first name **last name** **id#**

3 hr **closed book, no devices** **5 pages, 8 marks/page** **2019 CGT exam version** A

1. For a 2-player zero-sum game such as Go and a strategy S for player A, let $f(S)$ be the minimum score of S, against all possible opponent strategies. We call S a minimax strategy if it **(circle one)**

   a) maximizes $f(S_x)$, over all possible strategies S_x for A.

   b) minimizes $f(S_x)$, over all possible strategies S_x for A.

2. In Go, the positional superko rule is that **(circle one)**

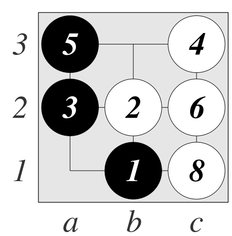
   a) a move that creates the previous position is illegal

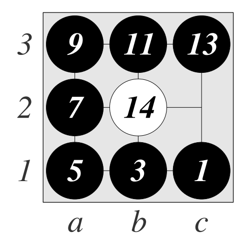
   b) a move that creates any earlier position is illegal

   c) a move that creates the previous state (position and player-to-move) is illegal

   d) a move that creates any earlier state (position and player-to-move) is illegal

3. Each diagram shows a Go state. At left, the move history so far is 1.B[b1] 2.W[b2] ...7.B[pass] 8.W[c1]. At right, each move 2,4,6,..., 12 was a White pass.





Assume positional superko and Tromp-Taylor rules and scoring. For each Go state with Black to move, give the current score, the final minimax score, and a brief justification of that final score.

current: Black score − White score is ________ current: Black score − White score is ________

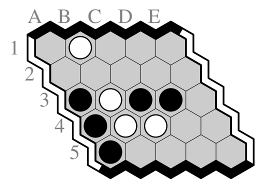
final: Black score − White score is ________ final: Black score − White score is ________

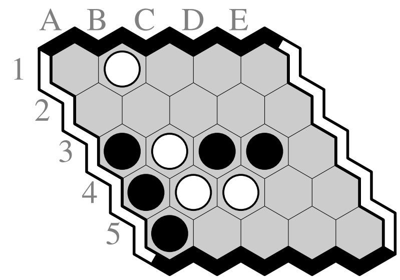
final score justification final score justification

Recall: for a game and a player, a move is *winning* if it is the first move in a winning strategy for that player. A *pairing strategy* is a 2nd-player strategy where empty cells are labelled in pairs: whenever the opponent plays at a cell, the player replies at the other cell with that label.

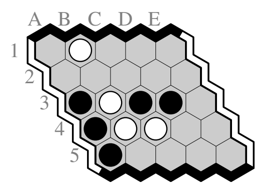
Put answers in the big diagrams. Use small diagrams for rough work.

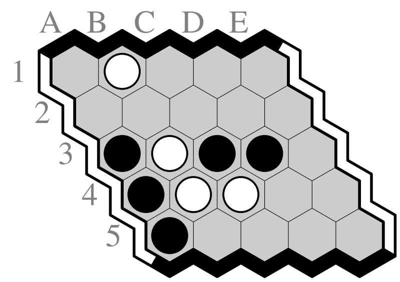
1. Put an X in each dead cell, put a B in each Black-captured cell, and put a W in each White-captured cell.



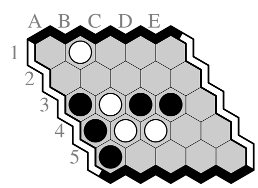


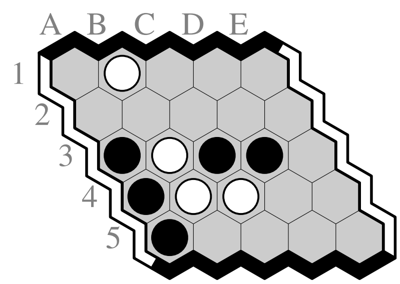
2. Put letters X X Y Y Z Z in six cells to show a pairing strategy that joins the Black stones at C3,D3 to the top.





