cmput 304 2023 study questions 9

`https://webdocs.cs.ualberta.ca/~hayward/304/asn/rectstein.pdf`

`https://webdocs.cs.ualberta.ca/~hayward/304/asn/GanleyC94.pdf`

1. Consider FDP code in the above paper. Explain why line 2 iterates $L_2 = \binom{k}{m}$ times ...
2. ... and why line 4 iterates mL_2 times ...
3. ... and why line 5 iterates $2^{m-1}L_4$ times.
4. The formula in the paper has a typo: it should be

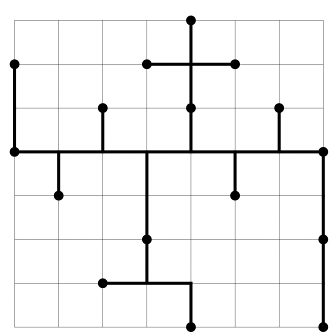
$$\sum_{m=2}^{k} \binom{k}{m} m2^{m-1} = k(3^{k-1} - 1)$$

   Prove this formula by induction on k. (Just kidding. You can skip this question.)

   Hint: start with the binomial coefficients theorem below with $x = 1$ and $y = 2$.

$$\sum_{j=0}^{n} \binom{n}{j} x^j y^{n-j} = (x + y)^n$$

5. For the first of the four RST problems on page 14 of the slides above, using A as the end of a spine (both straight spine and, if it applies, bent spine), give the r-caterpillar.
6. Repeat for B.
7. Are any of your answers to the previous two questions optimal? Justify.
8. See the questions on page 31 of the slides.
9. Below is an optimal rectilinear steiner tree with 16 terminals. Based on this, can you identify any subsets of terminals that are a) possibly full b) possibly not full c) definitely full d) definitely not full? Give at least one example. Explain



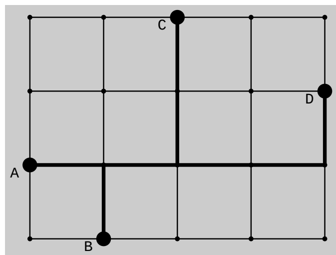
hints

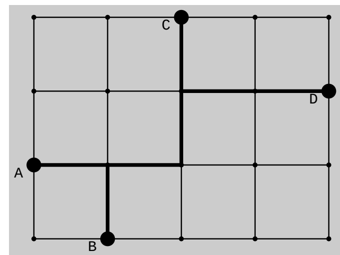
1. A set with k elements has $\binom{k}{m}$ subsets of size m.

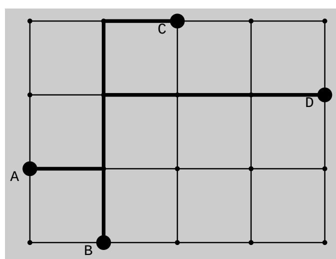
2. C has exactly m elements and so . . .

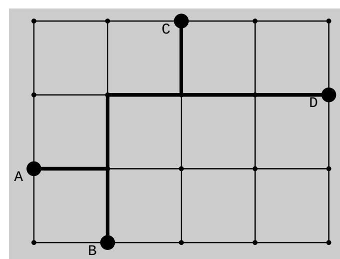
3. there are 2^{m-1} subsets of a set with $m - 1$ elements and so . . .

4. skip this question

5. 



6. 



7. In each case, the bend caterpillar has total number of edges equal to xspan + yspan, which is a lower bound on any rectilinear steiner tree. So those two trees are optimal. In each case, the straight caterpillar uses more edges, so neither of those are optimal.

8. hints are on page 32 of the slides

9. This is an optimal tree, so every subtree is an optimal subtree. In the diagram, the set of nodes from every subtree that has a cutpoint is *not* a full set (because that set has an optimal subtree in which at least one terminal is not a leaf). In the diagram, the set of nodes from every subtree with no cutpoint is *possibly* a full set. Examples: label points by (x,y) coordinates, e.g. rectangle's upper left corner is (0,7). Set of four terminals {(3,6),(4,5),(4,7),(5,6)} is full. Set of eight terminals with spine from (0,4) to (8,4) is possibly full. Set of 14 terminals that including neither (8,0) nor (8,2) is not full: cut point at (8,4) separates these two from the rest of the terminals, so the two subtrees (one with 14 nodes, one with 3 nodes) must each be optimal. But subtree with 14 nodes has a cutpoint, so its optimal and has a cutpoint, so its not full.