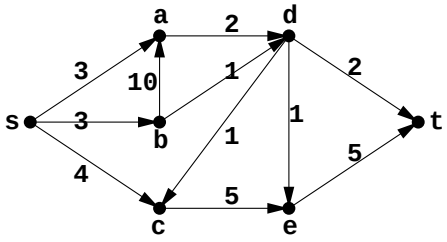
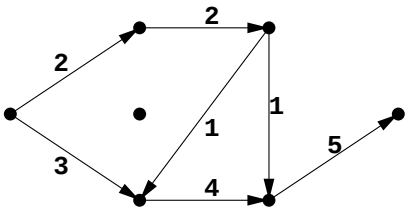
first name **last name** **id#**

30 min **30 marks** **closed book** **no devices** **2 pages**

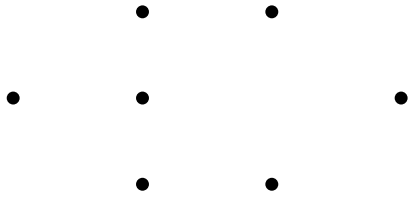
1. [12 marks] For this network, starting with flow 1, use the residual flow method to find a max flow: show residual networks 1,2,3 and flows 2 and 3.



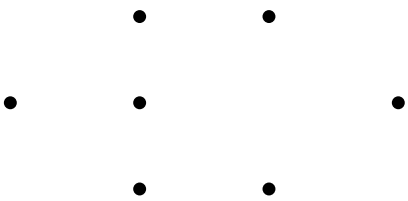
flow 1



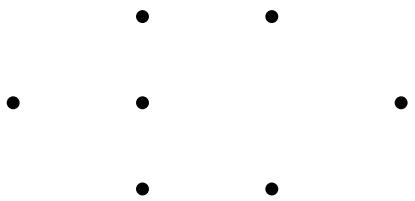
residual network 1



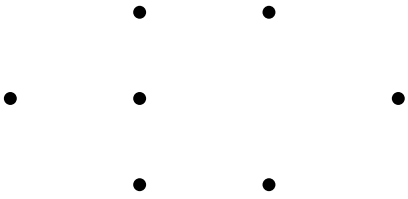
flow 2



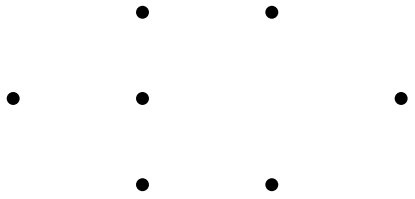
residual network 2



flow 3



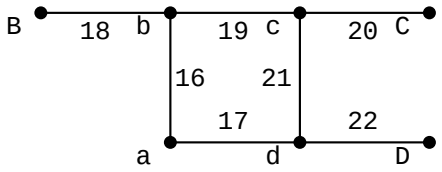
residual network 3



2. [3 marks] a) For the above network, give a cut whose capacity is equal to that of the flow that you found in the previous question. Answer like this: `8 { s, a }  {b, c, d, e, t}`

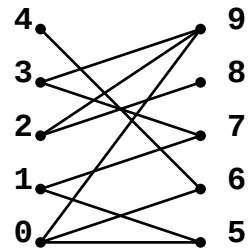
3. [2+5 marks] You manage a communications network with users B,C,D and bandwidths as shown in the diagram. You need to establish connections between B-C, B-D, and C-D: these pay you \$3, \$5, \$4 respectively per unit bandwidth. Between each pair of users at least 6 units must be routed. Each connection has two possible routes. For B-C: xBC is traffic volume along B-b-c-C, yBC is volume along B-b-a-d-c-C. For C-D: define xCD, yCD, similarly. For B-D: xBD, yBD is traffic along B-b-c-d-D, B-b-a-d-D respectively. You want to maximize the amount you are paid. Using these variables, formulate this problem as an LP.

   a) Give the objective function.

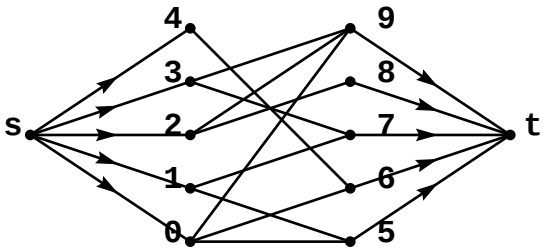


   b) Give the system of (in)equalities.

4. [3+5 marks] a) For this bipartite graph G, `{ (0,5) }` is a matching with size 1. Give a maximum matching for G.



   b) For this s-t flow network H, each arc has capacity 1 (arrows on middle arcs have been omitted, they are all from left to right). Notice that the cut C = `{s, 1}` `{t, 0, 2, 3, 4, 5, 6, 7, 8, 9}` has capacity 6.
   Prove/disprove: C is a min cut of H.
   (circle one) Proof Disproof Give your argument below.



