bipartite preference and stable matchings

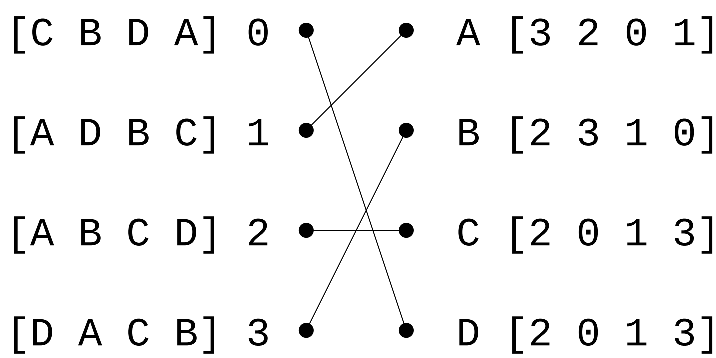
- see also `https://en.wikipedia.org/wiki/Stable_marriage_problem`

```
[C B D A] 0 •    • A [3 2 0 1]

[A D B C] 1 •    • B [2 3 1 0]

[A B C D] 2 •    • C [2 0 1 3]

[D A C B] 3 •    • D [2 0 1 3]
```

- ***bipartite preference system***
  - **hospitals** $H = \{0, 1, 2, 3\}$ **residents** $R = \{A, B, C, D\}$
  - **for each hospital, a *total preference order* of residents**

    e.g. hospital 0 **prefers** $C > B > D > A$
  - **for each resident, a TPO of doctors**

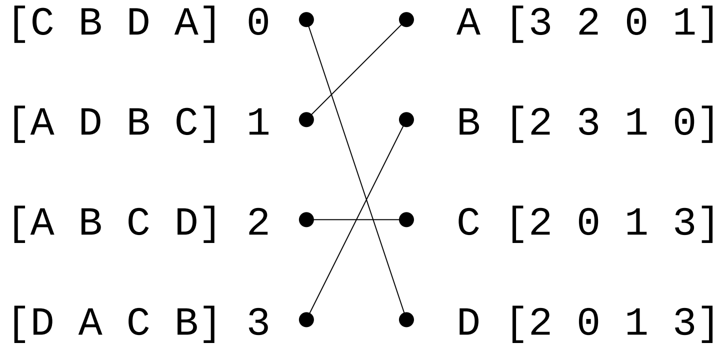
    e.g. resident A **prefers** $3 > 2 > 0 > 1$
- **a matching** $m : H \leftrightarrow R$



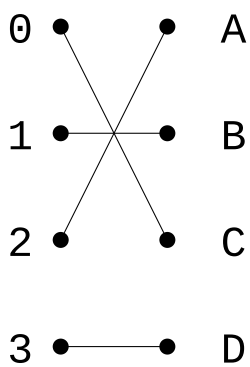
- **in** m**, couple** $\{2, A\}$ ***unhappy*****: prefer each other to their matches**

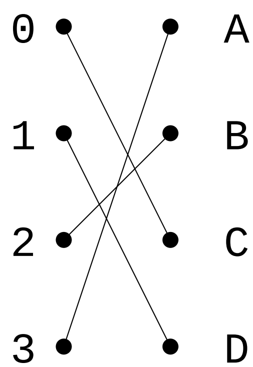
  2 **prefers** A **to** $m(2) = C$ A **prefers** 2 **to** $m^{-1}(A) = 1$

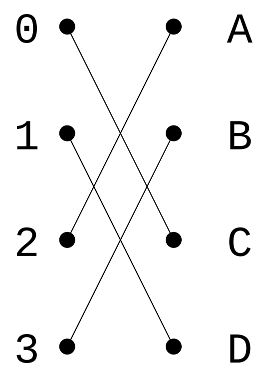
- **this m *unstable*: it has an unhappy couple**



