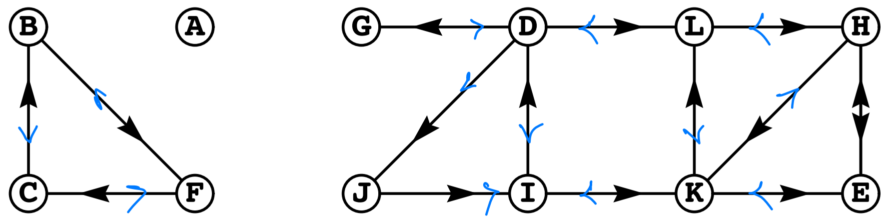
first name **last name** **id#**

each page 8 marks **30 min** **closed book** **no devices** **3 pages**

1. Give the postorder traversal sequence of the transpose of this digraph, assuming that its node list and neighbor lists are stored in alphabetic order.

   Write your answer here: Answer:
   AFCBJIDKLHEG



2. Using your answer to the previous question, list the strongly connected components of the digraph in the order that they are found by the algorithm from class.

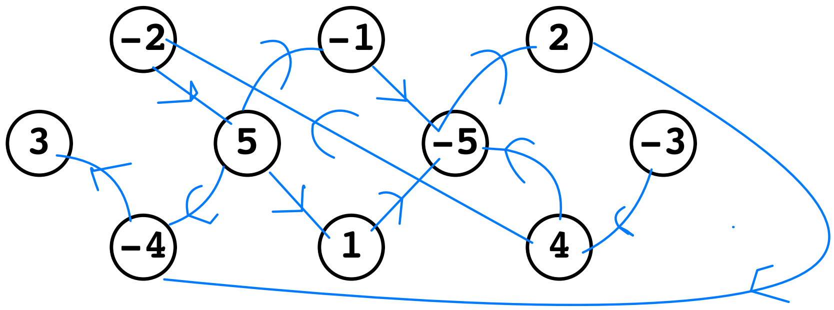
   Write your answer here: Answer: there are 5 connected component as listed below and they are found by algorithm in the order from **right** to **left**:
   A FCB IDJ KLEH G

3. Let D be a digraph with nodes x, y such that there is a directed path p_{xy} from x to y and a directed path p_{yx} from y to x. Prove/disprove: in the transpose D^T of D there is a directed path q_{xy} from x to y and a directed path q_{yx} from y to x.

   Answer:
   Prove: if there exists the path p from x to y in digraph D, then it is easy to show that the path p^T, path obtained by reversing the direction of edges in path p, exists in digraph D^T and is from the node y to x. This can be done by induction on the length of the path p.

4. (a) On the nodes below, draw the implication digraph for this 2-sat formula:

$f =$ `[1 -5] [-2 -4] [3 4] [-4 -5] [2 5] [-1 -5]` .



(b) Is the formula satisfiable? If yes, give a satisfying assignment; if no, explain.

Variable x_5 needs to be false because of the first and last clauses.

Based on clause number 5 from left, x_2 needs to be true.

Based on clause number 2 from left, x_4 needs to be False.

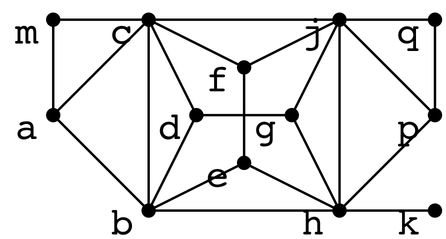
Based on clause number 3 from left, x_3 needs to be True.

Variable x_1 can have any value.

5. A node is *simplicial* in a graph if its set of neighbors is a clique.

(a) List all the simplicial nodes in this graph.

Write your answer here:



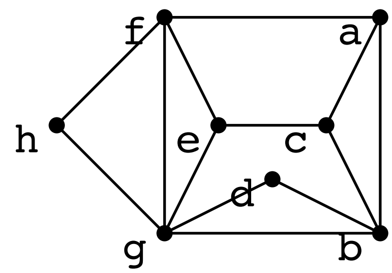
Answer: m,k,q

(b) In a graph G, let v be a simplicial node, let M be the non-neighbors of v, and let I_M be an independent set in $G[M]$. Prove that $\{v\} \cup I_M$ is an independent set in G.

Proof by contradiction:

Let the desired set don't be an independent set. Since I_M is already an independent set, then there should be an edge from the node v to one of the nodes in I_M, as a result on of the neighbours of node v should be in the independent set I_M, but this contradicts the fact that I_M in an independent set in graph G[M], where none of the neighbours of node v are included.

This is the graph G:



6. (a) Formulate the problem of finding a largest independent set in G as an integer program. Explain briefly.

$$\max x_a + x_b + x_c + x_d + x_e + x_f + x_g + x_h$$
$$x_a + x_b \leq 1$$
$$x_a + x_c \leq 1$$
$$x_a + x_e \leq 1$$
$$x_a + x_f \leq 1$$
$$x_b + x_c \leq 1$$
$$x_b + x_d \leq 1$$
$$x_b + x_g \leq 1$$
$$x_c + x_e \leq 1$$
$$x_d + x_g \leq 1$$
$$x_e + x_f \leq 1$$
$$x_e + x_g \leq 1$$
$$x_f + x_g \leq 1$$
$$x_f + x_h \leq 1$$
$$x_g + x_h \leq 1$$
$$x_a, x_b, x_c, x_d, x_e, x_f, x_g, x_h \in \{0, 1\}$$

$$\max x_a + x_b + x_c + x_d + x_e + x_f + x_g + x_h$$
$$x_a + x_b + x_c \leq 1$$
$$x_b + x_d + x_g \leq 1$$
$$x_e + x_f + x_g \leq 1$$
$$x_f + x_g + x_h \leq 1$$
$$x_a + x_f \leq 1$$
$$x_e + x_c \leq 1$$
$$x_a, x_b, x_c, x_d, x_e, x_f, x_g, x_h \in \{0, 1\}$$

Both integer programs formulate the maximum independent set problem.
In both integer linear programs above, we correspond to each node a decision variable. Each decision variable can have either value 0 or 1 which correspond to the decision that whether they are included in the independent set or not.
In the left integer program, to each edge a constraint is corresponded such that at most one of the end points corresponding to that edge be selected.
In the right integer program, to each maximal clique a constraint is corresponded such that at most one of the nodes in that clique be selected.

(b) If you solve your answer to (a) using the mixed-integer-program solver at sagemath, will the solution be all integer, or might there be some non-integer (fractional) values in the solution? Explain.

Answer: There might be some fractional values in the solution.