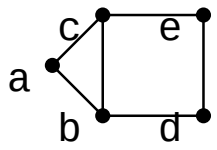
1. [3+3+3+3+1.5+1.5 marks] Below is the integer program (IP) (maximal clique version) for finding a maximum indendent set in this graph. Also below is the dual IP.




```
primal                                  dual
max x1 + x2 + x3 + x4 + x5 s.t.         min y1 + y2 + y3 + y4  s.t.
    x1 + x2 + x3           <= 1             y1                 >= 1
         x2      + x4      <= 1             y1 + y2            >= 1
              x3      + x5 <= 1             y1      + y3       >= 1
                   x4 + x5 <= 1                  y2      + y4  >= 1
                                                      y3 + y4  >= 1
x1, x2, x3, x4  in  {0, 1}              y1, y2, y3, y4  in  {0, 1}
```

What does primal variable x_3 represent?

What does dual variable y_2 represent?

Justify/explain this primal constraint: $x_1 + x_2 + x_3 \leq 1$.

Justify/explain this dual constraint: $y_1 + y_2 \geq 1$.

Is $x = (1, 0, 0, 0, 0)$ primal optimal? If yes, write it below: if no, find a primal optimal solution.

Is $y = (1, 1, 1, 1)$ dual optimal? If yes, write it below: if no, find a dual optimal solution.

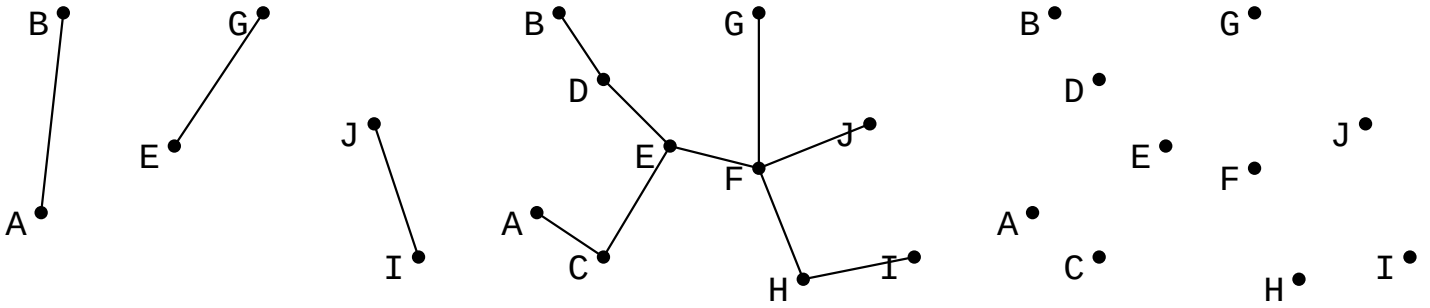
Your primal optimal solution:

Your dual optimal solution:

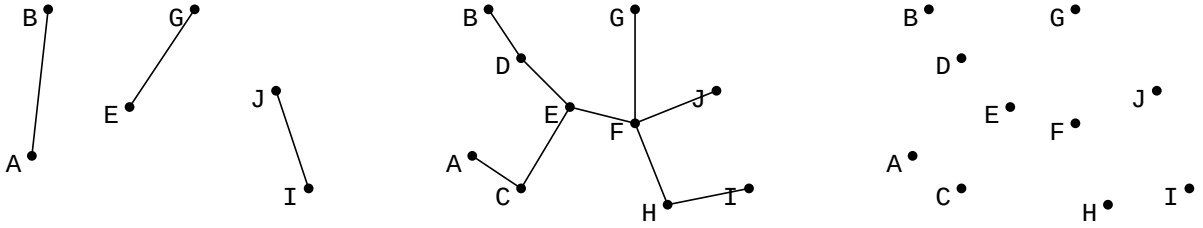
2. [2+3+5 marks] Give the weight of the matching M below left: _____

   For the graph with these edge weights, M below left and MST below middle, give the Hamiltonian cycle found by the `MST+M` method. Below middle: show MST+M. Below right: show your cycle.

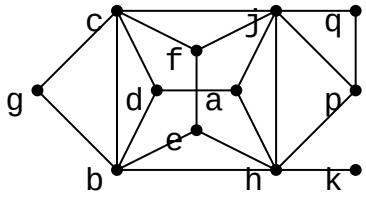
	B	C	D	E	F	G	H	I	J
A	5	2	4	4	6	7	7	9	9
B		6	2	4	6	5	9	11	8
C			4	3	4	7	5	8	7
D				2	4	4	7	9	7
E					2	4	5	7	5
F						4	3	4	3
G							7	7	4
H								3	4
I									4