- **which matchings below are stable? (use above prefs)**







- **problem: find stable matching, if one exists**

- **brute force?**

- **for each possible matching (there are n!):**

  – **check whether matching stable in $O(n^2 \lg n)$ time (how?)**

- **worstcase runtime $O(n!\ n^2 \lg n)$**

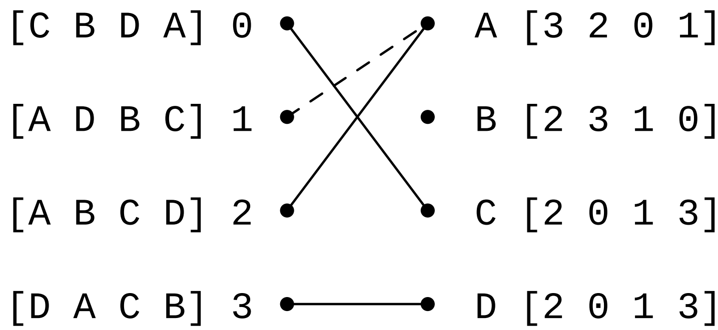
- **can we do better?**

Gayle-Shapley propose-maybe-reject alg'm

```
[C B D A] 0 •       • A [3 2 0 1]
[A D B C] 1 •       • B [2 3 1 0]
[A B C D] 2 •       • C [2 0 1 3]
[D A C B] 3 •       • D [2 0 1 3]
```

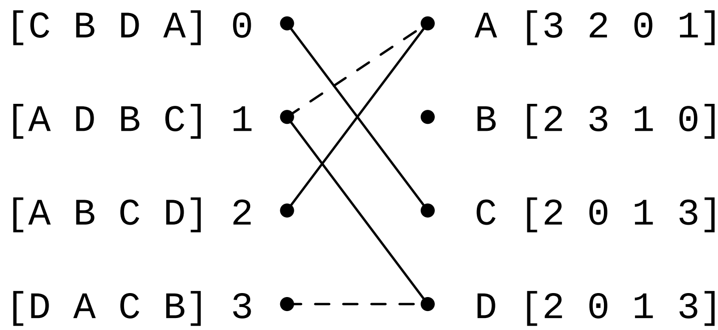
	A	**B**	**C**	**D**
0				
1				
2				
3				

Gayle-Shapley: after round 1



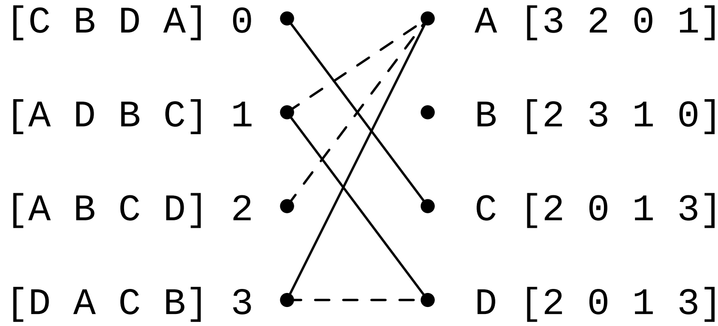
	A	**B**	**C**	**D**
0			?	
1	x			
2	?			
3				?

Gayle-Shapley: after round 2



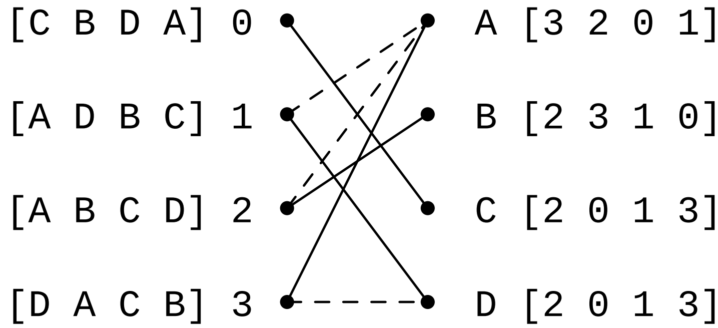
	A	**B**	**C**	**D**
0			?	
1	x			?
2	?			
3				x

Gayle-Shapley: after round 3



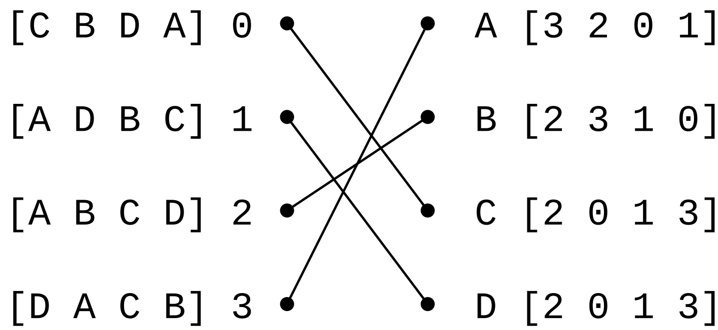
	A	**B**	**C**	**D**
0			?	
1	x			?
2	x			
3	?			x

Gayle-Shapley: after round 4



	A	B	C	D
0			?	
1	x			?
2	x	?		
3	?			x

Gayle-Shapley: matching



	A	**B**	**C**	**D**
0			?	
1				?
2		?		
3	?			