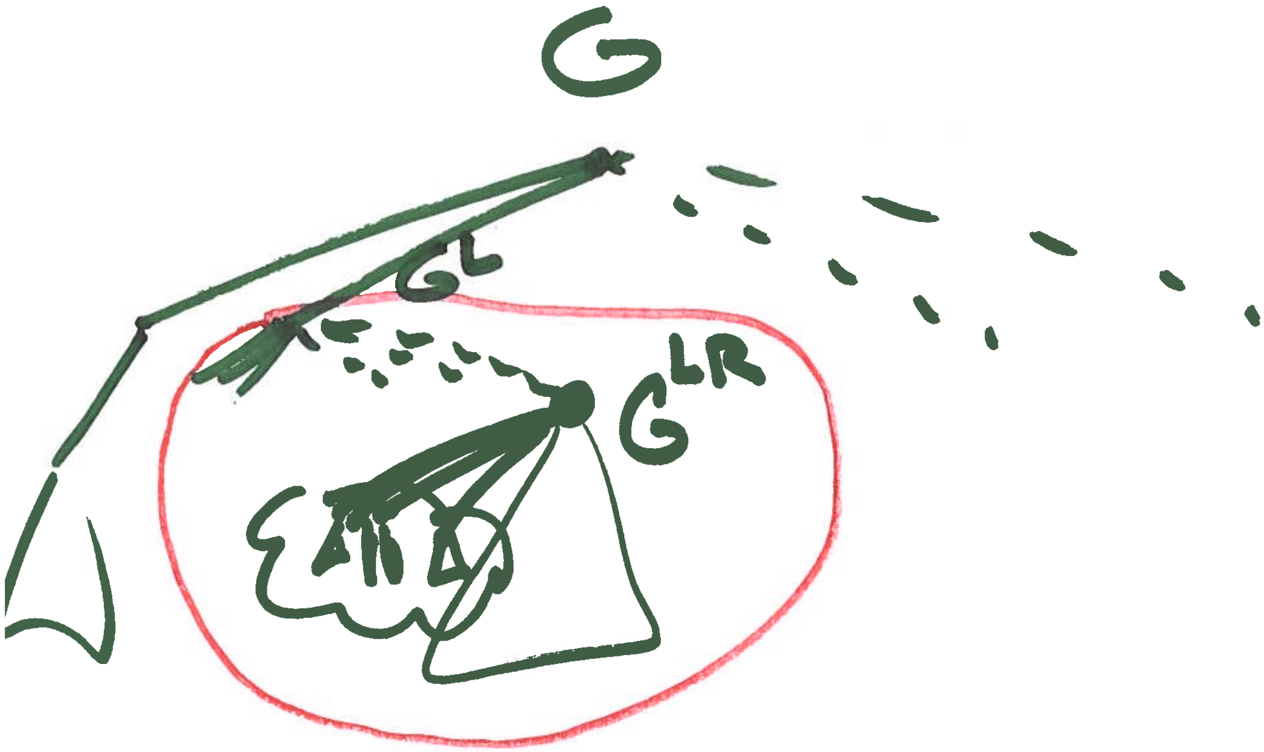
IFF $\uparrow^* \parallel **$ (WHY ?)

IFF $\uparrow^* \parallel \emptyset$ (" ?)

IFF $\uparrow^*$ IS A Π-PSN ?