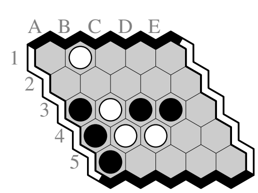
3. Put a dot in each cell of a 9-cell Black-plays-next-and-wins strategy. (Hint: this is also called a White must-play region. Use your answer from the previous question.)

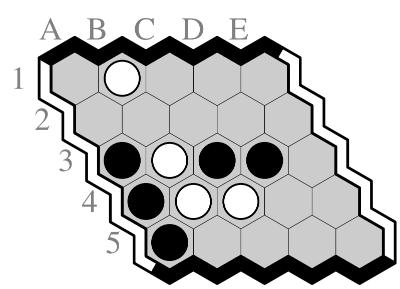




4. If White plays next, who wins?
   (circle one) White Black

   If your answer is White, put a dot at the winning move, and then show a winning pairing strategy after that move.
   If your answer is Black, show a winning pairing strategy that holds before White's move.





Recall: a P-position is a combinatorial game where the first player loses. For games G and H, we say that $G = H$ if $G + -H$ is a P-position. The canonical form of G is the game that equals G and has fewest nodes in its tree. Here are some game definitions: $0 = \{ \mid \}$, $* = \{0 \mid 0\}$, $\uparrow = \{0 \mid *\}$, $\downarrow = -\uparrow$.

1. For a combinatorial game G, we write $G = \{G^L \mid G^R\}$, were G^R is **(circle ALL that apply)**

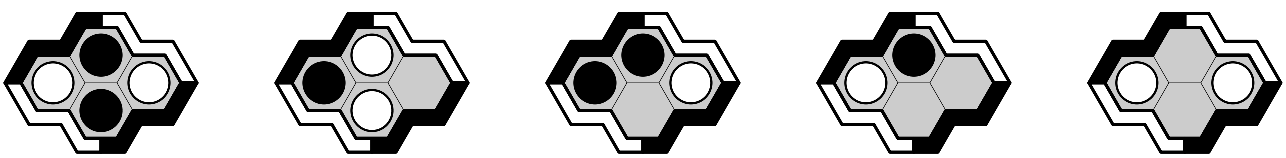
   a) the set of right options of G b) G added to itself R times

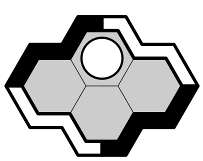
   c) G raised to the power R d) the canonical form of G's right options

2. For $G = \{G^L \mid G^R\}$ and the game $-1 = \{ \mid 0\}$, $-G$ satisfies these properties: **(circle ALL that apply)**

   a) $-G + G = 0$ b) $-G * -1 = G$ c) $-(-G) = G$ d) $-G = \{-G^L \mid -G^R\}$ e) $-G = \{-G^R \mid -G^L\}$

Each Hex position is a combinatorial game. Black is Left, White is Right. Below from the left, the Hex positions have canonical form 0, 0, *, ↑, ↓ respectively.



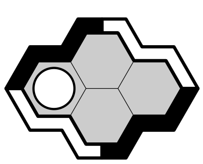
3. 

Here, give **all** Left options **(circle ALL that apply)** 0 1 -1 $*$ $\uparrow$ $\downarrow$

and **all** Right options **(circle ALL that apply)** 0 1 -1 $*$ $\uparrow$ $\downarrow$

and the canonical form **(fill in options)** $\{ \qquad \mid \qquad \}$

Justify briefly.

4. 

Here, give **all** Left options **(circle ALL that apply)** 0 1 -1 $*$ $\uparrow$ $\downarrow$

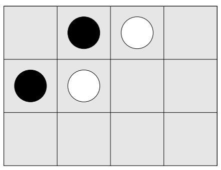
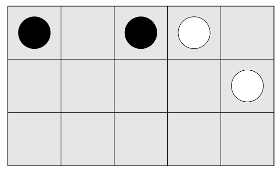
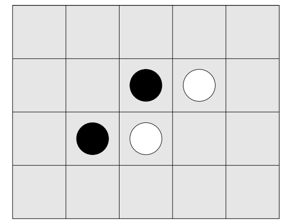
and **all** Right options **(circle ALL that apply)** 0 1 -1 $*$ $\uparrow$ $\downarrow$

and the canonical form **(fill in options)** $\{ \qquad \mid \qquad \}$

Justify briefly.

