cmput 304 2024 study questions 4 (with hints)

1. a) Give pseudocde for the dynamic-program 0-1 knapsack algorithm. b) Give pseudocde for the dynamic-program-by-value 0-1 knapsack algorithm. Hint: see `algs/hard/knap.py`.

2. Consider the 0-1 knapsack problem. Below, show the output from the dynamic-program-by-weight (DP) algorithm. Hint: use `algs/hard/knap.py`.

```
val [6, 7,10, 7, 8]
wt  [4, 3, 6, 5, 3] W  15
```

0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	--	--	--	--	--
4	0	--	--	--	--	--
5	0	--	--	--	--	--
6	0	--	--	--	--	--
7	0	--	--	--	--	--
8	0	--	--	--	--	--
9	0	--	--	--	--	--
10	0	--	--	--	--	--
11	0	--	--	--	--	--
12	0	--	--	--	--	--
13	0	--	--	--	--	--
14	0	--	--	--	--	--
15	0	--	--	--	--	--

3. Recall: a graph $G = (V, E)$ is defined by its node set V and edge set E. Recall: graphs $G = (V, E)$ and $H = (W, F)$ are *equal* if $V = W$ and $E = F$. Recall: graphs $G = (V, E)$ and $H = (W, F)$ are *isomorphic* if there is a bijection $f : V \leftrightarrow W$ such that $\{x, y\}$ is in E if and only if, for each edge $\{x, y\}$ in E, $\{f(x), f(y)\}$ is an edge in F.

   a) Let $G = (V, E)$ and $H = (W, F)$ with $V = \{1, 2, 3\}$, $W = \{a, b, c\}$, $E = \{\{1, 3\}, \{1, 2\}\}$, $F = \{\{a, c\}, \{b, c\}\}$. Prove/disprove: $G = H$.

   b) Continue from a): prove/disprove $G \cong H$.

4. Consider the 0-1 knapsack problem. Below, show the output from the dynamic-program-by-value (DPV) algorithm. Hint: use `algs/hard/knap.py`.

```
val [3, 4, 6, 5, 5]
wt  [4, 3, 6, 5, 3]  W 18
```

0	0	0	0	0	0
1	-	-	-	-	-
2	-	-	-	-	-
3	__	__	__	__	__
4	__	__	__	__	__
5	__	__	__	__	__
6	__	__	__	__	__
7	__	__	__	__	__
8	__	__	__	__	__
9	__	__	__	__	__
10	__	__	__	__	__
11	__	__	__	__	__
12	__	__	__	__	__
13	__	__	__	__	__
14	__	__	__	__	__
15	__	__	__	__	__
16	__	__	__	__	__
17	__	__	__	__	__
18	__	__	__	__	__
19	__	__	__	__	__
20	__	__	__	__	__
21	__	__	__	__	__
22	__	__	__	__	__
23	__	__	__	__	__

5. Let $S(n)$ be $\{1, 2, \ldots, n\}$. Let $P(n)$ be the set of all subsets of $S(n)$. Let $f(n)$ be the sum, over all p in P, of the size of p. E.g. $P(3) =$

`{ {}, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3} },`

so $f(3) = 0 + 1 + 1 + 1 + 2 + 2 + 2 + 3 = 12$. Prove that $f(n) = n2^{n-1}$. Hint: covered in Thursday's lecture.

6. Consider a 0-1 knapsack input with n items, each with a weight and value in $\{1, \ldots, 2\text{n}\}$. Consider the brute force (BF) algorithm.

   a) Explain why, over the whole algorithm, the number of the additions performed to sum the weight of each subset of items, is $n2^{n-1} - 2^n$. Hint: use the previous question.

   b) Explain why the worstcase total time to perform the additions in a) is in $\Theta(n2^n \lg n)$. Hint: covered in Thursday's lecture.