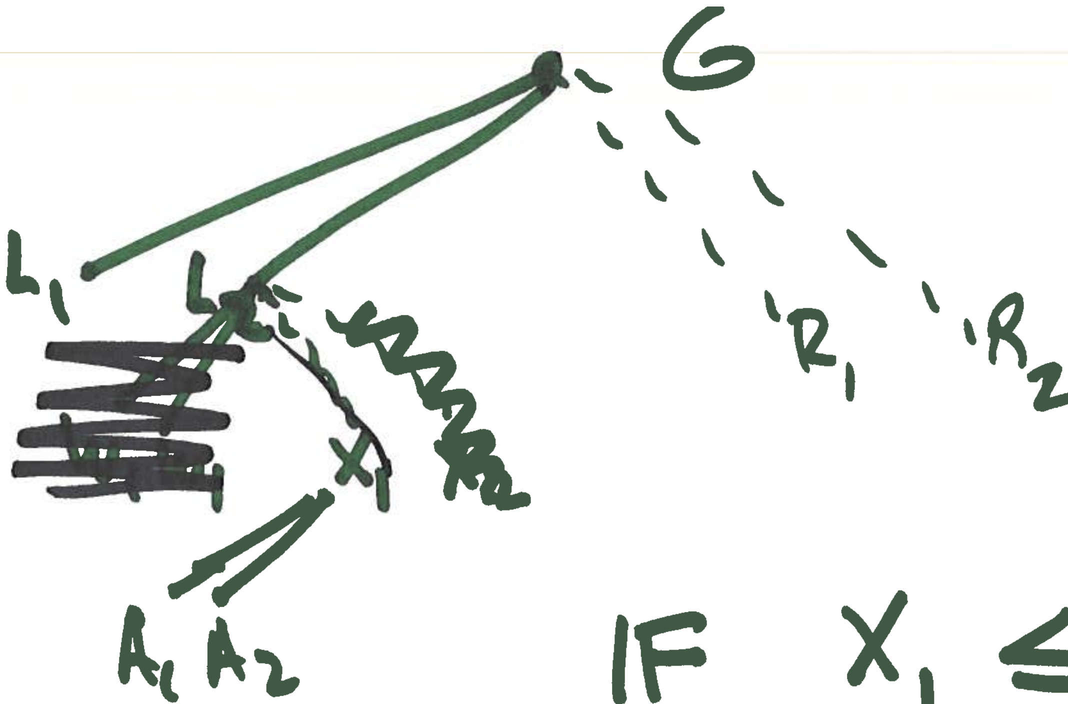


C.F. PART II



IF $G^{LR} \leq G$

THEN - IN G,
WE CAN REPLACE G^L
WITH SET OF
LEFT OPTIONS
OF G^{LR}



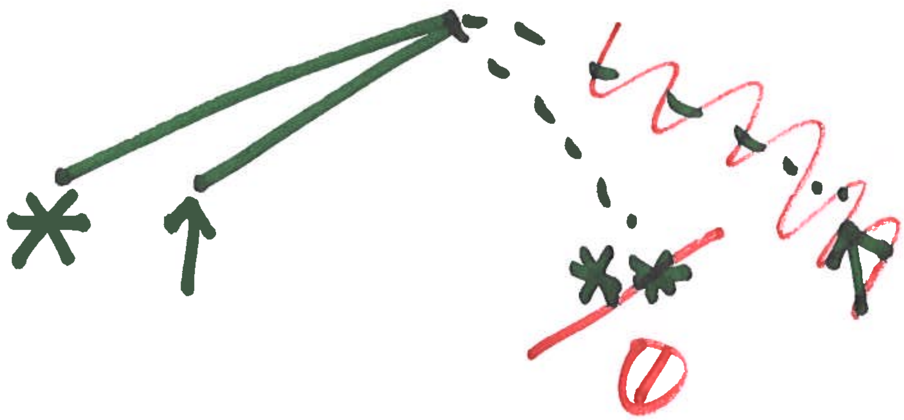
IF $X_1 \le G$

THEN

$$G = \{L_1, L_2 \mid R_1, R_2\}$$

$$= \{L_1, A_1, A_2 \mid R_1, R_2\}$$

SIMPLIFY $\uparrow^{*}$



PRUNE ? $\emptyset < \uparrow$

MORE PRUNING?

IS  $\leq \uparrow^{*}$?