1. Recall konane: a move consists of jumping (horizontally or vertically) one or more opponent stones, and then removing those opponent stones that were jumped. Your stone must land on an empty cell after each jump. Multiple jumps on the same move must continue in the same direction as the first jump.

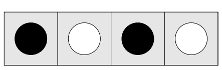
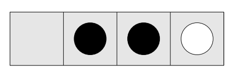
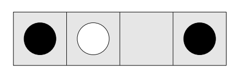
   For each Konane game give the notation (left options and right options), where each option is either nothing, 0, 1, -1 or $*$. Also, give the name of each game from this list: 0, 1, $*$, 2, $1*$, $1/2$, $\{1|*\}$, $\{1|0\}$, $\uparrow$, $\uparrow *$, $\{1|0, *\}$.

notation: { | } { | } { | }

name:

2. Recall clobber: on a move, a player takes one of her stones that touches an opponent stone, removes the opponent stone and move her stone to that place. Here is a clobber game and all games that result from a black move. Below each game give the requested information.

canonical form: name: name:

3. Give a clobber position with canonical form $G = 1 = \{0 \,|\, \}$ or explain why there is no such position.

1. For games G and H, $G \geq H$ if and only if **(circle ALL that apply)**

   a) the outcome class of $G - H$ is Left or P (Previous)

   b) the outcome class of $G - H$ is Left or N (Next)

   c) Left wins $G - H$ when playing first

   d) Left wins $G - H$ when playing second

   For example, recall $\Uparrow = \uparrow + \uparrow$. We have $\Uparrow \geq \uparrow$ because

   ____

2. For a game G with two left options A and A', we can prune A' if **(circle ALL that apply)**

   a) $A = A'$ b) A and A' are isomorphic c) $A < A'$ d) $A > A'$ e) $A \,||\, A'$

   For example, if $G = \{0, *, \uparrow \mid G^R\}$ then we can prune ____________ from the set of left options

   because ______________________________ .

3. If game G has a left option A, and A has a right option $B = \{B^L \mid B^R\}$ such that $G \geq B$, then $G = G'$ where G' is the game obtained from G by replacing ________ with ________ .

   For example, if $G = \{*, \uparrow \mid 0\}$ then we can replace $\uparrow$ with ________

   because ____

4. Define $M_0 = *$. For all positive integers t, define $M_t = \{0 \mid M_{t-1}\}$.

   So $M_1 = \{0 \mid 0\} = *$ and $M_2 = \{0 \mid *\} = \uparrow$.

   Claim: $M_3 = \Uparrow *$.

   Proof: Let $G = \Uparrow *$. If Left plays on G, Left can play on $\uparrow$ or $*$, leaving $0 + \uparrow *$ or $\Uparrow + 0$, so $G^L = \{\uparrow *, \Uparrow\}$.

   In G^L we can replace $L_1 = \uparrow *$ with the empty set since Right can move from L_1 to 0 and $G > 0$ and 0 has no left options. Also, we can replace $L_2 = \Uparrow$ with $\{\uparrow, *\}$ since Right can move from L_2 to to $L' = \uparrow *$ and ____________ and L' has those two left options. So we have replaced G^L with $\{\uparrow, *\}$. In similar fashion, we can replace $*$ with the empty set and we can replace $\uparrow$ with 0, so now $G^L = \{0\}$.

   If Right plays on G, Right moves to either ________ or ________. Right can prune one of these options since ________ $>$ ________. So $G = \{0 \mid \uparrow\} = \{0 \mid M_2\} = M_3$, and we have proved the claim.