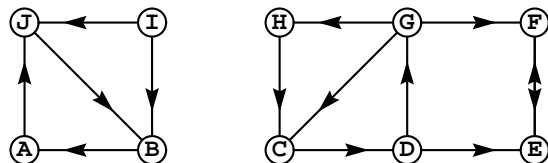1. [8 marks] For a dfs of the transpose T of this digraph D, in the order created (from left to right), draw the dfs trees and give the postorder sequence. (Assume that T's node and neighbor lists are in alphabetic order.)

Draw the dfs trees below:

Give the postorder sequence below:

Rough work here and below (will not be marked)

2. [4 marks] For a dfs of D that considers nodes in order EFCHGDABJI, draw the dfs trees. (Assume that D's neighbor lists are in alphabetic order.) **Draw the dfs trees below:**

Rough work here and below (will not be marked)

3. [4 marks] What property does the node order EFCHGDABJI have that guarantees that the dfs trees in the previous question are the sccs of the original digraph D?

**Answer.** This order has the property that _____

4. [4 marks] **Claim.** Let $T_1, T_2, \ldots, T_k$ be the trees, in the order they are created, from a dfs traversal of a digraph. Prove that there is no arc into T_k from any previous tree.

Proof. Argue by contradiction: for some $j < k$, assume that there is some arc from x in T_j to y in T_k. Before the dfs from x finishes, the dfs from any node reachable from x must finish (or have previously finished).
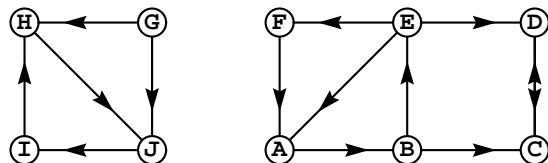
Now you finish the proof: _____

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5. [4+6 marks] (a) On the nodes below, draw the implication digraph for this 2-sat formula:

f = [1 -2] [1 3] [-2 -3] [2 4] [-3 -4] [3 -5] [3 5] .
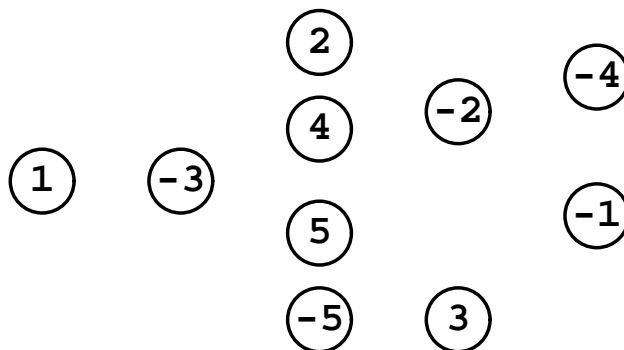
(b) Is the formula satisfiable? If yes, give a satisfying assignment. If no, find a variable x_j and give a sequence of implications that shows that x_j cannot be true, and another sequence of implications that shows that x_j cannot be false.

1. [8 marks] For a dfs of the transpose T of this digraph D, in the order created (from left to right), draw the dfs trees and give the postorder sequence. (Assume that T's node and neighbor lists are in alphabetic order.)

   Draw the dfs trees below:

   Give the postorder sequence below:

   Rough work here and below (will not be marked)

2. [4 marks] For a dfs of D that considers nodes in order HIJGCDAFEB, draw the dfs trees. (Assume that D's neighbor lists are in alphabetic order.) **Draw the dfs trees below:**

   Rough work here and below (will not be marked)

3. [4 marks] What property does the node order HIJGCDAFEB have that guarantees that the dfs trees in the previous question are the sccs of the original digraph D?

   **Answer.** This order has the property that _____

4. [4 marks] **Claim.** Let $T_1, T_2, \ldots, T_k$ be the trees, in the order they are created, from a dfs traversal of a digraph. Prove that there is no arc into T_k from any previous tree.