C.F. P. II

$$G = \{ *, \cancel{\uparrow} \mid 0 \}$$

$$= \{ *, 0 \mid 0 \}$$

IF $R_1 \le R_2$

THEN $G = \{L_1, L_2 \ldots \mid R_1, \cancel{R_2} \ldots\}$

SIMPLIFY

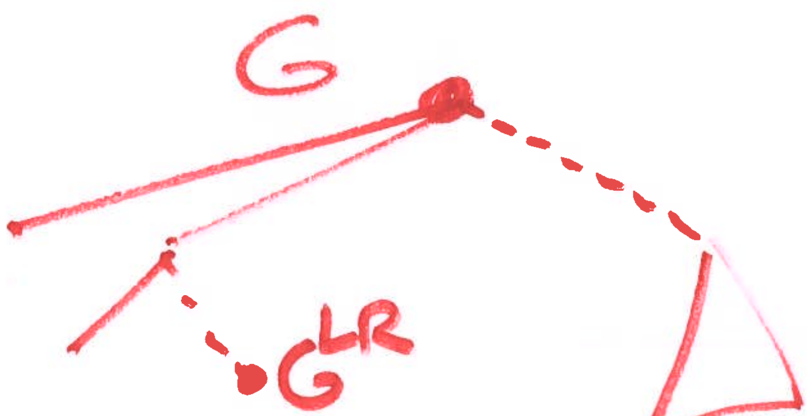
ODD $\rightarrow$ $\frac{m}{2^k} = \left\{ \frac{m-1}{2^k} \,\middle|\, \frac{m+1}{2^k} \right\}$

$\{\cancel{1}, 2 \mid 3\} = \frac{5}{2}$ $\quad \left\{ \frac{2}{1} \,\middle|\, \frac{3}{1} \right\}$

$\{\cancel{*}, 1* \mid -5\}$ $\quad \left\{ \frac{4}{2} \,\middle|\, \frac{6}{2} \right\}$

$\{\cancel{*}, \cancel{\uparrow}, 1 \mid -1\}$

$\{0, * \mid \cancel{0}, \Downarrow\}$



SIMPLIFY

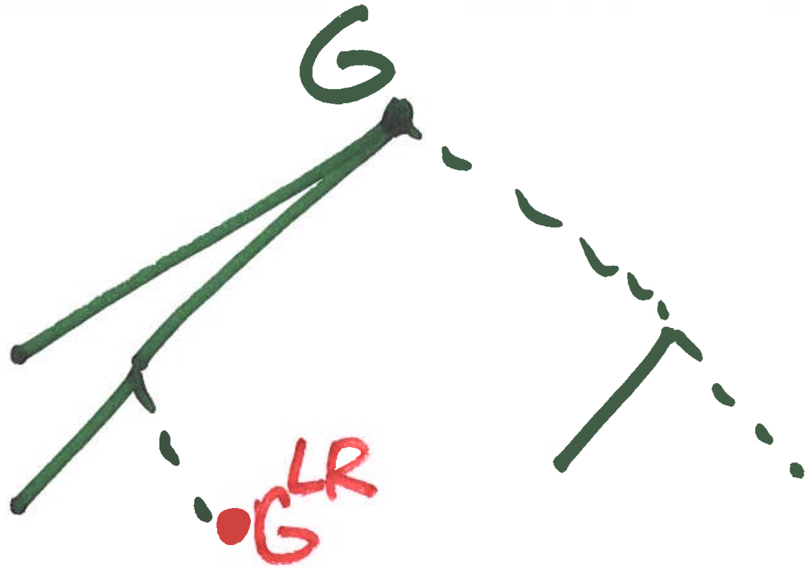
$\{* \mid \uparrow, *\}$ $\qquad 0$

P.II

$\{0, \cancel{*} \mid *\}$ $\qquad \uparrow$

$\{\uparrow \mid \downarrow\}$

$\{\Uparrow, \uparrow* \mid \Uparrow, \uparrow\}$



IF $G^{LR} \le G$

So $G = \{0, * \mid *\}$

$= \{0, \mid *\}$

$= \uparrow$ ☺