---------- rough work below here ----------



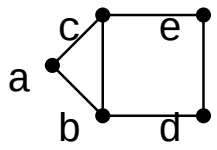
3. [5 marks] On this graph, show independent set IS after each of the first five iterations of this code. Always pick the first simplicial node in alphabetic order.




```
IS <- { }
while G has simplicial node v:
    add v to IS
    remove v and v's neighbors from G

after iteration 1: IS { __ }
after iteration 2: IS { __ __ }
after iteration 3: IS { __ __ __ }
after iteration 4: IS { __ __ __ __ }
after iteration 5: IS { __ __ __ __ __ }
```

1. [3+3+3+3+1.5+1.5 marks] Below is the integer program (IP) (maximal clique version) for finding a maximum indendent set in this graph. Also below is the dual IP.




```
primal                                  dual
max x1 + x2 + x3 + x4 + x5 s.t.         min y1 + y2 + y3 + y4  s.t.
    x1 + x2 + x3           <= 1             y1                 >= 1
         x2      + x4      <= 1             y1 + y2            >= 1
              x3      + x5 <= 1             y1      + y3       >= 1
                   x4 + x5 <= 1                  y2      + y4  >= 1
                                                      y3 + y4  >= 1
x1, x2, x3, x4  in  {0, 1}              y1, y2, y3, y4  in  {0, 1}
```

What does primal variable x_3 represent?

What does dual variable y_3 represent?

Justify/explain this primal constraint: $x_1 + x_2 + x_3 \leq 1$.

Justify/explain this dual constraint: $y_1 + y_3 \geq 1$.

Is $x = (1, 0, 0, 0, 0)$ primal optimal? If yes, write it below: if no, find a primal optimal solution.

Is $y = (1, 1, 1, 1)$ dual optimal? If yes, write it below: if no, find a dual optimal solution.

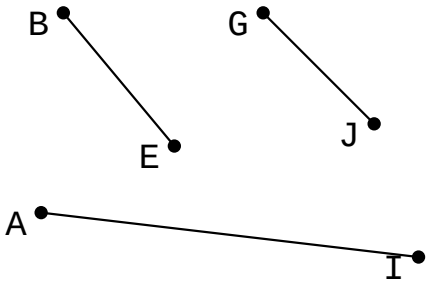
Your primal optimal solution:

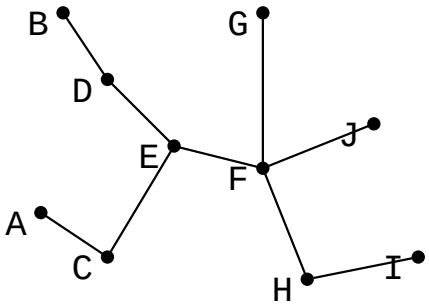
Your dual optimal solution:

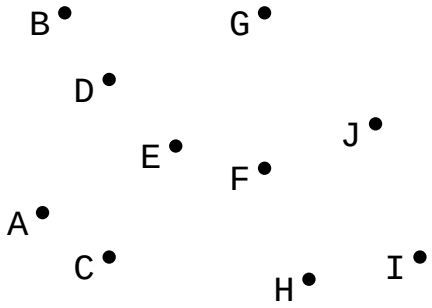
2. [2+3+5 marks] Give the weight of the matching M below left: _____

   For the graph with these edge weights, M below left and MST below middle, give the Hamiltonian cycle found by the `MST+M` method. Below middle: show MST+M. Below right: show your cycle.

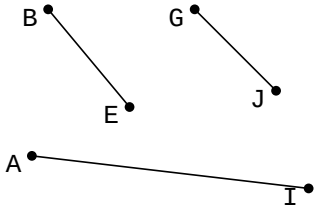
	B	C	D	E	F	G	H	I	J
A	5	2	4	4	6	7	7	9	9
B		6	2	4	6	5	9	11	8
C			4	3	4	7	5	8	7
D				2	4	4	7	9	7
E					2	4	5	7	5
F						4	3	4	3
G							7	7	4
H								3	4
I									4

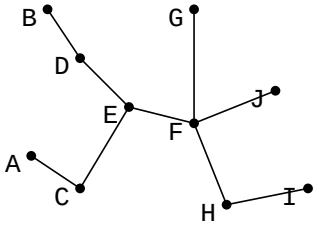


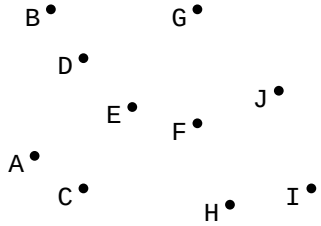




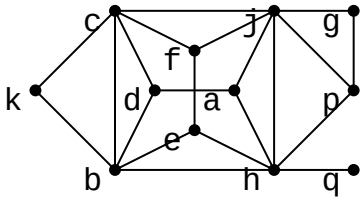
---------- rough work below here ----------







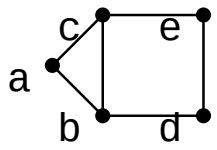
3. [5 marks] On this graph, show independent set IS after each of the first five iterations of this code. Always pick the first simplicial node in alphabetic order.




```
IS <- { }
while G has simplicial node v:
    add v to IS
    remove v and v's neighbors from G

after iteration 1: IS { __ }
after iteration 2: IS { __ __ }
after iteration 3: IS { __ __ __ }
after iteration 4: IS { __ __ __ __ }
after iteration 5: IS { __ __ __ __ __ }
```