Proof. Argue by contradiction: for some $j < k$, assume that there is some arc from x in T_j to y in T_k. Before the dfs from x finishes, the dfs from any node reachable from x must finish (or have previously finished).
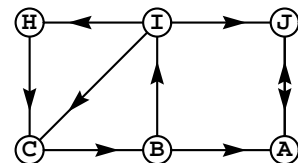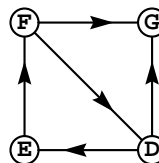
Now you finish the proof: ─────────────────────────────────────

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5. [4+6 marks] (a) On the nodes below, draw the implication digraph for this 2-sat formula:

f = [1 -2] [1 3] [-2 -3] [2 5] [-3 -5] [3 -4] [3 4] .
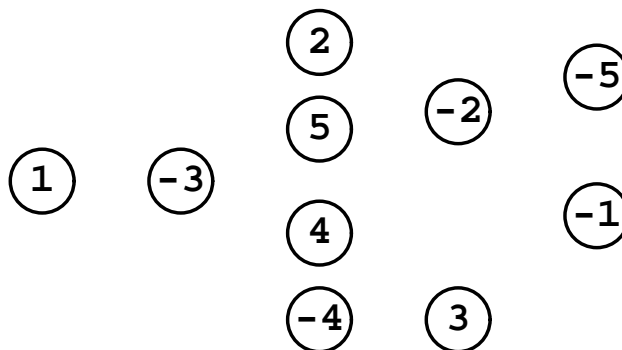
(b) Is the formula satisfiable? If yes, give a satisfying assignment. If no, find a variable x_j and give a sequence of implications that shows that x_j cannot be true, and another sequence of implications that shows that x_j cannot be false.

1. [8 marks] For a dfs of the transpose T of this digraph D, in the order created (from left to right), draw the dfs trees and give the postorder sequence. (Assume that T's node and neighbor lists are in alphabetic order.)

   Draw the dfs trees below:

   Give the postorder sequence below:

   Rough work here and below (will not be marked)

2. [4 marks] For a dfs of D that considers nodes in order GDFEAJBCHI, draw the dfs trees. (Assume that D's neighbor lists are in alphabetic order.) **Draw the dfs trees below:**

   Rough work here and below (will not be marked)

3. [4 marks] What property does the node order GDFEAJBCHI have that guarantees that the dfs trees in the previous question are the sccs of the original digraph D?

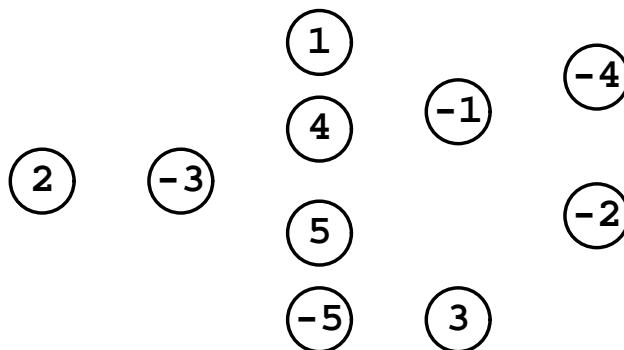   **Answer.** This order has the property that _____

4. [4 marks] **Claim.** Let $T_1, T_2, \ldots, T_k$ be the trees, in the order they are created, from a dfs traversal of a digraph. Prove that there is no arc into T_k from any previous tree.

   Proof. Argue by contradiction: for some $j < k$, assume that there is some arc from x in T_j to y in T_k. Before the dfs from x finishes, the dfs from any node reachable from x must finish (or have previously finished).

   Now you finish the proof: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

   ⎯⎯

   ⎯⎯

5. [4+6 marks] (a) On the nodes below, draw the implication digraph for this 2-sat formula:

   f = [-1 2] [2 3] [-1 -3] [1 4] [-3 -4] [3 -5] [3 5] .

   (1)   (−4)

   (4)   (−1)

   (2)   (−3)   (−2)

   (5)

   (−5)   (3)

   (b) Is the formula satisfiable? If yes, give a satisfying assignment. If no, find a variable x_j and give a sequence of implications that shows that x_j cannot be true, and another sequence of implications that shows that x_j cannot be false.