7. Call a function $f(n)$ *superpolynomial* if, for any positive integer k,

   $\lim_{n\to\infty}(n^k)/f(n) = 0$. Consider a 0-1 knapsack instance with n items, each weight and value n bits: for this instance, input size $f(n)$ is in $\Theta(n^2)$ and DPW worstcase runtime $t(n)$ is in $\Theta(n^4 2^n)$ (see

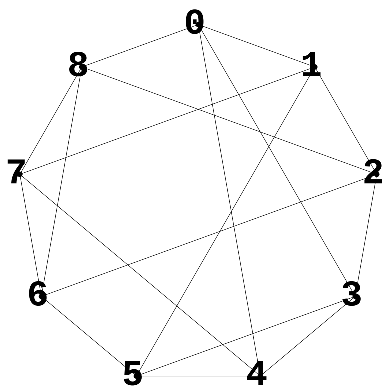
   `http://webdocs.cs.ualberta.ca/~hayward/204/jem/hard.html#knapdp`)

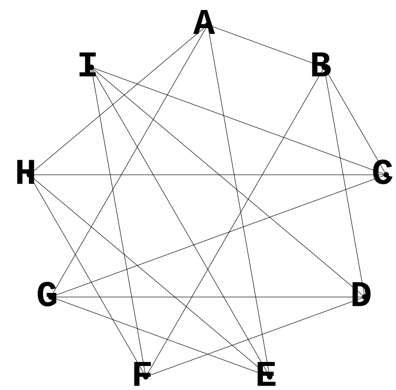
   Prove that $t(n)$ is superpolynomial. Hint: covered in Thursday's lecture.

8. Let G (left) and H (right) be the graphs below. a) Prove that this bijection $f : V \leftrightarrow W$ is not an isomorphism.

j	0	1	2	3	4	5	6	7	8
f(j)	a	b	c	d	e	f	g	h	j

b) Continue from a): prove/disprove $G \cong H$.





9. Let $f : V \to W$ be an isomorphism between graphs $G = (V, E)$ and $H = (W, F)$. (a) For each node v in G, explain why the degree of v in G equals the degree of $f(v)$ in H. (b) Explain why G and H have the same degree sequence. (c) Consider two graphs, each with degree sequence (1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4). How many possible permutations does the brute force isomorphism algorithm consider in a graph with this many nodes? Because of the degree sequence, how many permutations do you need to consider here when checking for isomorphism?

10. *Graph isomorphism* is this problem:

Input. Two graphs $G_0 = (V_0, E_0)$ and $G_1 = (V_1, E_1)$.

Query. Is there an isomorphism $f : V_0 \to V_1$?

Explain why graph isomorphism is in the class NP. Hint: covered in a lecture.

11. Find a minimum set cover of these subsets:

```
     0 1 2 3 4 5 6 7 8 910111213141516171819
 S0  - - - - - * - - * - - - - - - - - - - -
 S1  - - - - - - - - - - - - - - * - - * - -
 S2  - - - - * - - - * * - * - * * - - - - *
 S3  - - - - - - - - - - * * - * - - - - - -
 S4  - - - - - - - * * - - - - * - - - - * -
 S5  - - * - - - - - - - * - - - - - - - - -
 S6  - * * - - - * - * - - * - - - * * - * -
 S7  - - - - - - - - - * - - - * * - - - * -
 S8  - - - * - - - - - - * - - - - * - * - -
 S9  * - - - - - - - - - - - * - - - - - - -
S10  - - - - - - * - - - - * - - * - - - - -
```

12. a) Using the format of question 10, give a careful definition of decision set cover, the decision version of the set cover problem. Hint: you need to add an integer k to the input and then ask the appropriate yes/no question.

    b) Explain why decision set cover is in the class NP. Hint: covered in a lecture.

13. a) On this 4-pin graph, demonstrate the MST-approx algorithm to find a Steiner tree. b) What is the cost of the steiner tree you found? c) Find a min cost Steiner tree. Hint: covered in Thursday's lecture.