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30 min **30 marks** **closed book** **no devices** **2 pages**

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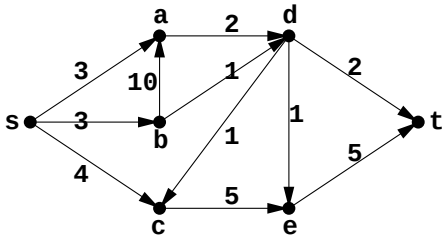
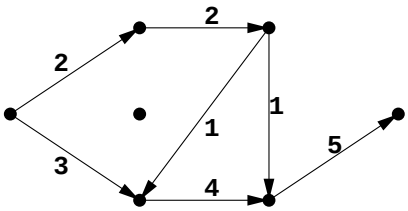
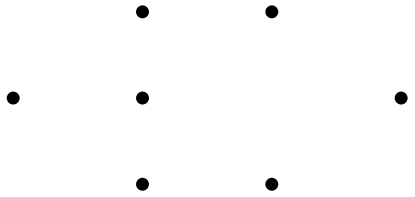


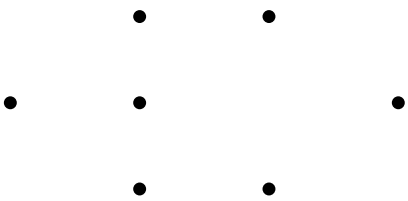
flow 1



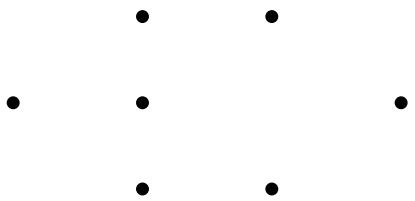
residual network 1



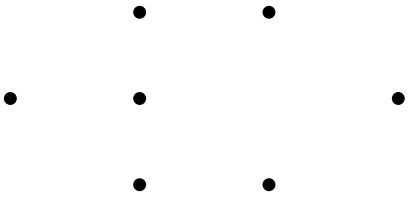
flow 2



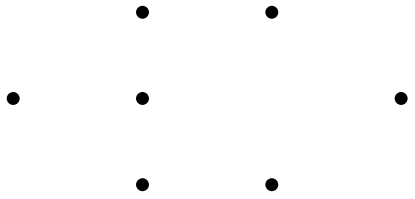
residual network 2



flow 3



residual network 3



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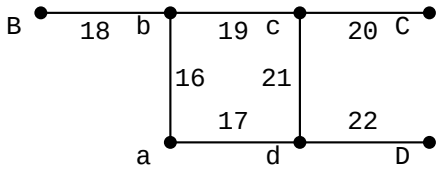

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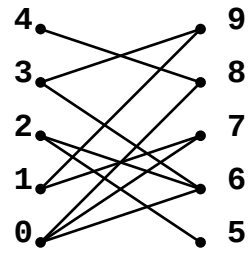

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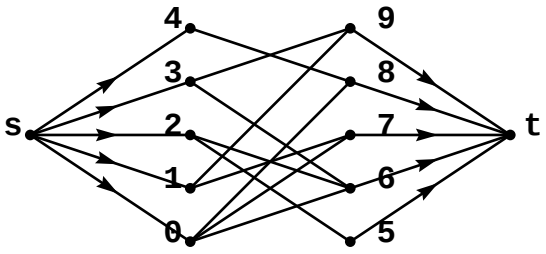

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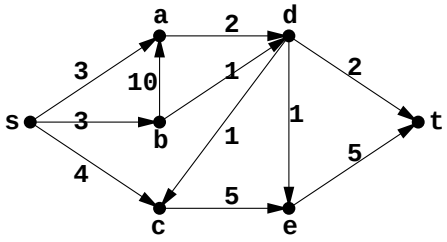

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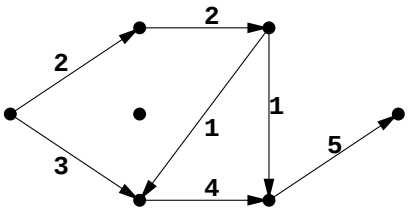
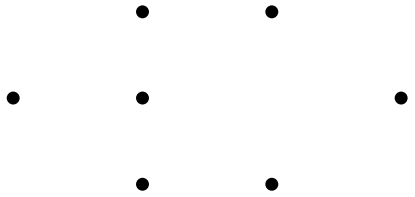
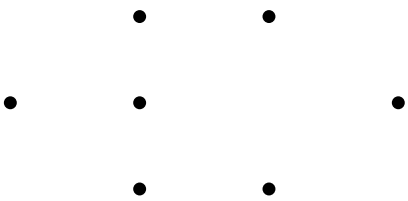


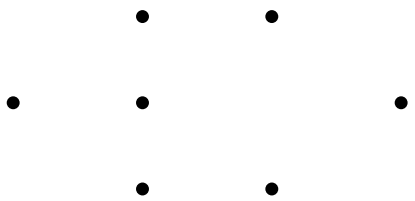
residual network 1



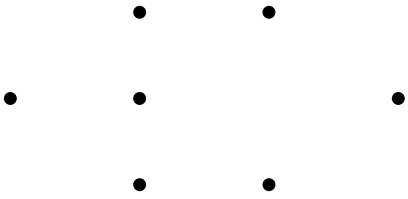
flow 2



residual network 2



flow 3



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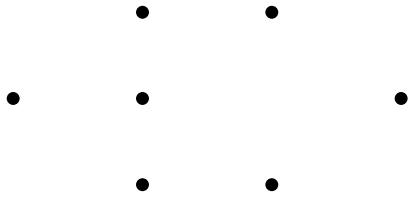

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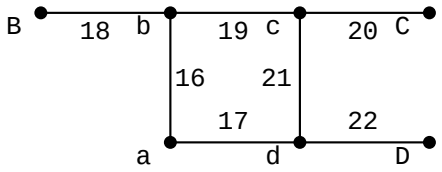

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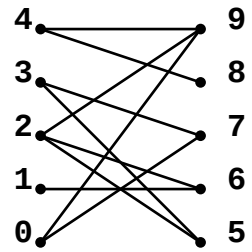

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