① input: $G=(V,E)$, $K \subseteq V$

query: is K a clique of G?

① In the set K, and for each node v

check if v is connected to $K-\{v\}$.

it takes $c^{\ 2})$

② Consider the induced subgraph $G[K]$.

If the degree of each node is $|K|-1$ while

$G[K]$ is simple (no loop and no parallel) the K is clique.

② Instance: $G = (V, E)$, integer k

Query: does G have a clique of size k?

to prove this is in fact in NP:

① is this a decision problem? yes. as the answer to the query is yes/no.

② Given a proof to yes instance can we verify it in poly-time?

yes! the proof would be a set S. the we check:

① if $|S| = k$

② using NSIC if S is a clique in polytime.

1. Let t be the transformation we saw in the lectures from cnf-sat to 3-sat. For the cnf-sat formula f represented below, give the corresponding 3-sat formula $t(f)$. In this question, write boolean clauses like this `[-1 2 4]` instead of like this $(\neg x_1 \vee x_2 \vee x_4)$.

clauses of f	corresponding clauses of t(f)
[2 3 -5 -6]	[2,3, 10] , [-10, -5, -6]
[2 3 -4 -5 -6]	[2,3, 11] , [-11, -4, 12] , [-12, -5, -6]
[1 -2 3 -4 5 6]	[1, -2, 13] , [-13, 3, 14] , [-14, -4, 15] , [-15, 5, 6]

ROUGH WORK BELOW THIS LINE

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2. Does there exist a polytime answer-preserving transformation from k-independent set to sat (satisfiability)? Write your answer here (yes/no/not known):

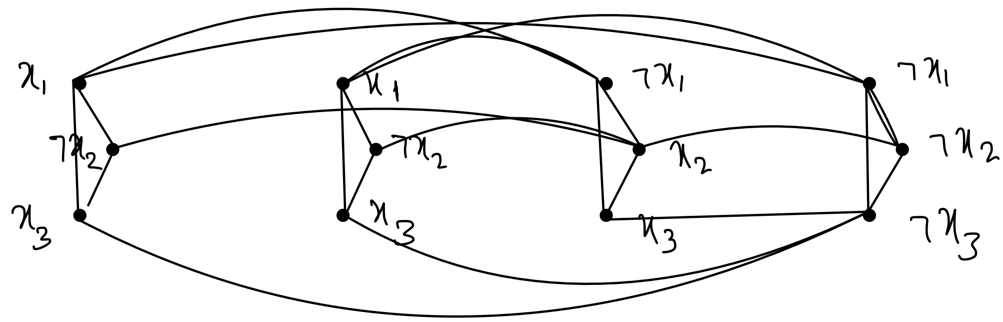
Justify your answer here:

Yes. We already know that sat is an NP-Complete problem.
A problem X ∈ NP is complete if for each problem A ∈ NP, there is a polytime answer-preserving transformation from A to X. As k-independent set is in NP (it is a decision problem and a k-independent set can be verified in polytime), then there is a transformation from k-independent set to sat.

5. Recall from the lectures the polytime answer-preserving transformation $t(\,)$ that maps any 3-sat instance z with k clauses to a k-independent set instance $t(z)$ where $t(z)$ is a graph.

a) On the nodes below, draw $t(z)$. Label each node with its corresponding literal.

$$z = (x_1 \vee \neg x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_3)$$



b) Assume that $t(z)$ has an independent set I of size k. For each statement below, write T (true) or F (false) and justify your answer.

i) If I contains a node labelled $\neg x_j$, then I does not contain a node labelled x_j.

T/F? T Reason?

A node labelled with $\neg x_j$ is always connected to a node labeled with x_j. So these two nodes can't be included in an independent set.

ii) I includes exactly one node from the first clause of z.

T/F? T Reason?

Since every two nodes of one clause are connected and the set **I** is an independent set, then **I** can contain at **most** one node from each of the clauses.
On the other hand, since there are **k** clauses and **I** has a size of **k**, then it should contain ***exactly*** one node from each of the clauses.

iii) Let A be the truth assignment that sets each literal that is a node of I to `true`. A satisfies z.

T/F? T Reason?

Based on the claim in part **(i)**, we can argue that **I** includes one node from each of the clauses, so the assignment **A** will assign truth to one literal from each of the clauses and all the clauses will be satisfied.
On the other hand, following from claim **(ii)**, no two nodes that correspond to negating literals can be in **I**. So, assignment **A** can't have a contradiction.