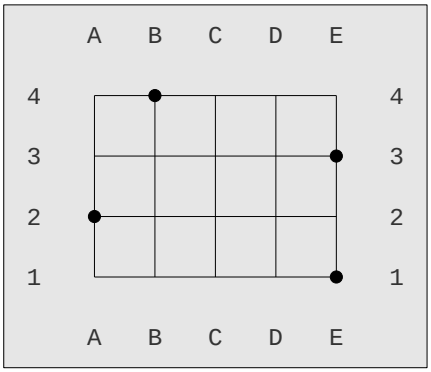
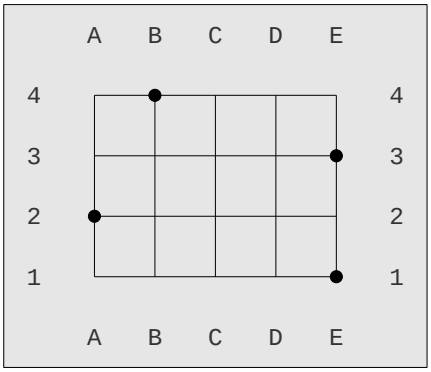
first name **last name** **id#**

each question 8 marks **30 min** **closed book** **no devices** **3 pages**

1. (a) Explain why any solution to this Steiner tree puzzle has cost at least 7.



(b) On the diagram, draw a min cost solution and write its cost.



2. a) In the box, show the output from this python program.

```
def collatz(n):
  print(n, end=': ')
  while n > 1:
    print(n, end=' ')
    if n % 2 == 0: n = n // 2
    else: n = n*3 + 1
  print(n)

for j in range(2,7): #from 2 to 6
  collatz(j)
```

b) Prove or disprove (if you can): if the Collatz conjecture fails for some integer, and if x is the smallest such integer, then x is odd.

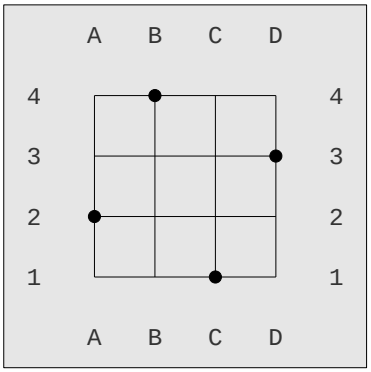
3. Explain why the runtime of `ifib(n)` is proportional to $\sum_{j=1}^{n} \lg(\ \mathrm{fib}(j)\ )$.

```
def ifib(n):
  a,b = 0,1
  for _ in range(n):
    a, b = b, a+b
  return a
```

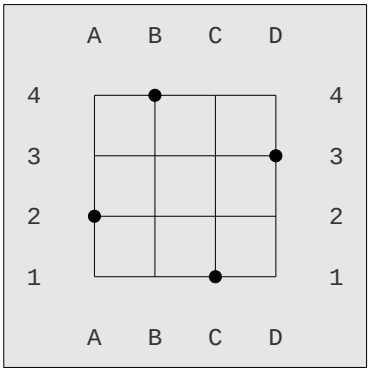
first name **last name** **id#**

each question 8 marks **30 min** **closed book** **no devices** **3 pages**

1. (a) Explain why any solution to this Steiner tree puzzle has cost at least 6.



(b) On the diagram, draw a min cost solution and write its cost.



2. a) In the box, show the output from this python program.

```
def collatz(n):
  print(n, end=': ')
  while n > 1:
    print(n, end=' ')
    if n % 2 == 0: n = n // 2
    else: n = n*3 + 1
  print(n)

for j in range(3,7): #from 3 to 6
  collatz(j)
```

b) Prove or disprove (if you can): if the Collatz conjecture fails for some integer, and if y is the smallest such integer, then y is odd.

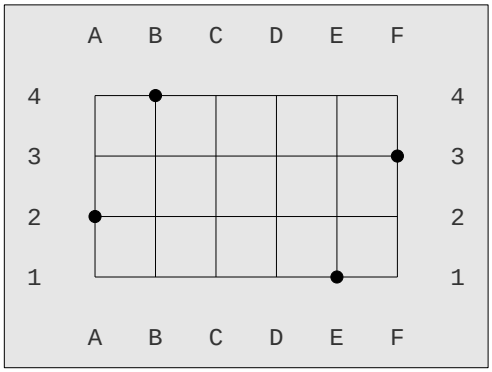
3. Explain why the runtime of `ifib(n)` is proportional to $\sum_{j=1}^{n} \lg(\ \mathrm{fib}(j)\ )$.

```
def ifib(n):
  a,b = 0,1
  for _ in range(n):
    a, b = b, a+b
  return a
```

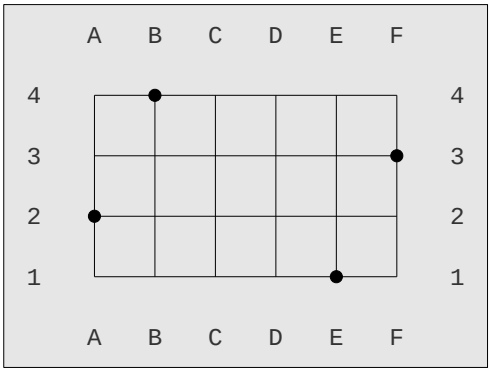
first name **last name** **id#**

each question 8 marks **30 min** **closed book** **no devices** **3 pages**

1. (a) Explain why any solution to this Steiner tree puzzle has cost at least 8.



(b) On the diagram, draw a min cost solution and write its cost.



2. a) In the box, show the output from this python program.

```
def collatz(n):
  print(n, end=': ')
  while n > 1:
    print(n, end=' ')
    if n % 2 == 0: n = n // 2
    else: n = n*3 + 1
  print(n)

for j in range(1,7): #from 1 to 6
  collatz(j)
```

b) Prove or disprove (if you can): if the Collatz conjecture fails for some integer, and if z is the smallest such integer, then z is odd.

3. Explain why the runtime of `ifib(n)` is proportional to $\sum_{j=1}^{n} \lg(\ \mathrm{fib}(j)\ )$.

```
def ifib(n):
  a,b = 0,1
  for _ in range(n):
    a, b = b, a+b
  return a
```