1. [3+3+3+3+1.5+1.5 marks] Below is the integer program (IP) (maximal clique version) for finding a maximum indendent set in this graph. Also below is the dual IP.




```
primal                                  dual
max x1 + x2 + x3 + x4 + x5 s.t.         min y1 + y2 + y3 + y4  s.t.
    x1 + x2 + x3           <= 1             y1                 >= 1
         x2      + x4      <= 1             y1 + y2            >= 1
              x3      + x5 <= 1             y1      + y3       >= 1
                   x4 + x5 <= 1                  y2      + y4  >= 1
                                                      y3 + y4  >= 1
x1, x2, x3, x4  in  {0, 1}              y1, y2, y3, y4  in  {0, 1}
```

What does primal variable x_3 represent?

What does dual variable y_4 represent?

Justify/explain this primal constraint: $x_1 + x_2 + x_3 \leq 1$.

Justify/explain this dual constraint: $y_3 + y_4 \geq 1$.

Is $x = (1, 0, 0, 0, 0)$ primal optimal? If yes, write it below: if no, find a primal optimal solution.

Is $y = (1, 1, 1, 1)$ dual optimal? If yes, write it below: if no, find a dual optimal solution.

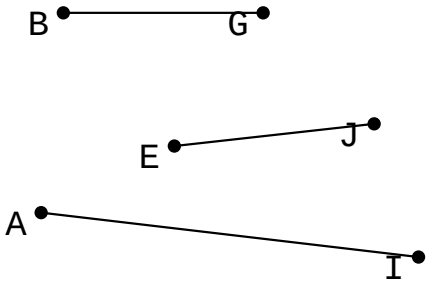
Your primal optimal solution:

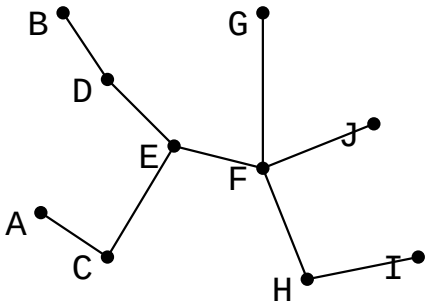
Your dual optimal solution:

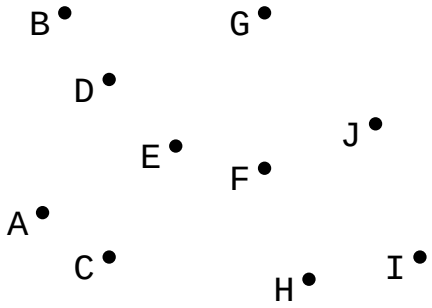
2. [2+3+5 marks] Give the weight of the matching M below left: _____

   For the graph with these edge weights, M below left and MST below middle, give the Hamiltonian cycle found by the `MST+M` method. Below middle: show MST+M. Below right: show your cycle.

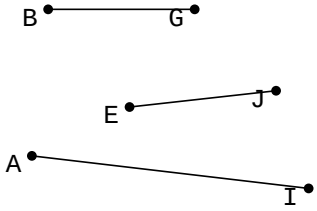
	B	C	D	E	F	G	H	I	J
A	5	2	4	4	6	7	7	9	9
B		6	2	4	6	5	9	11	8
C			4	3	4	7	5	8	7
D				2	4	4	7	9	7
E					2	4	5	7	5
F						4	3	4	3
G							7	7	4
H								3	4
I									4

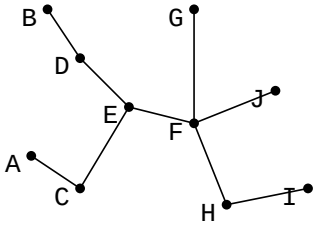


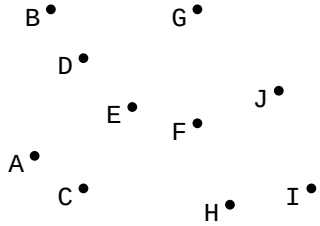




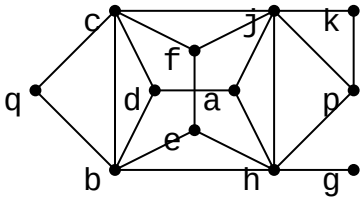
---------- rough work below here ----------







3. [5 marks] On this graph, show independent set IS after each of the first five iterations of this code. Always pick the first simplicial node in alphabetic order.




```
IS <- { }
while G has simplicial node v:
    add v to IS
    remove v and v's neighbors from G

after iteration 1: IS { __ }
after iteration 2: IS { __ __ }
after iteration 3: IS { __ __ __ }
after iteration 4: IS { __ __ __ __ }
after iteration 5: IS { __ __ __ __ __ }
```